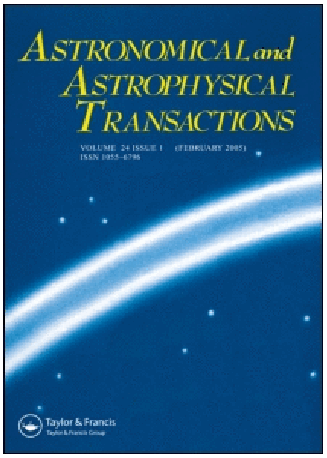

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Flattened γ models for galaxies

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Using the equipotential method we introduce a class of flattened γ models for galaxies and study the properties of their potential–density pairs and two-integral distribution functions.

Keywords: Stellar dynamics; Celestial mechanics; Structure of galaxies; Kinematics

1. Introduction

In this work, we construct a class of axisymmetric models for elliptical galaxies, obtained by flattening the spherical γ models ($\gamma \neq 2$) using the equipotential method [1–4] and then study the properties of their densities and two-integral even distribution functions.

The significance of the equipotential method is that it allows flattening to be achieved while maintaining a simple form of the potential. Furthermore, we easily determine whether the circular velocity generated from the simple potential of a flattened model can be used to describe the data of the rotation velocities of the flattened galaxies observed.

2. Potential–density pairs

The spherical γ model [5–7] is given by the potential–density pair

$$\Phi(r) = -\frac{GM}{r_J}\begin{cases} \dfrac{1}{2-\gamma}\left[1-\left(\dfrac{r}{r+r_J}\right)^{2-\gamma}\right] & \text{if } \gamma \neq 2, \\ \ln\left(\dfrac{r+r_J}{r}\right) & \text{if } \gamma = 2, \end{cases} \tag{1}$$

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and

$$\rho(r) = \frac{(3-\gamma)M}{4\pi r_J^3} \frac{r_J^4 r^{-\gamma}}{(r+r_J)^{4-\gamma}}, \tag{2}$$

where $0 \leqslant \gamma < 3$, M is the total mass and r_J is the scale radius. This model is a limiting case of a wide class of models described by Kuzmin *et al.* [8]. That class also includes generalized isothermic models (and Jaffe's [9] model as its limiting case) and generalized isochrones [10, 11].

Following the equipotential method we introduce a flattening idea, namely that flattening of the spherical γ models is achieved. It is known that the spherical γ models are Jaffe's [9] models when $\gamma = 2$, and their flattening can be achieved [4] by using the approach employed by Miyamoto and Nagais [12], i.e. by replacing the spherical radius r with an axisymmetric radius

$$\tau = \{R^2 + [(z^2+c^2)^{1/2} + d]^2\}^{1/2}. \tag{3}$$

This leads us to replace the radius r of the potentials of γ models (1) with the axisymmetric radius (3), thus giving a class of flattened γ models with the following potentials:

$$\Phi(R^2, z) = -\frac{GM}{r_J} \begin{cases} \dfrac{1}{2-\gamma}\left[1 - \left(\dfrac{\tau}{\tau + r_J}\right)^{2-\gamma}\right] & \text{if } \gamma \neq 2, \\ \ln\left(\dfrac{\tau + r_J}{\tau}\right) & \text{if } \gamma = 2. \end{cases} \tag{4}$$

The mass density $\rho(R^2, z)$ which is generated from the potential defined above is

$$\rho(R^2, z) = \frac{M}{4\pi r_J} \frac{A}{\tau^{2+\gamma}(\tau + r_J)^{4-\gamma} Y^3}, \tag{5}$$

with $A = c^2 d r_J \tau^3 + r_J^2 \tau^2[(3-\gamma)Y^3 + c^2 d] + (3\tau + \gamma r_J)c^2 r_J Y(Y+d)^2$ where $Y = (z^2+c^2)^{1/2}$. Obviously, $\rho(R^2, z)$ is positive and finite for any positive c and d when $0 \leqslant \gamma \leqslant 3$. At larger distances, the densities (5) fall radially according to r^{-4} like the oblate Jaffe's models [4] except on the major axis, and they decrease as r^{-3} on the major axis.

Next, we study the central axis ratio α of the model. The ellipticity of the edge-on projected surface density of the model is always greater than $1 - 1/\alpha$. It is easy to see that $\alpha \geqslant 1$, i.e. the models are oblate.

3. Two-integral even distribution functions

In this section, we study the properties of the two-integral even distribution functions of the oblate γ models for $\gamma \neq 2$, since the corresponding properties of Jaffe's oblate models have been given by Jiang [4]. The form of the potential allows its corresponding density to be also expressed as a function of $\psi \equiv -\Phi(R^2, z)$ and the radial coordinate R (the so-called augmented density). In fact, we can obtain

$$\rho(R^2, z) \equiv P(\psi, R^2) = \frac{E}{4\pi M^{-1} r_J^5 (w+1)^{4-\gamma} y^3}, \tag{6}$$

where $E = r_J^2 c^2\, \mathrm{d}w^6 + r_J^2 w^4[(3-\gamma)y^3 + c^2\, \mathrm{d}w^3] + c^2(3+\gamma w)w^5 y(y+\mathrm{d}w)^2$, $y = (r_J^2 - w^2R^2)^{1/2} - \mathrm{d}w$ and $w = [1 - r_J(2-\gamma)\psi/GM]^{1/(\gamma-2)} - 1$.

Thus, two-integral even distribution functions $f_+(\varepsilon, L_z)$ for the models with a known augmented density can be derived by using the Hunter–Qian [13] complex contour integrals, giving

$$f_+(\varepsilon, L_z^2) = \frac{1}{4\pi^2 \mathrm{i} 2^{1/2}} \int_{C(\varepsilon)} \frac{\mathrm{d}\psi}{(\psi - \varepsilon)^{1/2}} P_{11}\left[\psi, \frac{L_z^2}{2(\psi - \varepsilon)}\right], \tag{7}$$

where

$$P_{11}\left[\psi, \frac{L_z^2}{2(\psi - \varepsilon)}\right] = \frac{\partial^2 P}{\partial \psi^2}\left[\psi, \frac{L_z^2}{2(\psi - \varepsilon)}\right].$$

In this equation, ε is the relative energy and L_z is the angular momentum with respect to the z axis; $C(\varepsilon)$ is a contour of the complex integral; the contour is a loop which starts from the lower side of the real ψ axis at $\psi = 0$, passing through a window P in the real ψ axis to the right of $\psi = \varepsilon$, to the upper side of the real ψ axis at $\psi = 0$; the window P is determined by the positions of all the pole singularities of the integrand.

Using Jiang's [4] algorithm, we can estimate the values of $f_+(\varepsilon, L_z^2)$ in the physical domain of (ε, L_z^2), except at some low relative energy near the origin of ε on its physical boundary curve. We omit the rather cumbersome expression for it. Also, an analytical formula for the radial velocity dispersion σ_R^2 was found.

4. Conclusions

The flattened γ models are constructed by using the approach employed by Miyamoto and Nagai [12] for understanding the processes of galaxy formation and evolution. The flattened γ models are oblate and have finite total mass and finite densities which can be analytically expressed as equation (5). The central axis ratios of the flattened γ models can be analytically expressed. This analysis of the ratios leads to an understanding of the different shapes of elliptical galaxies near the galactic centre. Because of the simple form of the flattened γ models, it is easy to obtain the formula for the augmented density $P_{11}(\psi, R^2)$ (equation (6)) which is required by the two-integral systems. This in turn allows the application of Jiang's algorithm in the systems to be achieved.

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References

[1] S.A. Kutuzov and L.P. Ossipkov, Vest. Leningrad Univ. (1) 99 (1981) (in Russian).
[2] S.A. Kutuzov and L.P. Ossipkov, Astrophysics **25** 671 (1987).
[3] L.P. Ossipkov and J. Binney, in *Variable Stars as a Key for Understanding Structure and Evolution of the Galaxy*, edited by N.N. Samus and A.V. Mironov (Cygnus, Nizhnij Arkhyz, 2000), p. 219.
[4] Z. Jiang, Mon. Not. R. Astron. Soc. **319** 1067 (2000).
[5] P. Saha, Mon. Not. R. Astron. Soc. **262** 1062 (1993).
[6] W. Dehnen, Mon. Not. R. Astron. Soc. **265** 250 (1993).

[7] S. Tremaine, D.O. Richstone, Y.I. Byun, *et al.*, Astron. J. **107** 634 (1994).
[8] G.G. Kuzmin, Ü.-I.K. Veltmann and P.L. Tenjes, Publs W. Struve Tartu Astrofüüs. Observ. **51** 232 (1986) (in Russian).
[9] W. Jaffe, Mon. Not. R. Astron. Soc. **202** 995 (1983).
[10] G.G. Kuzmin and G.A. Malasidze, Publs W. Struve Tartu Astrofüüs. Observ. **38** 181 (1969) (in Russian).
[11] G.G. Kuzmin and Ü.I.-K. Veltmann, Publs W. Struve Tartu Astrofüüs. Observ. **40** 281 (1972) (in Russian).
[12] M. Miyamoto and R. Nagai, Publs Astron. Soc. Japan **27** 533 (1975).
[13] C. Hunter and E. Qian, Mon. Not. R. Astron. Soc. **262** 401 (1993).