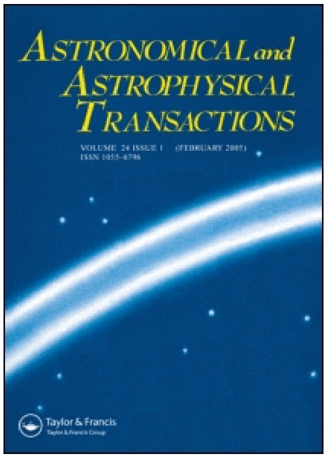

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Cosmic vacuum in the framework of macroscopic extra dimensions

Arthur D. Chernin [ab]
[a] Sternberg Astronomical Institute, Moscow University, Moscow, Russia
[b] Tuorla Observatory, University of Turku, Piikkiö, Finland

Online Publication Date: 01 February 2006
To cite this Article: Chernin, Arthur D. (2006) 'Cosmic vacuum in the framework of macroscopic extra dimensions', Astronomical & Astrophysical Transactions, 25:1, 1 - 6
To link to this article: DOI: 10.1080/10556790600849890
URL: http://dx.doi.org/10.1080/10556790600849890

Astronomical and Astrophysical Transactions
Vol. 25, No. 1, February 2006, 1–6

Cosmic vacuum in the framework of macroscopic extra dimensions

ARTHUR D. CHERNIN*†‡

†Sternberg Astronomical Institute, Moscow University, Moscow 119899, Russia
‡Tuorla Observatory, University of Turku, Piikkiö, FIN-21500, Finland

(Received 25 May 2006)

Basic cosmological parameters describing cosmic energies may have roots in extra-dimensional physics, if macroscopic spatial extra dimensions really exist.

Keywords: Theory of cosmology; Dark matter; Vacuum

1. Introduction

Modern developments in fundamental physics motivated by string theories have involved the recognition that additional spatial dimensions might exist and play a key role in determining the observable world. The potential implications range from experimental signatures of extra dimensions to understanding the nature of gravity and new insights into the evolution of the Universe. A framework suggested by Arkani-Hamed *et al.* [1] is based on the idea that the extra dimensions might have macroscopic sizes that are much larger than the Planck length usually considered in string theories. It is also assumed that there is one and only one 'truly fundamental' energy scale near the teraelectronvolt energy of electroweak symmetry breaking at which the electroweak, strong and gravitational forces would become uniform. One of the initial goals of this approach is to nullify the 'hierarchy problem' associated with the huge gap between the electroweak energy scale and the Planck scale.

It is expected that the idea of macroscopic extra dimensions will be directly tested with the Large Hadron Collider in the next few years. Meanwhile laboratory searches for extra dimensions are currently focused on possible deviations from Newton's law expected on scales less than the size of the extra dimensions. The measurements reach micron scales and no signs of the effect have been found from 1 cm down to 10^{-3} cm [2]. It has recently been demonstrated [3] that evidence for macroscopic extra dimensions may come from astrophysical phenomena of much larger scales; the self-interacting dark-matter model suggests that extra

*Email: chernin@sai.msu.ru

Astronomical and Astrophysical Transactions
ISSN 1055-6796 print/ISSN 1476-3540 online
http://www.tandf.co.uk/journals
DOI: 10.1080/10556790600849890

dimensions may reveal themselves in the spatial distribution of dark matter on galactic and subgalactic scales.

In this paper, new possible links between the compact macroscopic extra dimensions and the physics of cosmological scales are discussed. The recently discovered cosmic vacuum (dark-energy cosmological constant) [4, 5] is the focus of the discussion.

2. The hierarchy number in cosmology

The fact that gravity is exceedingly weak is quantified by the gravity characteristic energy which is the Planck energy $M_{\rm Pl} \approx 10^{16}$ TeV. The energy is related to Newton's gravitational constant G as $G = M_{\rm Pl}^{-2}$ (in units in which $\hbar = c = 1$). The Planck energy is much larger than the electroweak characteristic energy $M_{\rm EW} \approx 1$ TeV at which the symmetry between electromagnetic and weak interactions manifests itself. The big dimensionless number $X = M_{\rm Pl}/M_{\rm EW} \approx 10^{16}$ is referred to as the hierarchy number in particle physics. Why this number is so big is one of the most difficult problems, known as the hierarchy problem, in fundamental theory. The idea of compact macroscopic extra dimensions [1] does not eliminate this number from the theory but reformulates the problem in term of the 'truly fundamental' energy $M_* \approx M_{\rm EW} \approx 1$ TeV and the size R_* of the extra dimensions:

$$X = \frac{M_{\rm Pl}}{M_{\rm EW}} \approx \frac{M_{\rm Pl}}{M_*} \approx (M_* R_*)^{n/2}. \tag{1}$$

Here, n is the number of spatial extra dimensions, and their size R_* is proposed to be the same for all these dimensions.

It is argued [1] that the case $n = 2$ may be most promising; then the size of two extra dimensions is

$$R_* \approx 1\,\text{mm}, \quad n = 2. \tag{2}$$

Generally, the size of the compact extra dimensions depends on their number n:

$$R_* \approx 10^{32/n-17}\,\text{cm}. \tag{3}$$

In [5], $n = 3$ is preferred with the size $R_* \approx 1$ nm because of dark-matter considerations and in view of the negative results on searches for extra dimensions on the submillimetre scale [2–4]. In fact, the case $n = 2$ may not be excluded by these results, because all the number estimates in the framework [1] are valid only for the order of magnitude and may easily be modified to fit the experimental limitations.

Since the hierarchy number is so fundamental, it does not seem too surprising that the big number appears also in cosmology as a key quantity which determines the energy content of the Universe [6–8]. The four major forms of cosmic energy, namely vacuum (dark-energy cosmological constant), dark matter, baryons and radiation, may be represented by four constant parameters, the Friedmann integrals (FINTs) emerging from the Friedmann 'second' equation (which is the internal energy conservation law) for each of the energy forms in the adiabatic expansion:

$$\frac{\dot\rho}{\rho(1+w)} = -3\frac{\dot R}{R}. \tag{4}$$

Here, the constant pressure-to-density ratio $w = p/\rho = -1, 0, 0$ and $1/3$ for vacuum, dark matter, baryons and radiation respectively; $R(t)$ is the cosmological scale factor. The integral of the equation may be given in the form

$$A = \left[\kappa \rho R^{3(1+w)}\right]^{1/(1+3w)}, \tag{5}$$

where $\kappa = 8\pi/3c^2$.

It is easy to see that the FINTs describe the conservation of the total mass of the non-relativistic matter (dark matter and baryons) within a co-moving expanding volume, the conservation of the total number of photons (and other relativistic particles) in the same volume and also the 'conservation' of the vacuum density. The value of $R(t)$ may be identified here with the size $R_U(t)$ of the Universe, if its three-dimensional space is finite, as suggested by the Wilkinson Microwave Anisotropy Probe data [9]; in this case, one may consider the conservation of the total dark mass M_D and baryonic mass M_B, as well as the total number N_R of the relativistic particles in the Universe. If the Universe is spatially infinite, the value of $R(t)$ may be studied, say, as the size $R_M(t)$ of the Metagalaxy, our domain in the Universe. At the present ($t = t_0 \approx 14$ Gyears) epoch of the cosmic evolution, one has $R(t_0) \approx R_U(t_0) \approx R_M(t_0) \approx ct_0 \approx 10^{28}$ cm.

The FINTs are constant parameters of the Friedmann 'first' equation:

$$\dot{R}^2 = \left(\frac{A_V}{R}\right)^{-2} + \frac{A_D}{R} + \frac{A_B}{R} + \left(\frac{A_R}{R}\right)^2 - 2E. \tag{6}$$

Here, the constant $E < 0$, $E = 0$ and $E > 0$ for close, flat and open models respectively.

The numerical values of the integrals can be estimated with the use of the concordance observational data on the present-day cosmic densities with the present-day size $R(t_0)$ taken as indicated above:

$$\begin{aligned} A_V &= (\kappa \rho_V)^{-1/2} \approx 10^{61} M_{Pl}^{-1}, \\ A_D &= \kappa \rho_D R^3 \approx 10^{60} M_{Pl}^{-1}, \\ A_B &= \kappa \rho_B R^3 \approx 10^{59} M_{Pl}^{-1}, \\ A_R &= (\kappa \rho_R)^{1/2} R^2 \approx 10^{59} M_{Pl}^{-1}. \end{aligned} \tag{7}$$

Here, ρ_V, ρ_D, ρ_B and ρ_R are the densities of vacuum, dark matter, baryons and radiation respectively. Units are used in which $c = \hbar = 1$.

As can be seen, all the FINT values are numerically identical, within two orders of magnitude:

$$A_V \approx A_D \approx A_B \approx A_R \approx 10^{60\pm1} M_{Pl l}^{-1}. \tag{8}$$

Although the FINT identity of equation (12) is found with the data on the present (special) epoch of cosmic evolution, it is valid for all the epochs whenever the four energies exist in nature. (The FINT identity for radiation and 'ordinary matter' was first found [10] soon after the discovery of the cosmic microwave background (CMB).)

The FINT identity of equation (12) describes the similarity of the four cosmic energy ingredients. The ingredients are obviously different in many respects, and it is most essential that one of them is vacuum, while the three others are non-vacuum energies. Despite this and other differences, the four energies constitute a regular set (a quartet) with the Friedmann integral A as its common (approximately identical for all the members) genuine constant physical parameter. This similarity may be considered as 'cosmic internal symmetry' (COINS) which is time independent, covariant and robust. COINS is not an exact but an approximate symmetry, since the four FINT values differ within two orders of magnitude.

A standard freeze-out model for massive ($m_D \approx 1\,$TeV) weakly interacting dark-matter particles [11] expressed in terms of the FINT gives the result [6–8]

$$A_V \approx A_D \approx A_R \approx X^4 M_{Pl}^{-1}. \tag{9}$$

Thus, the equality of the three FINT values appears as an outcome of the interplay between gravity and electroweak-scale physics which controls the freeze-out kinetics in the early Universe. The model gives the Friedmann integral in terms of the hierarchy number X which provides the cosmic energies with a common natural quantitative measure.

To refine the quantitative estimates, one may introduce, as usual, a 'reduced Planck scale' $\bar{M}_{Pl} = 0.1 M_{Pl}$ which takes into account the effective number of the degrees of freedom that must be included in the freeze-out kinetics and also factors such as $8\pi/3$ or $32\pi/3$ in exact cosmological formulae. Then one has $X \approx \bar{M}_{Pl}/M_{EW} \approx 10^{15}$ and finally

$$A \approx X^4 M_{Pl}^{-1} \approx 10^{60} M_{Pl}^{-1}. \tag{10}$$

The agreement with the empirical relation (12) looks quite satisfactory here.

Note that the freeze-out model is not complete; it does not account for the FINT value for baryons. It may, however, be assumed, that baryons can be included in a more general model of gravity–electroweak interplay, which assumes that baryogenesis takes place at the electroweak temperatures. Electroweak baryogenesis was proposed by Kuzmin *et al.* [12] (see also [13, 14]).

Moreover, the gravity–electroweak interplay might also be responsible for the origin of cosmic vacuum via supersymmetry violation at teraelectronvolt temperatures [15]. If so, the epoch of electroweak energies is the real beginning of the evolution described by the current standard cosmological model. At that epoch, cosmic energies might come into existence owing to a common physical process. Their common origin in the 'electroweak Big Bang' might guarantee, in particular, the internal mutual correspondence among them which manifests itself as COINS, at a phenomenological level.

3. Extra dimensions and cosmic energies

In multidimensional space, all the physical fields, except gravity, are assumed to be confined in the three-dimensional space, or brane. The multidimensional physics affects cosmology on the brane via gravity, and therefore major cosmological parameters prove to be reformulated in terms of the true fundamental constants M_* and R_*. Since $X = \bar{M}_{Pl}/M_* \approx (M_* R_*)^{n/2}$, one has for the FINT

$$A \approx (M_* R_*)^{(3/2)n} M_*^{-1}. \tag{11}$$

In the case of two extra dimensions,

$$A \approx (M_* R_*)^2 R_*, \quad n = 2. \tag{12}$$

Accordingly, the vacuum density

$$\rho_V \approx (M_* R_*)^{-2n} M_*^4 \tag{13}$$

and, in the case of two extra dimensions,

$$\rho_V \approx R_*^{-4}, \quad n = 2. \tag{14}$$

As can be seen from the last relation, the vacuum density proves to be expressed via the size of the extra dimensions alone. The new relation is free from any signs of the hierarchy effect [16]. In this case, the hierarchy is effectively eliminated from the multidimensional physics of the vacuum.

In the case when $n = 3$, one has

$$A \approx (M_* R_*)^{7/2} R_*, \quad n = 3, \tag{15}$$

$$\rho_V \approx (M_* R_*)^{-2} R_*^{-4}, \quad n = 3. \tag{16}$$

In both case $n = 2$ and case $n = 3$, the numerical value of the vacuum density has the correct order of magnitude on the brane.

Finally, with the use of the relations above, the cosmological constant Λ can also be represented in terms of extra-dimensional constants:

$$\Lambda = \frac{3}{A_V^2} \approx X^{-8} M_{Pl}^2 \approx (M_* R_*)^{-3n} M_*^2. \tag{17}$$

According to the idea of extra dimensions, all that we observe in three-dimensional space are shadows of the true multidimensional entities. In particular, it may be assumed that true vacuum exists in multidimensional space, and the observed cosmic vacuum is not more than its three-dimensional 'projection' to the cosmological brane. This is possible only if vacuum is due to gravity (or gravity–geometry) alone and not related to the fields of matter. In this case, the true vacuum is defined in the multidimensional space and, say, for $n = 2$, its density. This 'true vacuum' is also free from the hierarchy effect.

4. Discussion and conclusions

Extra-dimensional physics may also be responsible for the origin of other major cosmological parameters. In terms of the FINTs, the Big Baryonic Number (the ratio of the number of the CMB photons to the number of baryons [17]) may be represented as

$$B \approx A_R^{3/2} A_B^{-1} m_B M_{Pl}^{-1/2}, \tag{18}$$

where $m_B \approx 1$ GeV is the baryon mass. If one puts, for a rough estimate, $A_R \approx A_B$ and also uses the relations of section 3, the Big Baryonic Number turns out to be

$$B \approx \frac{m_B}{M_{Pl}} X^2 \approx \frac{m_B}{M_{Pl}} (M_* R_*)^{n/2}. \tag{19}$$

If $n = 2$, the relation is especially simple: $B \approx m_B R_*$.

The Big Dark Number (the ratio of the number of the CMB photons to the number of dark-matter particles) may also be of interest:

$$D \approx A_R^{3/2} A_D^{-1} M_{EW} M_{Pl}^{-1/2}. \tag{20}$$

For $A_R \approx A_D$ and $m_D \approx 1$ TeV, this gives

$$D \approx X \approx (M_* R_*)^{n/2} \approx 10^{15}, \tag{21}$$

which is numerically not too far from the real value.

In a similar way, it can be seen that the total values (in the finite Universe or in the Metagalaxy with the present size $R \approx ct_0$) may be related to extra-dimensional physics. The total dark-matter mass

$$M_{\rm D} \approx X_4 M_{\rm Pl} \approx 10^{60} M_{\rm Pl} \approx (M_* R_*)^{5/2n} M_*. \tag{22}$$

The total number of teraelectronvolt dark-matter particles is

$$N_{\rm D} \approx X^5 \approx 10^{75} \approx (M_* R_*)^{5/2n}. \tag{23}$$

The total number of CMB photons is

$$N_{\rm R} \approx X^6 \approx 10^{90} \approx (M_* R_*)^{3n}. \tag{24}$$

Finally, one may try to relate the size of the Universe to the size of the extra dimensions, if the three-dimensional space is really finite. There is no theory that considers that the topology of the Universe has a relation with its energy content. However, if such a relation exists in nature, the equations of this unknown theory might have a simple solution for the value $R_{\rm U}(t)$. A candidate for such a solution may look like

$$R_{\rm U}(t) \approx A(1+z) \approx (M_* R_*)^{3/2n} M_*^{-1}(1+z), \tag{25}$$

where z is the red shift. It is taken into account here that numerically $R_{\rm U} \approx A$, at the present epoch. This coincidence indicates the special character of the present epoch.

Taking the relations above at face values, one may conclude that the basic cosmological parameters describing cosmic energies, namely FINTs, $\rho_{\rm V}$ and Λ, may have roots in the extra-dimensional physics—if extra dimensions really exist.

References

[1] N. Arkani-Hamed *et al.*, Phys. Lett. B **424** 263 (1998).
[2] C.D. Hoyle *et al.*, Phys. Rev. Lett. **86** 1418 (2001).
[3] B. Qin *et al.*, astro-ph/0508572 (2005).
[4] S. Perlmutter *et al.*, Astrophys. J. **918** 565 (1999).
[5] A. Ries *et al.*, Astron. J. 116 (1968).
[6] A.D. Chernin, Phys. Usp. **44** 1019 (2001).
[7] A.D. Chernin, New Astron. **7** 113 (2002).
[8] A.D. Chernin, New Astron. **8** 59 (2002).
[9] J.-P. Luminet *et al.*, Nature **425** 593 (2003).
[10] A.D. Chernin, Nature 270, 250 (1968).
[11] N. Arkani-Hamed *et al.*, Phys. Rev. Lett. **85** 4434 (2000).
[12] V.A. Kuzmin, JETP Lett. **62** 228 (1970).
[13] V.A. Rubakov and M.E. Shaposhnikov, Phys. Usp. **39** 461 (1996).
[14] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. B **955** 36 (1985).
[15] Ya.B. Zeldovich, Phys. Usp. **87** 124 (1968).
[16] A.D. Chernin, astro-ph//6206178 (2002).
[17] Ya.B. Zeldovich and I.D. Novikov, *The Structure and Evolution of the Universe* (The University of Chicago Press, Chicago, Illinois, 1983).