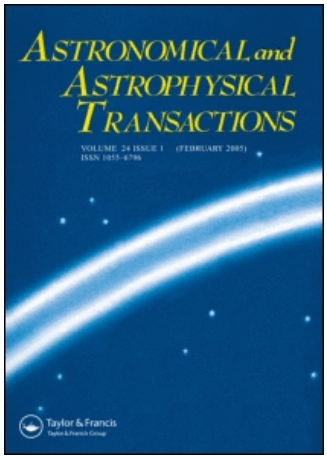

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MUSIC THEORY AND THE HARMONY METHOD IN J. KEPLER'S WORK *THE HARMONY OF THE UNIVERSE*

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In Kepler's book *The Harmony of the Universe*, edited in 1619, the theory of music as a science of that time is presented. Also the investigation of the correspondence between musical proportion and orbital parameters of the planets is presented. Kepler's book *The Harmony of the Universe* is a work that discloses the basic physical regularities of the developing Universe, which so far had not been definitively formulated. To explain the development process, Kepler introduced the concept of a "productive force" or "forming force" that directs the development of natural phenomena with the principles of world harmony, described by him. In addition to the four known natural interactions is a fifth one, that had never been studied fully. In this way we can explain the development of natural phenomena as alive and nonalive. Arising from the "productive force" that directs the flow of processes with the laws of harmony is an explanation of the existence of "anti-entropy" processes, a contradiction to the second law of thermodynamics, but playing a fundamental part in nature. The "golden section" apparatus defines space and time frames of process flow. The contents of the book give a notion about the way or "program" of development. Which basic law of nature is hiden in the contents of book is yet to be resolved (Kepler, 1939).

KEY WORDS Harmony of the Universe, theory of music, Plato's bodies, forming force, music intervals, Chladni figures, golden section, planet music composition, program of development

Early in Aristotle's time great importance was attached to harmony science and first of all to music theory. In the essay "Politics" Aristotle recommended the compulsory study of four topics: music, grammar, gymnastics and drawing (Aristotle, 1983). Apparently that is why the ideas of "world harmony" was close to the mentality of many ancient philosophers. From Pythagoras's, Plato's and Ptolemy's times the presence of knowledge, particularly the music proportions, was connected to the various ideas about the observed world. Kepler's fundamental work *The Harmony of the Universe* published in Frankfurt in 1619 was to play a particular part in the development of such ideas. Kepler was 48 years old (Kepler, 1939).

In this book, on the basis of the fundamental knowledge of his time, Kepler develop scientific information about geometry, astronomy and particularly music theory.

As is clearly shown in Fritjof Capra's book *The Tao of Physics*, the wisdom of ancient philosophers sometimes anticipates and exceeds the conclusions of modern theoretical physics. It is found that the dance of the ancient indian God Shiva personifies many physical processes or patterns, described is physics. (Capra, 1994). Apparently that is why we must return to Kepler's work which undeservedly undergoes criticism not only in atheist literature, but also in such work as *The History of Inductive Sciences* by V. Uevele (1869) and others.

There was uttered the opinion that the book had "still not found a reader" and contains much confusion. However, new information in astronomy and music have importance not only in its own time. Harmony method, which Kepler used to investigate the construction of the world and the depth of his generalizations also have a certain scientific interest nowadays.

In the introduction to the third part of the book, devoted to music theory, Kepler debates with the ancient philosophers. He adduces a legend about Pythagoras, who once passed by a smithy and heard the sounds, uttered by two large hammers, striking an anvil. By tone correlation this sound was formed into the music interval. Comparing the hammers Pythagoras found that the same correlation is formed when strings are sounded. In this way there were the beginnings of consonant and dissonant music interval theory. Analysing the information, Kepler agreed with Ptolemy and criticized Pythagoras in a dispute about the principles of music perception. Pythagoras considered the main problem to be explaining the consonants and correlation of numbers, corresponding to music intervals. Ptolemy showed the subjective sensitive role of the ear. He also found the string length ratios, which define the tone and half-tone, and studied large and small tertz, secst. In a description of musical intervals, Kepler use special geometric constructions, which easily follow from modern notions about standing waves. The sounding string Kepler identified with the chord, that spans the arc of a circle, in which the right figures are entered.

Considering the oscillating string proportions, Kepler implied the construction of corresponding geometric figures. In the chapter "On the cause of consonance" Kepler wrote "we think of the chord not as a segment that spans part of a circle, but as a length of string that utters the sound".

Thus, Kepler explained the sound of consonants by the presence of right within a polygons circle. For example, Kepler confirmed that because the ratio 1:2 ties with an octave, then the "semicircle is consonant with the circle".

Obviously, vertices of inscribed polygons correspond, as is known, to knots of standing waves along the circle. Therefore it follows from Kepler's notion that if part of the figure that inscribes the circle is abandoned then the remainder is dissonant with the other part of the circle.

In the light of modern data, Plato's description of world construction given by Kepler is curious:

(1) The Universe is material and sensual.

(2) Whole things in the Universe are connected indissolubly on basis of symmetry and analogy.

(3) Separate elements form a whole.

(4) Time is created by stellar rotation (it is interested to compare this with the ideas of the astronomer N. A. Kozyrev).

(5) Creatures adapt to four forms: the sky with the stars, the air with the birds, water with the fish, land with the four-legged animals.

According to Pythagoras's ideas, the evidence of the world as a whole is that the numbers 1, 2, 3, 4, that together compose the integer 10. Kepler apparently was the first to describe the phenomenon of overtone and resonance. In the sixth axiom of the third book Kepler pointed out: Quint and Quart sound simultaneously, when octave Gg is sounded. Kepler explained this phenomenon by "consonant string notion". Kepler further pointed out the possibility of string oscillations arising under the influence of another oscillated string, "when pushes of the same tones in a certain moment are coinsidered". Kepler also remarked that the tone pitch uttered by a string may be changed by altering the stretch.

In a special theorem Kepler underlined the connection of music and geometric relations. By that he introduced the proportions, known in science and in art as the "golden section" (see *Golden Section. Three views on the harmony of nature.* I. S. Shevelev, M. A. Marutaev, I. P. Shmelev, 1990). The theorem is formulated in the following way: "if two parts of a circle have a ratio of small to large equal to the ratio of large part to the whole circle, forming a double proportion, then the large part will be consonant with the whole circle, and the small part will the dissonant". Kepler adduced a special table for consonant and dissonant parts of the whole. In the chapter "An harmonic division of strings" Kepler wrote: "Harmonic proportions form raw rafters and stones, that may be used to construct a majestic harmonic system – the musical scale".

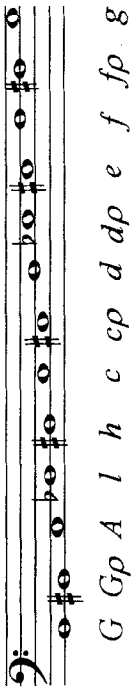
As earlier remarked by Pythagoras, string division in the prime ratios 1:1, 1:3, 1:4, 1:5, 2:3, 3:5 give consonant intervals. Kepler connects consonant harmonic intervals, as before, with geometric constructions. Kepler wrote: "As ancient scientists supposed, the basis of all melodious intervals is the unique consonant interval. But this is wide of the truth. Thinking in such a manner, we can say that a man is also composed of a separate homoncoulis covered with skin". Nevertheless, Kepler discovered a unique measure in geometrical figures that has a commensurable element. For instance, an inscribed triangle and pentagon with commensurable sides can be compared with musical intervals. As is known, modern tempered tones adopted in music may be order into an octave limit as rows of numbers, that divide an octave into 12 equal parts:

0	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	12/12
2	2	2	2	2	2	2	2	2	2	2	2	2

Such a construction also follows from an application of the "golden section" proportion to music.

Table 1. Table of music intervals making by notes sounding at lengths of strings

Lengths of the string	*Interval*	*Notes*
1080.....		
1152.....	half-tone.....	
1215.....	limma.....	
1296.....	half-tone.....	
1350.....	diesis.....	
1440.....	half-tone.....	
1536.....	half-tone.....	
1620.....	limma.....	
1728.....	half-tone.....	
1800.....	diesis.....	
1920.....	half-tone.....	
2048.....	half-tone.....	
2160.....	limma.....	



Kepler's sequence of musical intervals is: prime, second, tertz, quart, quint, than again prime, second, tertz, quart. Such a sequence was better connected with the number of planets in the Solar system, known at that time. Eight tones in the sound row has the form of an octave with the first tone. "The voice of a singing man has come back to himself, and put the beginning to a new sequence of sound tones". Kepler built octave systems in major and minor parts. By that he referred to the work of Vincenzo Galilei – Galileo Galilei father – *Dialogue about ancient and modern music*, published in Florence in 1581. Kepler formulated the law that all tones may be formed by three sounds. Kepler remarked that three sounds is not the origin of harmony, but an action that leads to harmonic sounds. "A triple number carries a cosmic origin and such coordination that it takes one's breath away and brings astonishment and amazement".

J. Kepler introduced the concept of the "mean" quantity and "harmonic average". "If only a few formed proportions define pure harmony, and all other have nothing in common with harmony, then all that is created from arithmetic and geometric proportions are selected correctly.

As a result, Kepler built an octave by means of thirteen strings in the following way (see Table 1).

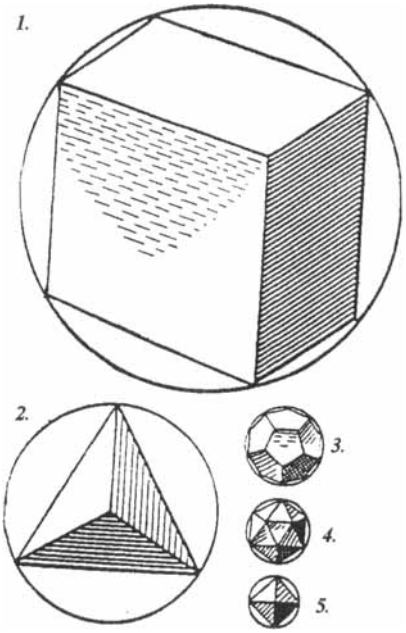


Figure 1 Plato's right bodies, that J. Kepler have operate. According to numeration: cube, tetrahedron, dodecahedron, exahedron, octahedron (from J. Kepler's book *Harmony of Universe*).

Kepler contributed to the elaboration of musical note symbols. He stopped on the note titles, given in the year 1000 by Guido: "ut", "re", "me", "fa", "soul", "lya" and studied the building of four-soundings – tetrachords. Kepler had also studied melody transposition from one key to another. He created a theory of musical interval resolution: "rising from the base tone, melody must return to it". Kepler give 12 examples of sound rows, and studied keys like "skeletons of octave systems".

In his book Kepler also studied various musical methods for expressing feeling and hues. For example, a quint leap, corresponding to an inscribed triangle, where vertices divide the circle, reflect "fitness, courage, and bravery".

Melody, going through an octave, was found to be impassioned. Minor tertz show depression and dejection. Major ones characterize energy and bravery. The last feature Kepler connected with the birth of the major tertz from an inscribed pentagon. It is interesting that Kepler's inimitable humour came through when describing the small tertz, created to get over obstacles: "She is like a hen, ever ready for the cock, that wants to jump on her".

More detailed information about music theory in Kepler's book *Welt Harmonik* are given by the author and V. A. Shvets, *Music Theory Questions in Johannes Kepler's "Harmony of the World"* (1987).

In his works Kepler remembers repeatedly what Pythagoras, Plato and Euclid infer, that God adorns the world in the likeness and manner of five right bodies.

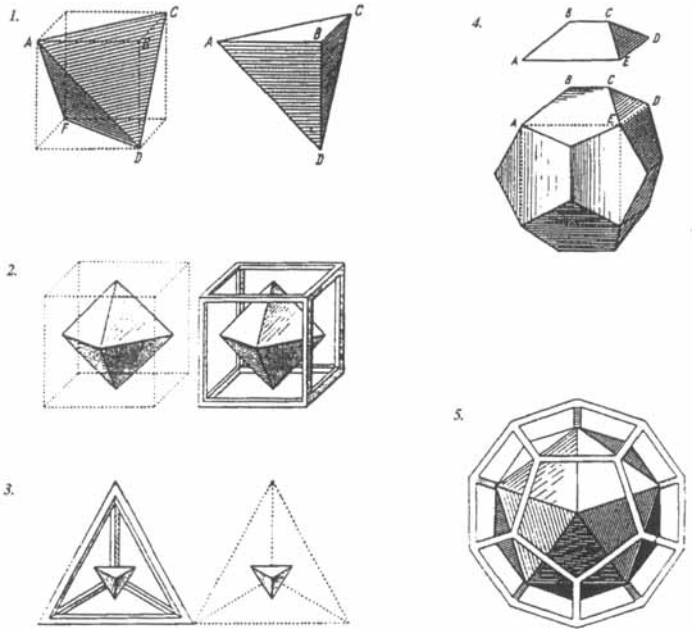


Figure 2 Space combinations of right geometric figures according to J. Kepler. Such figures have allow to imagine planet orbits in Solar system construction. By numeration: tetrahedron in cube, octahedron in cube, tetrahedron in tetrahedron, cube in dodecahedron, exahedron in dodecahedron.

Kepler defined his role in the study of this problem in the following way: if Copernicus in his works posed the question "how?" and partly answered it, then Kepler answered the question "why?" (see Kepler, 1982).

In his work, Kepler treated the world construction by means of five right geometric bodies with criticism. The truth is more complex, Kepler remarked.

But he persisted in looking for a common foundation for the different phenomena – the arising of musical proportions in intervals and geometric constructions that can help us to describe the motion of the known planets, as ancient philosophers think.

When he was 23 years old, J. Kepler carried out calculations of planetary orbits on the basis of Copernicus's theory. In his book *The Mystery of World Building*, published in 1597, the structure of planetary orbits was already shown. Kepler repeatedly refers to this book in "The Harmony of the World". Such a "plan" of the Solar system was as follows: if we build a dodecahedron over the Earth's orbit, which is the common measure of other orbits, and a sphere over the dodecahedron, then we shall obtain the sphere Jupiter's orbit. If over Jupiter's orbit we build a cube and a sphere over the cube, then we shall obtain the sphere of Saturn's orbit.

An exahedron built on the Earth's orbital sphere with the sphere built in them will give the Venusian orbital sphere. Finally, the octahedron, built on the Venusian sphere with the sphere built in it, will give Mercury's orbit. Such constructions are

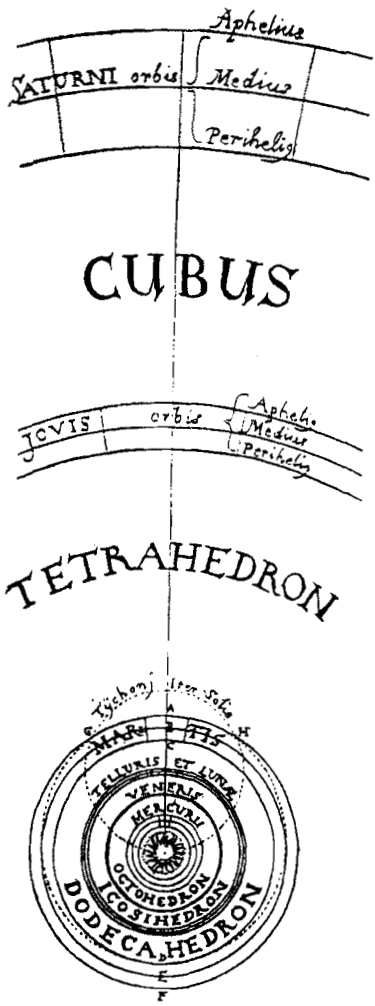


Figure 3 Planet orbit construction according to J. Kepler from book *Harmony of Universe*).

shown in illustrations from Kepler's book (see Figures 1, 2, 3). We see here the common mathematical basis of music interval construction as well as planetary orbits, by mean of figures built on the sphere.

Kepler also analysed the complications in such a construction. In the third chapter of part 5 of the book he adduced the discovered law, according to which planets move along an ellipse where centre coincides with the Sun. In this way motion parameters at aphelion and perihelion differ as in visible day motion and in distance from the centre of the body.

These parameters give proportions which Kepler compared with musical intervals. As an original astronomer-composer, Kepler gets, as a result, the grand planet contrapunt.

Apparently such "music of the planets" allowed Kepler to comprehend and formulate "the basic astronomy equation" according to which "the proportion between any two planet circulation periods is exactly equal to one and a half times the proportion of their average distances".

Kepler analysed planet movement parameters with music proportions. There exists no contradiction in such a method. It is known that the same different equations of oscillatory movement, for example, describe diverse physical phenomena. The mathematical basis of such a "musical" description of astronomical phenomena was the so-called "God's proportion" or "golden section".

Let us consider Kepler's planet music composition in more detail. To "tune" certain notes, connected with the planets, Kepler used astronomical data of planet circulatory periods. Using the dichotomy principle, i.e. dividing the known planet circulation periods into octave intervals, Kepler gets 4 points to Saturn, and 3 points to Jupiter. In this way, the circulation period is divided into a certain limit number, that least of all differs from Earth's period. Doubling the Venus and Mercury periods in such a manner, Kepler gets one and two "octaves" by means of circulation period data.

To infer musical correlations, Kepler also used planet aphelion and perihelion distances from the Sun and daily average planet movement in degrees and in agreed units of length. In this way , the average Earth-to-Sun distance is adopted as 1000.

Forming proportions for every individual planet by means of aphelion and perihelion location data, as well as for other planets, Kepler obtained a table of ratios, which often approaches musical interval ratios. The most characteristic ratios are obtained by composing the visible day's motion of the six planets of aphelion and perihelion. Let us cite this calculations in brief:

Planet	*Visible day's motion*		*Harmony for every planet*
Saturn	aphelion	1′46″	
	perihelion	2′15″	4/5 big tertz
Jupiter	aphelion	4′30″	
	perihelion	5′30″	5/6 small tertz
Mars	aphelion	26′14″	
	perihelion	38′01″	2/3 quint
Earth	aphelion	57′03″	
	perihelion	61′18″	15/16 half-tone
Venus	aphelion	94′50″	
	perihelion	97′37″	24/25 diesis
Mercury	aphelion	164′00″	
	perihelion	384′00″	5/12 octave with small tertz

Music proportions can also be obtained from the ratio of neighbour planet angle velocities in aphelion and perihelion.

Kepler paid considerable attention to composite music intervals, to calculate the number of octaves in various intervals of numbers. For example, he gives a calculation according to which 5 octaves are contained between one day's angular movements of Saturn and the Earth in aphelion. If we divide one day's angular path

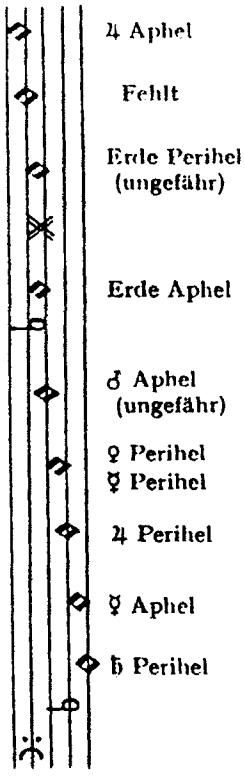


Figure 4 Scale of minor key, building by proportions, determined by planets day's angle movements.

of the Earth, $57'03''$, by 2^5, then we shall get Saturn's angular movement in aphelion, $1'46''$. Kepler chose Saturn's parameters as a zero-point of the construction of the planet scale.

To get the remainder which is nearly a zero-point of the scale, Mercury's daily angular movement in perihelion must be divided by 2^7, and in aphelion by 2^5. For the perihelion and aphelion of Venus and the Earth these are at five octaves each, because the daily angular path must be divided by 2^5 to get the same result. For Mars in perihelion the divisor is 2^4; in aphelion, 2^3. Finally, for Jupiter in perihelion, 2^2, and in aphelion 2^1. On such a scale basis, Kepler built a sound row, in which every note corresponds to the movement of either planet in aphelion or perihelion (Figure 4).

Kepler compared the same notes-planets with string lengths obtained in music theory, and also with right geometric figures. It is curious that a simple physic experiment enables us to be convinced of Kepler's correctness. For this, by means of a violin fiddlestick, one may excite the standing waves on a metallic plate, that is first powdered with fine sand or groats. When sounding a certain tone, the steady right geometric figures – Chladni figures – arise on this plate in the knot lines. A

more magnificent symmetrical picture would be in three-dimensional space. If one excites some space wave, then, obviously, in knot lines we shall see figures that form planetary orbits as Chladni figures. In this way we can also be convinced that Kepler saw just the same picture in Plato's multisides and just these musical proportions between parameters of this figures suggest that Kepler discovered his "basic astronomy equation".

As mentioned above, on the basis of such "world building" lies the universal wave principle and space-time frames of the "golden section". In other words, Kepler's method of Universe harmony, described in his book, shows itself in such a manner. Kepler intensified his music-astronomy research by comparing the known planets with note groups, which connected with planetary orbit parameters by means of music proportions. With that, for every planet Kepler chose its own musical key and tone. On top of these analogies is the general harmony of all six planets for major and minor consonance.

Comparing the planets' daily angular movements with notes, Kepler gives a common sound of all notes, inherent to planets. So, Kepler imagined his planet contrapunt in a certain tune. Kepler proved to be a composer-astronomer, who showed world harmony in his own interpretation. In the final part of the book Kepler again returns to right figures, which delineate planetary orbits.

So, in his book *The Harmony of the Universe* he uses a unique method of research that is defined in the title. Kepler is successful not only in researching of various fields of his time, but also in generalizing this knowledge by means of harmonic methods, which allow him to clear up the casual connection of phenomena.

Kepler writes: "On music proportions I write, first of all for astronomy and metaphysics, because all the harmonic apparatus of music is needed for celestial proportions and its causal explanation". "Astronomy and music are, so to speak various nationalities of the common fatherland – geometry".

In today's scientific literature there exist repeated attempts to follow research similar to Kepler's work. Some of this research is in sufficiently detail and convincingly describes, in particular, an application of a common mathematical basis of various physical phenomena with the "golden section" (see *The Gold Section in the Solar System*, by K. P. Butusov, 1978).

The essence of the mathematical analysis of this problem consists in dividing some segment in the middle and extreme ratio to get the equation:

$$X^2 - X + 1 = 0,$$

where solution gives a root – the F number. The series, composed from sequent degrees of this number, have multiplicative properties, as well as additive properties.

Considering the hexagonal snowflake growth process in the book *On hexagonal snowflakes* published in 1611, Kepler explains the growth mechanism by using such the series in nature. Flower petals, and the arrangement of leaves, etc. grow according to the same low.

As the author thinks, besides many applications of the "golden section" shown in the cited sources, a striking example of the same harmony is the Fresnel equal zone building which explains the straightforward spreading of light.

As far back as 1202 the Italian mathematician Fibonacci used such series to solve problems concerning rabbit reproduction.

To explain the development processes, Kepler introduced the concept of "productive force" or "forming force" that directs the natural development of phenomena with the principles of world harmony, described by him. Here, in addition to four known natural interactions is a fifth one, that has never been studied in detail. (J. Kepler, 1982 or J. Kepleris *Strena, seu de nive sexangula. Francofurti ad Moenum: apud Tampach*, 1611, 21p.).

As it follows from *The Harmony of the Universe*, in the same way we can explain the development of natural phenomena as alive and non-alive.

In the well-known popular book *Universe, Life, Mind* I. S. Shklovsky writes that the probability of a living "preDNA" molecule arising in nature is equal to the probability that "some marmoset that randomly drums on a typewriter keyboard will accidentally type Shakespeare's 66 sonnet" (Shklovsky, 1976).

As is known, natural development in time proceeds corresponding to the second thermodynamics principle, according to which systems transit to a state that is realized with greater probability which is equivalent to the growth of universal entropy. However, the predominance of energy dissipation arid dispersion contradicts observed reality. In such a theoretical course of events, it is impossible to understand the growth of the Universe from the beginning of a primary singularity.

Also the growth of the crown of the tree cannot be understood on the basis of event probabilities, etc.

The arising of the "productive or forming force" which directs process flow with the laws of harmony is explained by the existence of "antientropy" processes, contradicting the second thermodynamics principle, but playing a fundamental part in nature.

According to this law the separate parts form a whole, that have an insignificant probability of arising by statistical laws.

The "golden section" apparatus defines space and time frames of process flow as with the given program.

So Kepler's book *The Harmony of the Universe* is a work that discloses the basic physical regularities of the developing Universe, which so far had not been definitively formulated. The contents of the book reflect the pathos of indivisibility of regularities in the developing Universe and give a notion about the way or "program" of this development.

Which basic law of nature is hidden in the contents of the book must be resolved in future. However, it is clear that Kepler's "harmony method" will play a fundamental part in the definition of these regularities.

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