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# SUPERNOVAE AS A POWERFUL SOURCE OF A GRAVITATIONAL RADIATION 

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We consider the gravitational radiation in the framework of the non-spherical symmetrical evolution of SN. The scenario was considered by Imshennik recently. Unlike the gravitational radiation analysis, which was considered by Imshennik and Popov in the framework of the Peters and Mathews formalism, the gravitational radiation is analysed in the framework of the $(P N)^{5 / 2}$ approximation by Damour and Deruelle, and Lincoln and Will in our paper. It is shown that the eccentricity is more than 0.1 at the moment of filling by a low mass component of a Roche lobe; thus the conclusion of Imshennik and Popov is incorrect (that final eccentricity is less than 0.1). If the SN lies in the Large Magellanic Cloud ( $R=50 \mathrm{Kpc}$ ), then we have the following estimation for the amplitude of gravitational waves: $h \approx 8 \times 10^{-20}$. The frequency of emitted gravitational waves is about 1 kHz .

KEY WORDS Gravitational waves, supernovae

## 1 INTRODUCTION

In the work of Imshennik (1992) a model of non-spherical symmetrical evolution of a pre-SN is considered. We recall the main stages of Imshennik's scenario.

Stage I. This stage consists of the formation of a rapidly rotating protoneutron star as a result of gravitational collapse.

Stage II. The formed protoneutron star can be unstable or becomes unstable if there are small perturbations, therefore a close binary system of neutron stars is formed; we define the system parameters from the laws of conservation of mass and angular momentum.

Stage III. There is a mutual decrease of the distance between the components of a binary system (since there is gravitational radiation), until the filling of the low mass component of a Roche lobe.

Stage IV. There are mass losses of the low mass component and an unstable neutron star with a mass of about $0.1 M_{\odot}$ is formed.

Stage V. The unstable neutron star with mass $0.1 M_{\odot}$ explodes and emits energy of about $10^{51}$ ergs, according to calculations by Blinnikov et al. (1990a, b).

We consider the gravitational radiation of a binary system of neutron stars, and the parameters of a system are equal to values which were considered in Imshennik's paper (1992), namely, the mass of the protoneutron star is equal to $m=M_{t}=2 M_{\odot}$, and the momentum moment is equal to $J_{0}=8.81 \times 10^{49} \mathrm{ergs} / \mathrm{s}$. Unlike in the paper by Imshennik and Popov (1994), where the gravitational radiation was considered in the framework of the Peters (1964) and Peters and Mathews formalism (1963), in this paper the gravitational radiation is analysed in the framework of the approach of Damour and Deruelle (1981), using expressions of Lincoln and Will (1990).

## 2 BASIC EQUATIONS AND EXPRESSIONS

We recall the basic equations for the description of the motion of a coalescing binary system of neutron stars, and the equations include (post) ${ }^{5 / 2}$-Newtonian correction terms, since only the (post) $)^{5 / 2}$-Newtonian correction term represents the dominant - radiation - reaction effects (Lincoln and Will, 1990). The following equation (Lincoln and Will, 1990) can be obtained for a relative acceleration of components of a binary system

$$
\begin{equation*}
\mathbf{a}=\left(m / r^{2}\right)[(-1+A) \mathbf{n}+B \mathbf{v}] \tag{1}
\end{equation*}
$$

where $\mathbf{a}$ is the relative acceleration of components of a binary system, $m=m_{1}+$ $m_{2}, m_{1}, m_{2}$ are the masses of the components of a binary system, $r=|\mathbf{x}|, \mathbf{x}=r \mathbf{n}$, $\mathbf{v}=\mathbf{v}_{1}-\mathbf{v}_{2}, A$ and $B$ define post, post-post and (post) $)^{5 / 2}$-Newtonian correction terms. Writing these terms in the form $A=A_{1}+A_{2}+A_{5 / 2}$ and $B=B_{1}+B_{2}+B_{5 / 2}$, we obtain the following expressions for these variables (Lincoln and Will, 1990)

$$
\begin{align*}
A_{1} & =2(2+\eta) \frac{m}{r}-(1+3 \eta) v^{2}+\frac{3}{2} \eta \dot{r}^{2}  \tag{2}\\
A_{2} & =-\frac{3}{4}(12+29 \eta)\left(\frac{m}{r}\right)-\eta(3-4 \eta) v^{4}-\frac{15}{8} \eta(1-3 \eta) \dot{r}^{4} \\
& +\frac{3}{2} \eta(3-4 \eta) v^{2} \dot{r}^{2}+\frac{1}{2} \eta(13-4 \eta) \frac{m}{r} v^{2}+\left(2+25 \eta+2 \eta^{2}\right) \frac{m}{r} \dot{r}^{2}  \tag{3}\\
A_{5 / 2} & =\frac{8}{5} \eta \frac{m}{r} \dot{r}\left(3 v^{2}+\frac{17}{3} \frac{m}{r}\right)  \tag{4}\\
B_{1} & =2(2-\eta) \dot{r},  \tag{5}\\
B_{2} & =\frac{1}{2} \dot{r}\left[\eta(15+4 \eta) v^{2}-\left(4+41 \eta+8 \eta^{2}\right) \frac{m}{r}-3 \eta(3+2 \eta) \dot{r}^{2}\right]  \tag{6}\\
B_{5 / 2} & =-\frac{8}{5} \eta \frac{m}{r}\left(v^{2}+3 \frac{m}{r}\right) \tag{7}
\end{align*}
$$

where $\eta=\mu / m, \mu=m_{1} m_{2} / m$, and the dot over a variable means differentiation with respect to time ( $d / d t$ ). Using osculating orbit elements (Abalakin et al., 1976) we obtain equations for a description of the motion of the binary system according
to Lincoln and Will (1990). We recall that osculating orbit elements are used for an analysis of small perturbations of Keplerian orbits (Abalakin et al., 1976). In the general case the Keplerian orbit is determined by six parameters: $i$, the inclination angle of the orbit relative to a reference plane; $\Omega$, the angle to the line of the ascending node; $\omega$, the angle between a line of node and the pericentric line; $a$, the semimajor axis; $e$, the eccentricity, and $T$, the time of pericentric passage. There are the following definitions (Lincoln and Will, 1990):

$$
\begin{align*}
x^{\prime} & \equiv r \cos \psi  \tag{8}\\
y^{\prime} & \equiv r \sin \psi  \tag{9}\\
z^{\prime} & \equiv 0  \tag{10}\\
p / r & \equiv 1+e \cos (\psi-\omega)  \tag{11}\\
\dot{r} & \equiv(m / r)^{1 / 2} e \sin (\psi-\omega)  \tag{12}\\
r^{2} \dot{\psi} & \equiv(m / r)^{1 / 2} \tag{13}
\end{align*}
$$

where $\psi$ is the angle in the orbit from the ascending node, $p \equiv a\left(1-e^{2}\right)$, and the relations between the coordinates $x, y, z$ and coordinates $x^{\prime}, y^{\prime}, z^{\prime}$ in terms of the angles $i, \Omega$ are

$$
\begin{align*}
& x \equiv x^{\prime} \cos \Omega-y^{\prime} \sin \Omega \cos i+z^{\prime} \sin \Omega \sin i,  \tag{14}\\
& y \equiv x^{\prime} \sin \Omega+y^{\prime} \cos \Omega \cos i-z^{\prime} \cos \Omega \sin i,  \tag{15}\\
& z \text { 曰 } y^{\prime} \sin i+z^{\prime} \cos i . \tag{16}
\end{align*}
$$

we resolve the perturbing acceleration into a radial component $R$, a component $S$ perpendicular to $R$ in the direction of advancing angle $\psi$ and the component of acceleration $W$, which is perpendicular to the orbital plane. So long as the acceleration is a linear combination of the vectors $\mathbf{n}$ and $\mathbf{v}$, then $W=0$, so the orbital motion is flat. Using osculating elements, we write the equations of motion of a binary system (Lincoln and Will, 1990)

$$
\begin{align*}
d i / d t & =0  \tag{17}\\
\dot{\Omega} & =0  \tag{18}\\
\dot{a} & =\frac{2 a^{2}}{(m p)^{1 / 2}}[e R \sin (\psi-\omega)+(p / r) S]  \tag{19}\\
\dot{e} & =(p / m)^{1 / 2} R \sin (\psi-\omega)+[e(r / p)+(1+r / p) \cos (\psi-\omega)] S  \tag{20}\\
e \dot{\omega} & =(p / m)^{1 / 2}[-R \cos (\psi-\omega)+(1+r / p) \sin (\psi-\omega) S]  \tag{21}\\
m \dot{T} & =a\left[2 r-(p / e) \cos (\psi-\omega)-3(m / p)^{1 / 2} e(t-T) \sin (\psi-\omega)\right] R \\
& +(a / e)\left[(r+p) \sin (\psi-\omega)-3(m p)^{1 / 2} e(t-T) / r\right] S \tag{22}
\end{align*}
$$

We therefore choose the orbital plane to be the plane $x-y$ and the line of ascending nodes is the $x$-axis. Thus, $i=0, \Omega=0$. According to Lincoln and Will (1990), it is possible to choose the angle $\psi=\phi$ ( $\phi$ is the polar angle in the plane of
the orbit). In this case the orbit is described by the following expressions (Lincoln and Will, 1990)

$$
\begin{align*}
\mathbf{x} & \equiv r\left(\mathbf{e}_{x} \cos \phi+\mathbf{e}_{y} \sin \phi\right)  \tag{23}\\
\mathbf{v} & \equiv(m / p)^{1 / 2}\left[-\mathbf{e}_{x}(\sin \phi+e \sin \omega)+\mathbf{e}_{y}(\cos \phi+e \cos \omega)\right]  \tag{24}\\
p / r & \equiv 1+e \cos f  \tag{25}\\
\dot{r} & \equiv(m / r)^{1 / 2} e \sin f,  \tag{26}\\
r^{2} \dot{\phi} & \equiv(\mathrm{~m} / r)^{1 / 2} \tag{27}
\end{align*}
$$

where $f \equiv \phi-\omega$. Clearly, we have following expressions for the components of acceleration in this case

$$
\begin{align*}
R & =\left(m / r^{2}\right)(A+\dot{r} B)  \tag{28}\\
S & =(m / r) \dot{\phi} B \tag{29}
\end{align*}
$$

The equation (28) can be used for changing the independent variable (instead of the time $t$ it is possible to use the polar angle $\phi$ ). Then we have following equations for the motion of a binary system

$$
\begin{align*}
\frac{d(a / m)}{d \phi} & =2\left(\frac{a}{m}\right)^{2} \frac{m}{p}\left[e A \sin f+\left(\frac{m}{p}\right)^{1 / 2} B\left(1+e^{2}+2 e \cos f\right)\right]  \tag{30}\\
\frac{d e}{d \phi} & =A \sin f+2\left(\frac{m}{p}\right)^{1 / 2} B(e+\cos f)  \tag{31}\\
e \frac{d \omega}{d \phi} & =-A \cos f+2\left(\frac{m}{p}\right)^{1 / 2} B \sin f \tag{32}
\end{align*}
$$

Substituting expressions for the perturbations $A$ and $B$ in equations (31-33) we obtain equations for the variables $e, \omega, p$ (Lincoln and Will, 1990)

$$
\begin{aligned}
\frac{d e}{d \phi} & =\frac{m}{p}[(3-\eta) \sin f+(5-4 \eta) e \sin 2 f \\
& \left.+\frac{e^{2}}{8}[(56-47 \eta) \sin f-3 \eta \sin 3 f]\right] \\
& -\left(\frac{m}{p}\right)^{2}\left[\frac{1}{4}\left(36+73 \eta-8 \eta^{2}\right) \sin f\right. \\
& +\left(11+31 \eta-3 \eta^{2}\right) e \sin 2 f+\frac{e^{2}}{16}\left[\left(60+245 \eta-64 \eta^{2}\right) \sin 3 f\right. \\
& \left.+\left(92+181 \eta-32 \eta^{2}\right) \sin f\right] \\
& +\frac{e^{3}}{8}\left[\left(2+25 \eta-16 \eta^{2}\right) \sin 4 f+4\left(3-11 \eta-10 \eta^{2}\right) \sin 2 f\right] \\
& +\frac{\eta}{128} e^{4}[15(1-3 \eta) \sin 5 f-3(73+53 \eta) \sin 3 f
\end{aligned}
$$

$$
\begin{align*}
& -2(477+161 \eta) \sin f]] \\
& \text { - } \frac{\eta}{15}\left(\frac{m}{p}\right)^{5 / 2}[192 \cos f+16 e(19+20 \cos 2 f) \\
& +2 e^{2}(91 \cos 3 f+269 \cos f) \\
& \left.+e^{3}(121+180 \cos 2 f+35 \cos 4 f)+6 e^{4}(3 \cos 3 f+5 \cos f)\right] \text {, }  \tag{33}\\
& e \frac{d \omega}{d \phi}=\frac{m}{p}[-(3-\eta) \cos f+e[3-(5-4 \eta) \cos 2 f] \\
& \left.+\frac{e^{2}}{8}[3 \eta \cos 3 f+(8+21 \eta) \cos f]\right] \\
& \times\left(\frac{m}{p}\right)^{2}\left[\frac{1}{4}\left(36+73 \eta-8 \eta^{2}\right) \cos f+e\left[\left(7+5 \eta-7 \eta^{2}\right)\right.\right. \\
& \left.+\left(11+31 \eta-3 \eta^{2}\right) \cos 2 f\right] \\
& +\frac{e^{2}}{16}\left[\left(84+79 \eta-224 \eta^{2}\right) \cos f+\left(60+245 \eta-64 \eta^{2}\right) \cos 3 f\right] \\
& +\frac{e^{3}}{8}\left[\left(2+25 \eta-16 \eta^{2}\right) \cos 4 f-2 \eta(1+24 \eta) \cos 2 f\right. \\
& \left.-\left(2-21 \eta+48 \eta^{2}\right)\right] \\
& +\frac{\eta}{128} e^{4}[15(1-3 \eta) \cos 5 f+3(33-19 \eta) \cos 3 f \\
& +10(27-41 \eta) \cos f]] \\
& -\frac{\eta}{15}\left(\frac{m}{p}\right)^{5 / 2}\left[192 \sin f+320 e \sin 2 f+2 e^{2}(91 \sin 3 f+115 \sin f)\right. \\
& +5 e^{3}\left(7 \sin 4 f+26 \sin 2 f+18 e^{4}(\sin 3 f+\sin f)\right] \text {, }  \tag{34}\\
& \left.\frac{d(p / m)}{d \phi}=4(2-\eta) e \sin f+e \frac{m}{p}\left[-2\left(2+13 \eta+2 \eta^{2}\right) \sin f\right)\right] \text {, } \\
& -\frac{1}{2}(4+11 \eta) e \sin 2 f+\frac{1}{4} \eta(33-2 \eta) e^{2} \sin f \\
& \left.+\frac{3}{4} \eta(3+2 \eta) e^{2} \sin 3 f\right] \\
& -\frac{8}{5} \eta\left(\frac{m}{p}\right)^{3 / 2}\left(8+18 e \cos f+7 e^{2}+5 e^{2} \cos 2 f+2 e^{3} \cos f\right) \text {. } \tag{35}
\end{align*}
$$

## 3 QUASICIRCULAR ORBITS

According to the approach of Lincoln and Will we give a definition of quasicircular orbits of a binary system (Lincoln and Will, 1990). It is easy to see that the solution of the equations of motion with vanishing eccentricity is impossible, since at $e=0$
we have from equations (34-36) that $\frac{d e}{d \phi} \neq 0$. First we define of the quasicircular orbit in the framework of the $(\mathrm{PN})^{2}$-approach as follows: $\frac{d \omega}{d \phi}=1$, i.e. the particle and osculating ellipse are rotated by the same velocity. We have $f=\phi-\omega=$ const in this case; using the choice of initial value $\phi$ it is possible to consider the following value: $f=\pi$. Thus, we obtain in the framework of the ( PN$)^{2}$-approach, that $\frac{d p}{d \phi}=\frac{d e}{d \phi}=0$. It is easy to see that we have in the (PN) ${ }^{2}$-approach

$$
\begin{equation*}
e \approx(3-\eta)(m / p)-\left(15+\frac{17}{4} \eta+2 \eta^{2}\right)(m / p)^{2} . \tag{36}
\end{equation*}
$$

It is easy to see also that the approximation for $e$ is the solution also in the $(\mathrm{PN})^{5 / 2}$ approach (if we assume that)

$$
\begin{equation*}
f=\pi+\frac{64}{5} \frac{\eta}{3-\eta} u^{3 / 2} . \tag{37}
\end{equation*}
$$

We have the following equation in the (PN) $)^{5 / 2}$-approach for the coordinate $u \equiv \frac{m}{p}$ which depends on the angle $\phi$ :

$$
\begin{equation*}
\frac{d u}{d \phi} \approx 16 u^{7 / 2}[4 / 5+(1-\eta) u] . \tag{38}
\end{equation*}
$$

It is easy to see that we have from the definition of quasicircular orbits, that the inequality $\frac{d r}{d u}<0$ is valid during the evolution of a binary system. Therefore the distance between the components continuously decreases. Clearly, that this inequality is invalid at some points of an elliptical orbit (if we assume an elliptical motion of binary system components). In fact, as long as

$$
\begin{aligned}
\hat{r} & \equiv \frac{r}{m} \approx\left[1+(3-\eta) u-\left(6+\frac{41}{4} \eta+2 \eta^{2}\right) u^{2}\right] / u \\
\frac{d \hat{r}}{d u} & =\frac{1}{u^{2}}-\left(6+\frac{41}{4} \eta+2 \eta^{2}\right)<0
\end{aligned}
$$

$\frac{d r}{d u}<0$, thus a binary system moves on an orbit as a spiral so that the distance between the components continuously decreases.

## 4 EVOLUTION OF A BINARY SYSTEM WITH RADIATION OF GRAVITATIONAL WAVES

We considered the evolution of a binary system which emits gravitational waves. We use the approach of quasicircular orbits. In fact, Lincoln and Will showed that the solution of the equations of motion of a binary system is general enough, since the general solution evolves into quasicircular orbits (Lincoln and Will, 1990). The rapid decreasing of eccentricity to small values was also shown in the paper (Imshennik and Popov, 1994).

We consider the evolution of a binary system until the moment of filling by a low mass component of a Roche lobe, according to Imshennik's model (1992). The critical value of the orbit radius is connected with the radius of the low mass component. Namely, we determine the value from the approximation by Masevich and Tutukov (1988):

$$
\begin{equation*}
\frac{r_{\mathrm{cr}}}{R_{2}}=\frac{1}{0.52\left(m_{2} / m\right)^{0.44}} . \tag{39}
\end{equation*}
$$

In this case the radius of the Roche lobe is determined as a spherical radius, and the sphere has the Roche lobe volume.

## 5 RESULTS AND DISCUSSION

We choose values of the mass of a protoneutron star and the momentum moment, which are equal to corresponding values from the papers by Imshennik (1992) and by Imshennik and Popov (1994), namely $M=M_{t}=2 M_{\odot}$, and the momentum moment is equal to $J_{0}=8.81 \times 10^{49} \mathrm{ergs} / \mathrm{s}$. We recall that the limiting value (see (Imshennik and Popov, 1994) $\delta=m_{2} / m=0.205(\eta=0.163)$ ) was obtained from the condition that the time of decreasing distance between components of a binary system (which is connected with the emission of gravitational waves) is equal to about 1 hour, and therefore $\eta=0.2$, i.e. the value of this constant is about the limiting value. The gravitational wave form for different polarizations is determined by Lincoln and Will's expressions (1990)

$$
\begin{align*}
h_{+} & \approx-\frac{2 \mu}{R}\left[\left(\frac{m}{p}\right)\left(1+\cos ^{2} \Theta\right) \cos 2 \Psi-\frac{1}{2}\left(\frac{m}{p}\right)^{3 / 2} \frac{\delta m}{m} \sin \Theta[\sin \Psi\right. \\
& \left.+\frac{1}{4}\left(1+\cos ^{2} \Theta\right)(\sin \Psi+9 \sin 3 \Psi)\right] \\
& -\frac{1}{3}\left(\frac{m}{p}\right)^{2}\left\{\frac{1}{2}(37-9 \eta)\left(1+\cos ^{2} \Theta\right) \cos 2 \Psi+(1-3 \eta) \sin ^{2} \Theta[2 \cos 2 \Psi\right. \\
& \left.\left.\left.+\left(1+\cos ^{2} \Theta\right)(\cos 2 \Psi+4 \cos 4 \Psi)\right]\right\}\right]  \tag{40}\\
h_{\times} & \approx \frac{2 \mu}{R} \cos \Theta\left[2\left[\frac{m}{p}\right] \sin 2 \Psi+\frac{3}{4}\left[\frac{m}{p}\right]^{3 / 2} \frac{\delta m}{m} \sin \Theta(\cos \Psi+3 \cos 3 \Psi)\right. \\
& \left.-\frac{1}{3}\left[\frac{m}{p}\right]^{2}\left[(37-9 \eta) \sin 2 \Psi+4(1-3 \eta) \sin ^{2} \Theta(\sin 2 \Psi+2 \sin 4 \Psi)\right]\right] \tag{41}
\end{align*}
$$

where $\Psi=\Phi-\phi$. The angles $\Phi, \Theta$ determine the observer position, respectively the binary system. So if $\Theta=0, \pi$ then the observer position is on the pole; if $\Theta=\pi / 2$, this corresponds to the observer position on the equatorial plane. The angle $\Phi$ describes the rotation of the binary system.


Figure 1 The gravitational wave for the polarization $h_{+}$(the solid line) and for the polarization $h_{\times}$(the dashed line) ( $\eta=0.2$ ) for angles $\Phi=0, \Theta=\pi / 4$.

The radius of a low mass star with mass $m_{2}=1.1 \times 10^{33} \mathrm{~g}$ (similarly to Imshennik and Popov (1994)) is equal to about 13.32 km (see, for example Zeldovich and Novikov (1971)). We assumed that there is a coalescence of a binary system according to the approach of quasicircular orbits.

It is known that the amplitude waves formed at the Earth will have the amplitude (Lincoln and Will, 1990; Zakharov, 1996)

$$
\begin{equation*}
h_{\mathrm{obs}} \approx 7 \times 10^{-23}(4 \eta) \frac{m}{2.8 M_{\odot}} \frac{100 \mathrm{Mpc}}{R} \tag{42}
\end{equation*}
$$

therefore if the SN lies in the Large Magellanic Cloud ( $R=50 \mathrm{Kpc}$ ), then $h \approx 8 \times$ $10^{-20}$. The frequency of the emitted gravitational waves is about 1 kHz (Zakharov, 1996) (Figure 1). Thus the source of the gravitational radiation may be observed using the VIRGO detector, at least in principle.

The gravitational radiation luminosity in the PN -approximation is equal to the following expression (Lincoln and Will, 1990)

$$
\begin{aligned}
L_{\mathrm{WW}} & =\frac{8}{15} \frac{\mu^{2} m^{2}}{r^{4}}\left\{\left(12 v^{2}-11 \dot{r}^{2}\right)+12 v^{2}[14 \eta \tilde{E}-(6-9 \eta) m / r]\right. \\
& -2 \dot{r}^{2}\left[2(33+43 \eta) \tilde{E}+3(8+21 \eta) m / r-\frac{3}{2}(20+3 \eta) \dot{r}^{2}\right] \\
& +4 \eta(1-6 \eta)\left[\frac{v^{2}}{2}(6 \tilde{E}+7 m / r)-\frac{1}{3} \dot{r}^{2}\left(21 \tilde{E}+23 m / r-6 \dot{r}^{2}\right)\right]
\end{aligned}
$$



Figure 2 The ratio of $L_{\text {PM }}$ (gravitational luminosity in the framework of the Peters and Mathews approximation) and $L_{\mathrm{WW}}$ (gravitational luminosity in the framework of the Wagoner and Will approximation) as the function of the number of revolutions.

$$
\begin{align*}
& +\frac{1}{7}(1-3 \eta)\left[8 v^{2}(17 \tilde{E}-10 m / r)-\frac{1}{3} \dot{r}^{2}\left(144 \tilde{E}-440 m / r+105 \dot{r}^{2}\right)\right] \\
& +\frac{1}{7}(1-4 \eta)\left[\frac{v^{2}}{2}(345 \tilde{E}+397 m / r)\right. \\
& \left.\left.+4(m / r)^{2}-\dot{r}^{2}\left(319 \tilde{E}-349 m / r+297 \dot{r}^{2} / 4\right)\right]\right\} \tag{43}
\end{align*}
$$

where

$$
\begin{align*}
\tilde{E} & =\frac{v^{2}}{2}-\frac{m}{r} \\
v^{2} & =u\left[1-(3-\eta) u+\left(15+\frac{17}{4} \eta+2 \eta^{2}\right) u^{2}\right]^{2} \\
\dot{r} & =\frac{d \hat{r}}{d u} \frac{d u}{d \phi} \frac{d \phi}{d \hat{t}} \tag{44}
\end{align*}
$$

The ratio of $L_{\mathrm{PM}}$ and $L_{\mathrm{WW}}$ is presented in Figure 2. We see that the PetersMathews approximation gives a greater value of the gravitational radiation than the Wagoner-Will approximation for the problem.

In Figure 3 the dependence of the eccentricity on the number of revolutions of a binary system is presented. Thus, the eccentricity is about 0.12 at the moment when the distance is minimal. Thus, the conclusion of Imshennik and Popov (1994),


Figure 3 The dependence of the eccentricity on the number of revolutions $(\eta=0.2)$.
that the eccentricity is smaller than 0.1 at the final moment, is incorrect. This is a natural consequence of the non-vanishing post-Newtonian parameter value (which is about $7 \%$ ). The final eccentricity is about 0.11 in the case of the limiting value of the mass ratio $\delta=m_{2} / m=0.205$. Nevertheless, we have a decrease of the eccentricity for binary systems with small post-Newtonian parameters, for example such as the binary pulsar PSR $1913+16$, where a post-Newtonian parameter of about $10^{-6}$ is obtained.

We remark that the considered system is the sample in which the post-Newtonian parameter is not too small, especially at the moment of minimal distance between components and certainly it is necessary to take into account post-Newtonian terms. If we consider a vanishing value of the post-Newtonian parameter in the problem there is the possibility of obtaining incorrect conclusions, similar to the results of the paper (Imshennik and Popov, 1994), namely the conclusion about the monotonie reduction of the eccentricity during the evolution and the conclusion that the final value of the eccentricity is less than 0.1.

More detailed discussion of the problem was presented in the recent paper (Zakharov, 1996).

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