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# Analytical and numerical investigations of relativistic particle acceleration in a current layer 

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#### Abstract

We investigate the motion of a charged particle in the process of acceleration up to relativistic energies in a superhot turbulent current layer with three components of the magnetic field. We solve analytically the relativistic equation of motion that has been averaged over the particle gyration in the magnetic field. The obtained results are compared with numerical solution of the ordinary (without averaging) equation of motion. The analytical solution describes a stable motion, i.e. when a particle remains in the reconnecting current layer until it reaches its edges. The stability conditions are found imposed on the electric and magnetic fields in the layer. Particles with positive and negative charges have different behaviours. The numerical solution has confirmed the conclusions of the analytical approach. Applications to solar flares are discussed.


Keywords: Acceleration of particles; Solar flares; Solar magnetic fields

## 1. Introduction

Acceleration of charged particles up to relativistic energies is one of the most interesting problems of modern astrophysics (see, for example, [1-3]). According to solar observations, accelerated electrons with energies $20 \mathrm{keV}-1 \mathrm{GeV}$, which produce hard X -rays and $\gamma$-rays via bremsstrahlung, may contain as much as $10 \%$ of the flare energy, i.e. about $10^{31} \mathrm{erg}$. The energy content of protons with energies from 1 MeV to several gigaelectronvolts can exceed $10^{30} \mathrm{erg}$; the energization rate is about $10^{34}-10^{35} \mathrm{~s}^{-1}$ [4-6].

At present, the almost conventional viewpoint is that the energy release in solar flares is due to magnetic reconnection in reconnecting current layers (CLs). Here, the magnetic field energy is converted into the thermal and kinetic energy of the plasma and accelerated particles (see, for example, [7, 8]). The CL formation is confirmed by the theory of the superhot turbulent CL $[8,9]$, as well as by laboratory experiments (see, for example, $[10,11]$ ) and by recent Reuven Ramaty High Energy Solar Spectroscopic Imager observations of solar flares [12, 13].

Inside the CL, the inductive electric field is directed along the current; this strong field exerts positive work on charged particles, thus increasing their energy.

[^0]The problem of particle motion in a CL has been considered many times. In a neutral CL, characterized by a single magnetic-field component $B_{0}$, a charged particle can spend an infinite time and can take an infinite energy from the electric field [14]. However, under real conditions, the magnetic field also has a non-zero transverse (to the CL plane) component $B_{\perp}$ and longitudinal (parallel to the electric field) component $B_{\|}$. Speiser [14] showed that even a small transverse field changes the particle motion in such a way that the particle leaves the CL after a finite time. This time is short and the energy is not sufficient in the context of solar flares.

Litvinenko and Somov [15] found that the high longitudinal magnetic field increases the acceleration time and, in this way, increases the efficiency of electron acceleration, thus allowing one to explain the first step of electron acceleration in flares. The physical meaning is that $B_{\|}$tends to keep particles 'frozen' and to confine them inside the CL.

Efthymiopoulos et al. [16] also considered the same problem. They argued that the condition given by Litvinenko and Somov [15] is not sufficient to ensure stability of the orbits of all accelerated electrons. Zones of instability exist for arbitrary high values of the longitudinal magnetic field $B_{\|}$. However, the width of these zones decreases as the value of the longitudinal field increases. For super-Dreicer electric fields, which are typical for solar flares [8], these zones are very narrow so that the criterion proposed by Litvinenko and Somov is an acceptable approximation.

Another role of the longitudinal field $B_{\|}$was demonstrated by Zhu and Parks [17] for a magnetotail-like CL. These authors showed, in particular, that, when the ratio $B_{\|} / B_{\perp}$ is greater than a certain value, the ejection direction of particles becomes sign dependent. A similar effect in a solar CL was obtained in numerical computations by Zharkova and Gordovskyy [18]; for the ratio $B_{\|} / B_{0}>10^{-2}$, there is separation of particles with opposite charges into the opposite halves from the CL midplane.
Wood and Neukirch [19] presented the results of charged-particle orbit calculations in a CL. They showed that an initially Maxwellian distribution function in the inflow region can develop a beam-like component in the outflow region. A partial separation of accelerated electrons and ions is also obtained.

Note that all the above-described studies consider non-relativistic particle motion in a CL. The aim of our study is to investigate the trajectories of charged particles accelerating up to relativistic energies in a superhot turbulent CL with three components of the magnetic field [8].

## 2. Mathematical description of the model

Let us consider the relativistic motion of a particle with mass $m$ and charge $q$ in a specified electric field $\boldsymbol{E}(\boldsymbol{r})$ and a specified magnetic field $\boldsymbol{B}(\boldsymbol{r})$. The equation of motion is

$$
\begin{equation*}
\frac{\mathrm{d} \boldsymbol{p}}{\mathrm{~d} t}=q\left(\boldsymbol{E}+\frac{1}{c}(\boldsymbol{v} \times \boldsymbol{B})\right), \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
\boldsymbol{p}=\frac{m \boldsymbol{v}}{\left(1-v^{2} / c^{2}\right)^{1 / 2}} \tag{2}
\end{equation*}
$$

is the momentum of the particle, $\boldsymbol{v}(t)$ is its velocity and $c$ is the speed of light. It is required to determine the particle trajectory $\boldsymbol{r}(t)$.

Equation (1) can be rewritten for dimensionless variables and functions, indicated by asterisks:

$$
\begin{equation*}
t^{*}=\frac{t}{t_{0}}, \quad r^{*}=\frac{r}{r_{0}}, \quad v^{*}=\frac{v}{c}, \quad B^{*}=\frac{B}{B_{0}}, \quad E^{*}=\frac{E}{B_{0}} . \tag{3}
\end{equation*}
$$

Here, the timescale is

$$
\begin{equation*}
t_{0}=\omega_{0}^{-1} \tag{4}
\end{equation*}
$$

where $\omega_{0}=|q| B_{0} / m c$ is the gyrofrequency of a non-relativistic particle in the characteristic magnetic field $B_{0}$; the length scale is

$$
\begin{equation*}
r_{0}=c t_{0}=\frac{m c^{2}}{|q| B_{0}} . \tag{5}
\end{equation*}
$$

Thus, equation (1) becomes

$$
\begin{equation*}
\frac{\mathrm{d} \boldsymbol{p}^{*}}{\mathrm{~d} t^{*}}=(\operatorname{sgn} q)\left[\boldsymbol{E}^{*}+\left(\boldsymbol{v}^{*} \times \boldsymbol{B}^{*}\right)\right], \tag{6}
\end{equation*}
$$

where $\operatorname{sgn} q=q /|q|$ and $p^{*}=v^{*} /\left(1-v^{* 2}\right)^{1 / 2}$.
Let us choose the coordinate system so that the plane $(x, z)$ coincides with the CL plane, the $z$ axis is parallel to the electric field, and the main component of the magnetic field is coaligned with the $x$ axis (figure 1). The electric and magnetic fields inside the layer are approximated by the following expressions:

$$
\begin{equation*}
\boldsymbol{E}=(0,0, E), \quad \boldsymbol{B}=B_{0}\left(-\frac{y}{a},-\xi_{\perp} \operatorname{sgn} x, \xi_{\|}\right) . \tag{7}
\end{equation*}
$$

Here, $a$ is the half-thickness of the $\mathrm{CL}, \xi_{\perp}$ and $\xi_{\|}$are the dimensionless constants defining the transverse magnetic field component $B_{y}$ and longitudinal magnetic field component $B_{z}$, respectively. $\xi_{\perp}>0$. Hence

$$
\begin{equation*}
\boldsymbol{E}^{*}=(0,0, \varepsilon), \quad \boldsymbol{B}^{*}=\left(-y^{*} \delta,-\xi_{\perp} \operatorname{sgn} x^{*}, \xi_{\|}\right), \tag{8}
\end{equation*}
$$

where

$$
\begin{equation*}
\varepsilon=\frac{E}{B_{0}}, \quad \delta=\frac{r_{0}}{a}=\frac{m c^{2}}{|q| B_{0} a}, \tag{9}
\end{equation*}
$$

are dimensionless parameters of the problem.


Figure 1. Reconnecting CL and coordinate system.

The dimensionless equation (6) takes the coordinate form (hereafter asterisks indicating dimensionless values will be omitted)

$$
\begin{align*}
\dot{p}_{x} & =(\operatorname{sgn} q)\left(\dot{y} \xi_{\|}+\dot{z} \xi_{\perp} \operatorname{sgn} x\right)  \tag{10}\\
\dot{p}_{y} & =(\operatorname{sgn} q)\left(-\dot{z} y \delta-\dot{x} \xi_{\|}\right)  \tag{11}\\
\dot{p}_{z} & =(\operatorname{sgn} q)\left(\varepsilon-\dot{x} \xi_{\perp} \operatorname{sgn} x+\dot{y} y \delta\right) . \tag{12}
\end{align*}
$$

These equations determine the particle trajectory in the CL for the specified input parameters $\varepsilon, \delta, \xi_{\perp}$ and $\xi_{\|}$, and initial conditions $x(0), p_{x}(0), y(0), p_{y}(0), z(0)$ and $p_{z}(0)$.

## 3. Analytical approach: searching for a stable solution

Let us search for a solution describing a stable motion of particles in the CL, i.e. the case when a particle leaves the layer only as a result of either the limited length $l$ along the $z$ axis or limited width $2 b$ along the $x$ axis.
In general, a charged particle in a magnetic field follows a spiral trajectory, which can be presented as a combination of forward and gyratory motions: $\boldsymbol{R}(t)=\boldsymbol{r}(t)+\boldsymbol{r}_{\mathrm{L}}(t)$, where $\boldsymbol{r}_{\mathrm{L}}(t)$ is a periodic function of time. Below we shall consider this forward motion, i.e. the motion averaged over the particle gyration in magnetic field: $\langle\boldsymbol{R}(t)\rangle=\boldsymbol{r}(t)$ and $\left\langle\boldsymbol{r}_{\mathrm{L}}(t)\right\rangle=0$.

### 3.1 First iteration

The particle does not leave the CL through its surface $y= \pm a$ (in dimensional units) if it gyrates in the vicinity of the plane $y_{0}=$ constant $<a$ inside the layer. Moreover, its mean velocity and force balance in the $y$ direction, i.e. perpendicular to the layer plane, must be zero:

$$
\begin{gather*}
\langle y\rangle=\text { constant }=y_{0},  \tag{13}\\
\left\langle p_{y}\right\rangle=0, \quad\left\langle v_{y}\right\rangle=0, \quad\left\langle\dot{p}_{y}\right\rangle=0 .
\end{gather*}
$$

Therefore, equation (11) becomes (on omitting the angular brackets)

$$
\begin{equation*}
0=(\operatorname{sgn} q)\left(-\dot{z} y_{0} \delta-\dot{x} \xi_{\|}\right) \tag{14}
\end{equation*}
$$

Hence,

$$
\begin{align*}
& \dot{z}=-\dot{x} \frac{\xi_{\|}}{y_{0} \delta},  \tag{15}\\
& z=-x \frac{\xi_{\|}}{y_{0} \delta}+c_{y} . \tag{16}
\end{align*}
$$

So, the requirement of zero force in the $y$ direction leads to the relation between the velocities along the $x$ and $z$ axes given by equation (15). The proportionality coefficient $-\xi_{\|} / y_{0} \delta$ eqauls the ratio of the magnetic field components, $B_{z} / B_{x}$, where $B_{x}$ depends on the coordinate $y_{0}$.

Taking into account equations (15) and (16), we find expressions for $\dot{x}, x, \dot{z}$ and $z$ from the equations of motion (10) and (12):

$$
\begin{align*}
\dot{x} & =\frac{\varepsilon}{\xi_{\perp}} \frac{\operatorname{sgn} x}{1+\left(\xi_{\|} / y_{0} \delta\right)^{2}},  \tag{17}\\
x & =\frac{\varepsilon}{\xi_{\perp}} \frac{\operatorname{sgn} x}{1+\left(\xi_{\|} / y_{0} \delta\right)^{2}} t+c_{1}, \quad \text { where } c_{1}=\text { constant }=x(0),  \tag{18}\\
\dot{z} & =-\dot{x} \frac{\xi_{\|}}{y_{0} \delta}=-\frac{\varepsilon}{\xi_{\perp}} \frac{\operatorname{sgn} x}{1+\left(\xi_{\|} / y_{0} \delta\right)^{2}} \frac{\xi_{\|}}{y_{0} \delta},  \tag{19}\\
z & =-\frac{\varepsilon}{\xi_{\perp}} \frac{\operatorname{sgn} x}{1+\left(\xi_{\|} / y_{0} \delta\right)^{2}} \frac{\xi_{\|}}{y_{0} \delta} t+c_{2}, \text { where } c_{2}=\text { constant }=z(0) .  \tag{20}\\
v^{2} & =\dot{x}^{2}+\dot{y}^{2}+\dot{z}^{2} \approx \dot{x}^{2}+\dot{z}^{2}=\left(\frac{\varepsilon}{\xi_{\perp}}\right)^{2} \frac{1}{1+\left(\xi_{\|} / y_{0} \delta\right)^{2}} . \tag{21}
\end{align*}
$$

Thus, in the first iteration, the velocities $\dot{x}$ and $\dot{z}$ do not depend on time. Note that $y_{0}$ in the equalities (17)-(21) remains unknown. It will be defined in the next iteration.

### 3.2 Second iteration

Generally speaking, the velocities $\dot{x}$ and $\dot{z}$ can be non-constants but with time they approach these constants. Let us solve the equations of motion (10) and (12) in the second iteration; we substitute the solution of the first iteration (17)-(20) in the right-hand sides of equations (10) and (12) and find the new values of $\dot{x}, x, \dot{z}$ and $z$.

Since we consider the relativistic case $v \approx 1$, equation (21) can be rewritten as

$$
\begin{equation*}
v^{2}=\left(\frac{\varepsilon}{\xi_{\perp}}\right)^{2} \frac{1}{1+\left(\xi_{\|} / y_{0} \delta\right)^{2}} \approx 1 \tag{22}
\end{equation*}
$$

Let, in the denominator of this formula, the ratio

$$
\begin{equation*}
\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{2} \gg 1 \tag{23}
\end{equation*}
$$

Then, from equation (22), we obtain

$$
\begin{align*}
& v^{2}=\left(\frac{\varepsilon}{\xi_{\perp}}\right)^{2}\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{-2} \approx 1,  \tag{24}\\
&\left(\frac{\varepsilon}{\xi_{\perp}}\right)^{2} \approx\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{2} \gg 1 . \tag{25}
\end{align*}
$$

Thus, condition (23) corresponds to the case of a strong electric field:

$$
\begin{equation*}
\varepsilon^{2} \gg \xi_{\perp}^{2} . \tag{26}
\end{equation*}
$$

The solution of the first iteration (17)-(20) at $\left(\xi_{\|} / y_{0} \delta\right)^{2} \gg 1$ takes the simpler form

$$
\begin{align*}
\dot{x} & =\frac{\varepsilon}{\xi_{\perp}}\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{-2} \operatorname{sgn} x,  \tag{27}\\
x & =\frac{\varepsilon}{\xi_{\perp}}\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{-2}(\operatorname{sgn} x) t+c_{1}, \quad \text { where } c_{1}=\text { constant }=x(0),  \tag{28}\\
\dot{z} & =-\frac{\varepsilon}{\xi_{\perp}}\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{-1} \operatorname{sgn} x,  \tag{29}\\
z & =-\frac{\varepsilon}{\xi_{\perp}}\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{-1}(\operatorname{sgn} x) t+c_{2}, \quad \text { where } c_{2}=\text { constant }=z(0) . \tag{30}
\end{align*}
$$

With allowance for equation (24), we note that

$$
\begin{equation*}
|\dot{x}|=|v|\left|\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{-1}\right| \ll 1, \quad|\dot{z}| \approx|v| \approx 1 \tag{31}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
v^{2}=\dot{x}^{2}+\dot{y}^{2}+\dot{z}^{2} \approx \dot{z}^{2} . \tag{32}
\end{equation*}
$$

This means that the particle is predominantly accelerated in the electric field direction along the CL.

Let us rewrite the equation of motion (12), integrating it over time and then substituting in the right-hand side the function $x(t)$ obtained in the first iteration (see equation (28)):

$$
\begin{equation*}
p_{z}=(\operatorname{sgn} q)\left[\varepsilon t-\varepsilon\left(\frac{\xi_{\|}}{y_{0} \delta}\right)^{-2} t-c_{1} \xi_{\perp} \operatorname{sgn} x-\frac{1}{2}\left\langle y^{2}\right\rangle \delta\right]+c_{z} . \tag{33}
\end{equation*}
$$

In the course of time, the terms containing $t$ will exceed the constant terms on the right-hand side. Moreover, under the condition (23), the first term in the square brackets is much larger than the second term. Therefore, the equation of motion (33) takes the form

$$
\begin{equation*}
p_{z}=(\operatorname{sgn} q) \varepsilon t . \tag{34}
\end{equation*}
$$

As $p_{z}=\dot{z} /\left(1-v^{2}\right)^{1 / 2} \approx \dot{z} /\left(1-\dot{z}^{2}\right)^{1 / 2}$ (see equation (32)), equation (34) can be rewritten in the following manner:

$$
\begin{equation*}
\frac{\dot{z}}{\left(1-\dot{z}^{2}\right)^{1 / 2}}=(\operatorname{sgn} q) \varepsilon t \tag{35}
\end{equation*}
$$

From this, we obtain the expressions for $\dot{z}(t)$ and $z(t)$ that are the result of the second iteration:

$$
\begin{align*}
& \dot{z}=\frac{\varepsilon t}{\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2}} \operatorname{sgn} q \longrightarrow 1 \operatorname{sgn} q \text { for } t \longrightarrow \infty  \tag{36}\\
& z=\frac{\operatorname{sgn} q}{\varepsilon}\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2}+c_{2} \longrightarrow t \operatorname{sgn} q \text { for } t \longrightarrow \infty \tag{37}
\end{align*}
$$

Thus, the velocity in the $z$ direction is close to the speed of light: $|\dot{z}| \rightarrow 1$. Particles with different signs of charges move in opposite directions along the $z$ axis.

Let us now determine the function $x(t)$. For this, let us integrate the equation of motion (10) over time and then substitute the obtained function $z(t)$ :

$$
\begin{equation*}
p_{x}=(\operatorname{sgn} q)\left[y_{0} \xi_{\|}+\left(\frac{\operatorname{sgn} q}{\varepsilon}\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2}+c_{2}\right) \xi_{\perp} \operatorname{sgn} x\right]+c_{x} . \tag{38}
\end{equation*}
$$

In the course of time, we shall have only the term with $t$ on the right-hand side:

$$
\begin{equation*}
p_{x}=\frac{\xi_{\perp}}{\varepsilon}\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2} \operatorname{sgn} x . \tag{39}
\end{equation*}
$$

Taking into account equations (32) and (36), we note that

$$
\begin{equation*}
p_{x}=\frac{\dot{x}}{\left(1-v^{2}\right)^{1 / 2}} \approx \frac{\dot{x}}{\left(1-\dot{z}^{2}\right)^{1 / 2}}=\dot{x}\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2} \tag{40}
\end{equation*}
$$

Hence, the equation of motion (39) takes the following form:

$$
\begin{equation*}
\dot{x}\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2}=\frac{\xi_{\perp}}{\varepsilon}\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2} \operatorname{sgn} x . \tag{41}
\end{equation*}
$$

Its solution (the result of the second iteration) is of the form

$$
\begin{align*}
\dot{x} & =\frac{\xi_{\perp}}{\varepsilon} \operatorname{sgn} x,  \tag{42}\\
x & =\frac{\xi_{\perp}}{\varepsilon}(\operatorname{sgn} x) t+c_{1} . \tag{43}
\end{align*}
$$

So, the velocity in the $x$ direction is low: $|\dot{x}| \ll 1$. The motion does not depend on the sign of the charge; particles move from the centre of the layer to its edges.

Now we find the plane $y=y_{0}$, in the vicinity of which the particle executes gyrations. From the force balance in the $y$ direction (equation (14)) with allowance for the obtained functions $\dot{x}(t)$ (equation (42)) and $\dot{z}(t)$ (equation (36)) it follows that

$$
\begin{equation*}
y_{0}=-\frac{\dot{x}}{\dot{z}} \frac{\xi_{\|}}{\delta}=-\frac{\xi_{\perp} \xi_{\|}}{\varepsilon \delta} \frac{\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2}}{\varepsilon t}(\operatorname{sgn} x)(\operatorname{sgn} q) \longrightarrow-\frac{\xi_{\perp} \xi_{\|}}{\varepsilon \delta}(\operatorname{sgn} x)(\operatorname{sgn} q) \tag{44}
\end{equation*}
$$

A particle remains in the layer if the condition $\left|y_{0} \delta\right| \leqslant 1$ is fulfilled. It corresponds to the inequality

$$
\begin{equation*}
\varepsilon^{2} \geqslant\left(\xi_{\perp} \xi_{\|}\right)^{2} \tag{45}
\end{equation*}
$$

The relativistic factor $\gamma$ can be estimated using equality (32) and the obtained function $\dot{z}(t)$ (equation (36)):

$$
\begin{equation*}
\gamma=\frac{1}{\left(1-v^{2}\right)^{1 / 2}} \approx \frac{1}{\left(1-\dot{z}^{2}\right)^{1 / 2}}=\left(1+\varepsilon^{2} t^{2}\right)^{1 / 2} \longrightarrow \varepsilon t \tag{46}
\end{equation*}
$$

Note that the solution of the first iteration (17)-(20) for $y_{0}$ determined by the equality (44) is identical with those of the second iteration for $t \rightarrow \infty$, i.e. this fact is considered as evidence of stability of the solution.

Considering other variants (relativistic case for $\left(\xi_{\|} / y_{0} \delta\right)^{2} \ll 1$ and also the non-relativistic case [15]) in the same manner, we conclude that the results of the first and second iterations do not coincide for any input parameters. So, stable solutions under these conditions are not found. Thus, a stable trajectory of a particle in the CL exists in the strong electric field under the conditions

$$
\begin{equation*}
\varepsilon^{2} \gg \xi_{\perp}^{2} \quad \text { and } \quad \varepsilon^{2} \geqslant\left(\xi_{\perp} \xi_{\|}\right)^{2} \tag{47}
\end{equation*}
$$

The motion along the $x$ axis is described by equations (42) and (43), and along the $z$ axis by equations (36) and (37); the particle gyrates near the plane $y=y_{0}$ (equation (44)) inside the layer.

## 4. Numerical solution of the ordinary motion equation

The task of this section is to test the results of the analytical approach by an independent method. We have solved numerically the ordinary (without averaging over the gyration) equations of motion (10)-(12).

The computations were performed for an electron and proton (table 1). We have obtained two trajectories for each particle: the first is for the weak electric field $E=3 \mathrm{~V} \mathrm{~cm}^{-1}$, which does not satisfy the stability conditions (47); the second is for the strong electric field $E=$ $30 \mathrm{~V} \mathrm{~cm}^{-1}$, for which the stability conditions are fulfilled.

At the moment $t=0$, particles are on the layer surface: $y=a / r_{0}$ (in dimensionless units). The initial velocity $v(0)=0$, which corresponds to the acceleration of initially thermal particles.

Figures 2-5 present the numerical results for an electron, and figures 6-9 those for a proton. The grey curves correspond to the weak electric field, and the black curves to the strong field. The time dependences are shown of the particle coordinates $x(t), y(t)$ and $z(t)$, the velocity components $v_{x}(t), v_{y}(t)$ and $v_{z}(t)$, the total velocity $v(t)=\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}\right)^{1 / 2}$ and the relativistic factor $\gamma=1 /\left(1-v^{2}\right)^{1 / 2}$. Note that all values are in dimensionless units. To estimate the corresponding dimensional value, we have to multiply the time by $t_{0}$, the coordinates by $r_{0}$, and the velocity by the speed of light.

The numerical results confirm the analytical conditions of stability (47). Let us look at the curves $y(t)$ in figure 3. At the initial time the electron is on the layer surface: $y(0)=1.87$. Then

Table 1. Values of the parameters for the numerical computations.

|  | Value for the following variants of the computations |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Electron |  | Proton |  |
|  | First trajectory | Second trajectory | First trajectory | Second trajectory |
| Constants |  |  |  |  |
| $q$ (CGSE units) | $-4.8 \times 10^{10}$ | $-4.8 \times 10^{10}$ | $4.8 \times 10^{10}$ | $4.8 \times 10^{10}$ |
| Input parameters |  |  |  |  |
| $E$ (CGSE units) [20] | $10^{-2}$ | $10^{-1}$ | $10^{-2}$ | $10^{-1}$ |
| $B_{0}(\mathrm{G})$ [20] | 100 | 100 | 100 | 100 |
| $\xi_{\perp}=B_{z} / B_{0}$ | $5 \times 10^{-4}$ | $5 \times 10^{-4}$ | $5 \times 10^{-4}$ | $5 \times 10^{-4}$ |
| $\xi_{\\|}=B_{y} / B_{0}$ | 0.1 | 0.1 | 0.1 | 0.1 |
| $a(\mathrm{~cm})$ [21] | 32 | 32 | 32 | 32 |
| Characteristic values |  |  |  |  |
| $\omega_{0}=\|q\| B_{0} / m c\left(\mathrm{~s}^{-1}\right)$ | $1.8 \times 10^{9}$ | $1.8 \times 10^{9}$ | $9.6 \times 10^{5}$ | $9.6 \times 10^{5}$ |
| $t_{0}=\omega_{0}^{-1}(\mathrm{~s})$ | $5.7 \times 10^{-10}$ | $5.7 \times 10^{-10}$ | $1.0 \times 10^{-6}$ | $1.0 \times 10^{-6}$ |
| $r_{0}=c t_{0}(\mathrm{~cm})$ | 17 | 17 | $3.1 \times 10^{4}$ | $3.1 \times 10^{4}$ |
| Initial conditions |  |  |  |  |
| $x(0)$ | 100 | 100 | 100 | 100 |
| $y(0)=a / r_{0}$ | 1.9 | 1.9 | $10^{-3}$ | $10^{-3}$ |
| $z(0)$ | 0 | 0 | 0 | 0 |
| $v_{x}(0), v_{y}(0), v_{z}(0)$ | 0 | 0 | 0 | 0 |
| Dimensionless parameters |  |  |  |  |
| $\delta=r_{0} / a$ | 0.53 | 0.53 | 980 | 980 |
| $\varepsilon=E / B_{0}$ | $10^{-4}$ | $10^{-3}$ | $10^{-4}$ | $10^{-3}$ |
| Stability conditions | Not fulfilled | Fulfilled | Not fulfilled | Fulfilled |



Figure 2. Electron coordinate $x(t)$ and velocity $v_{x}(t)$ obtained from the numerical solution of the ordinary (without averaging) equation of motion: grey curves, unstable trajectory; black curves, stable trajectory.


Figure 3. Electron coordinate $y(t)$ and velocity $v_{y}(t)$.


Figure 4. Electron coordinate $z(t)$ and velocity $v_{z}(t)$.


Figure 5. Electron velocity $v(t)$ and relativistic factor $\gamma(t)$.


Figure 6. Proton coordinate $x(t)$ and velocity $v_{x}(t)$ obtained from the numerical solution of the ordinary (without averaging) equation of motion: grey curves, unstable trajectory; black curves, stable trajectory.


Figure 7. Proton coordinate $y(t)$ and velocity $v_{y}(t)$.


Figure 8. Proton coordinate $z(t)$ and velocity $v_{z}(t)$.


Figure 9. Proton velocity $v(t)$ and relativistic factor $\gamma(t)$.
it moves to its centre. In the unstable case (the grey curve), the electron turns back and leaves the layer at the moment $t=2.8 \times 10^{4}$. In the stable case (the black curve), it continues the movement inside the layer in the vicinity of the plane $y=0.1$, which is in good agreement with the analytical estimation (44). As follows from the plot $v_{y}(t)$, this movement has a gyratory character. However, in the unstable case, the gyratory amplitude increases with time, and the mean value $v_{y}(t)$ is non-zero. In the stable case, the amplitude decreases with time, and the mean velocity along the $y$ axis is zero. The numerical results for protons confirm the stability conditions, too (see figure 7).

The results of the stable motion (black curves) in the $x$ and $z$ directions also agree well with the analytical predictions. The main acceleration occurs along the electric field ( $z$ axis), $v_{z} \approx-0.9$ for the electron and $v_{z} \approx 0.9$ for the proton, i.e. it is close to the speed of light, and particles with different charges move in opposite directions (figures 4 and 8). The velocity $v_{x} \approx 0.5$ agrees with the analytical estimation $\dot{x}=\left(\xi_{\perp} / \varepsilon\right) \operatorname{sgn} x$ (equation (42)), the electron and proton move in the same direction (figures 2 and 6). The relativistic factor $\gamma$ (figures 5 and 9 ) is well described by equation (46): $\gamma=\varepsilon t$.

## 5. Application to solar flares

Continuum emission in solar flares in hard X-rays ( $20 \mathrm{keV}-1 \mathrm{MeV}$ ) and $\gamma$-rays ( $1 \mathrm{MeV}-1 \mathrm{GeV}$ ) is produced primarily by bremsstrahlung from non-thermal electrons with kinetic energies $\mathcal{K}=$ $20 \mathrm{keV}-1 \mathrm{GeV}$. $\gamma$-ray emission is produced also by high-energy protons with $\mathcal{K}=10 \mathrm{MeV}$ 10 GeV (see, for example, pp. 149 and 165 of [5]).

The total energy $\mathcal{E}$ of a relativistic particle is composed of the rest mass $m c^{2}$ and the kinetic energy $\mathcal{K}$ :

$$
\begin{equation*}
\mathcal{E}=m c^{2}+\mathcal{K}=m c^{2} \gamma . \tag{48}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\gamma=1+\frac{\mathcal{K}}{m c^{2}} . \tag{49}
\end{equation*}
$$

For relativistic particles, $\gamma$ greater or similar 2. So, relativistic electrons accelerated during a flare are characterized by the factors $\gamma=2-2 \times 10^{3}$, and protons by the factors $\gamma=2-11.6$.

According to our model, $\gamma=\varepsilon t=\left(E / B_{0}\right) t=10^{-3} t$. Therefore, the observed values $\gamma$ are acquired by electrons during the dimensionless time $t=\gamma / \varepsilon=2 \times 10^{3}-2 \times 10^{6}$ or the actual (real) time $1.1 \times 10^{-6}-1.2 \times 10^{-3} \mathrm{~s}$. During this time, relativistic electrons overcome the distance $3.4 \times 10^{2}-3.6 \times 10^{7} \mathrm{~cm}$. For protons, the acceleration time is $2.0 \times 10^{3}-1.2 \times 10^{4}$ in dimensionless units or $2.0 \times 10^{-3}-1.2 \times 10^{-2} \mathrm{~s}$. The corresponding distance is $6.0 \times$ $10^{7}-3.6 \times 10^{8} \mathrm{~cm}$.

## 6. Discussion

The analytical results obtained here remain valid as long as changes in the magnetic field are on timescales and length scales much larger than the period and gyroradius. The gyroradius of the proton in the general case turns out to be too large and does not satisfy these conditions. Nevertheless, the numerical solution of the ordinary (without averaging over the gyration) equation of motion for proton confirms all the analytical results: the stability conditions and the character of the stable motion.

In our work, we draw a conclusion about different trajectories of protons and electrons that could lead to charge separation, but it would be accompanied by a modification of the electric
and magnetic fields. Allowance for this effect is outside the scope of this work. It will be taken into account in future.

## 7. Conclusion

An analytical solution of the equation of relativistic motion is obtained. The solution describes the stable motion of a charged particle in a reconnecting CL, when a particle leaves the layer through its edges, and not through the surface.

The stability conditions are found corresponding to a sufficiently strong electric field:

$$
\varepsilon^{2} \gg \xi_{\perp}^{2} \quad \text { and } \quad \varepsilon^{2} \geqslant\left(\xi_{\perp} \xi_{\|}\right)^{2} .
$$

They are rewritten in dimensional values:

$$
E^{2} \gg B_{y}^{2} \quad \text { and } \quad E^{2} \geqslant\left(\frac{B_{y} B_{z}}{B_{0}}\right)^{2} .
$$

The character of the stable motion is found. Acceleration dominates along the electric field, up to the speed of light: $\dot{z} \rightarrow 1 \operatorname{sgn} q$. Particles with positive and negative charges move in opposite directions along the $z$ axis. The velocity in the perpendicular direction in the layer plane is non-zero but significantly lower:

$$
\dot{x}=\frac{\xi_{\perp}}{\varepsilon} \operatorname{sgn} x \ll 1 .
$$

Motion in this direction does not depend on the charge sign; all particles move from the centre of the layer to its edges. Also, particles gyrate in the vicinity of the plane

$$
y_{0}=-\frac{\xi_{\perp} \xi_{\|}}{\varepsilon \delta}(\operatorname{sgn} x)(\operatorname{sgn} q) .
$$

The relativistic factor $\gamma=\varepsilon t$.
The analytical results can be applicable for a wide range of physical conditions: from active regions in the solar atmosphere to coronae of accretion discs and so on.

For the case of magnetic reconnection in the solar corona, the analytical results have been compared with the numerical solutions of the ordinary (without averaging over the gyration) equation of motion. It has confirmed the conclusions of the analytical approach.

The obtained results can be used in the future to develop more detailed models, which will be able to obtain spectra of accelerated particles, their dependence on the total energy of a flare and so on.

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