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# ON THE ABSORPTION OF LIGHT FROM THE BRIGHTEST STARS IN M 31 

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The classical Q-method for the brightest OB stars, the period-luminosity ( $\mathrm{P}-\mathrm{L}$ ) relation for the Cepheids and the maximum magnitude-rate of decline (MMRD) relation for novae were used to study the absorption variations in M 31. An extinction map with 34 arcsec resolution was constructed. This resolution corresponds to a characteristic size of about 100 pc at which the colour excesses correlate most strongly. The mean colour excesses are $E(B-V)=0.51 \pm 0.27 \mathrm{mag}$ from the OB stars, $0.21 \pm 0.09 \mathrm{mag}$ from Cepheids and $0.14 \pm 0.10 \mathrm{mag}$ from novae. The colour excess shows no big radial dependence within 12 kpc from the centre. The distributions of the colour excesses determined from OB stars (located predominantly in spiral arm regions) in six bins along the radius were compared with distributions generated via Monte Carlo simulation to estimate the optical thickness of the disk at different galactocentric distances. The total optical thickness of the disk is $\tau_{B}=11.5$ near the bulge and decreases to 2 in the outermost parts ( $R / R_{25}=0.62$, where $R_{25}$ is the isophote diameter at $B=25 \mathrm{mag} \mathrm{arcsec}^{-2}$ ). We obtain a radial dust scale factor $R_{D}=4.8 \pm 0.4 \mathrm{kpc}$. The constant ratio of the dust to OB star scale factors in the $Z$-direction was found to be ( $Z_{D} / Z_{S}=1.25 \pm 0.25$ ). We derived vertical scale factors $Z_{S}=76 \pm 36 \mathrm{pc}$ for the OB stars and $Z_{D}=95 \pm 45 \mathrm{pc}$ for the dust. The visible OB stars lay on a protruding conical surface with an angle of about $2^{\circ}$ between the base plane and the surface of the cone as a sequence of the optical thickness gradient toward the centre in the spiral arm regions.

KEY WORDS M 31, the light extinction, the dust distribution, the optical thickness

## 1 INTRODUCTION

Dust is an important part of the interstellar medium because of its influence on the magnitudes (absorption) and colours (reddening). The first photographs of M 31 taken in the 19th century revealed the presence of dust clouds and prolonged complexes of them. The quantitative characteristics of the dust and its spatial distribution could be studied from both the absorption and the reddening of different objects or from the change of their statistics and hereafter we will call those two approaches respectively photometrical and statistical.

In the photometrical approach all the objects of interest should be luminous enough, have sufficient numbers, and posses known intrinsic absolute magnitudes and colours. To construct a complete picture of the distribution of the dust these objects have to be distributed homogeneously over the whole body of the galaxy. Until recently these conditions were satisfied only by Cepheids and globular clusters but now there are comprehensive sets of photometry of the brightest OB stars in M 31 that can be used as well.

Vetesnik (1962) was one of the first to use multicolour photometry of M 31's globular clusters to examine the extinction at various positions in the galaxy. Cepheids have been used by van Genderen (1973) who determined the optical half-thickness of the disk in the four Baade variable stars fields. According to Efremov (1985) the deviations from the Cepheid $\mathrm{P}-\mathrm{L}$ relation in spiral arm S4 are consistent with the absorption determined from the H I surface brightness. Later on, Iye and Richter (1985) used a homogeneous set of UBV photometry of globulars in M 31 not only to determine the reddening as a function of position but also to study the nature of the extinction law in Andromeda galaxy.

The presence of a large number of early type stars (especially main sequence stars) in the disk of M 31 makes possible much more detailed mapping of the reddening in the disk than from globular clusters and Cepheids. For the first time the classical Q-method was applied to these stars in M 31 galaxy by van den Berg (1964a). The estimates of extinction in the disk from individual stars were based on photographic material until the mid-1980 s when extensive studies begun with the new "CCD" era (Massey et al., 1986; Hodge and Lee, 1988; Cananzi, 1992; Magnier et al., 1992; Haiman et al., 1994a; Hunter et al., 1996). More recently the available spectra of the hot, luminous stars in M 31 combined with photometry allowed investigation of the internal reddening within the selected regions in the disk of M 31 (Humphreys et al., 1990; Bianchi et al., 1991; Hutchings et al., 1992; Herero et al., 1994; Massey et al., 1995).

In this work we employed a large sample of $O B$ stellar photometry combined with some Cepheids and novae to construct a more detail absorption map of M 31.

## 2 SAMPLES AND METHODS

We used blue stars from the Catalogue of the Brightest Stars in M 31 of Berkhujsen et al. (1988) and brightest blue stars from Nedialkov et al. (1989). Although these surveys suffered from incompleteness below limited magnitudes of $V=19 \mathrm{mag}$, they supplemented each other properly in the severely crowded regions of associations where the plate scale of the 2 m telescope of the Bulgarian NAO-Rozhen used in the later survey is more favourable than the plate scale of the Tautenburg-Smidth telescope used in the former one. In order to avoid the foreground contamination from the Milky Way only stars which lay below the reddening line for the B9I star $\left((U-B)_{0}=-0.5\right)$ were taken into account (Figure 1). Later on we excluded 15 more stars with luminosities greater than $M_{V}=-9$ which are obviously unresolved clusters due to severe crowding. Some stars with "negative" reddening were omitted


Figure 1 Two-colour diagram for the selected OB sipergiants lying above the reddening line for the B9I star $\left((U-B)_{0}<-0.5\right)$ in M 31. Two different slopes of the reddening line, 0.72 (short dashes) and 0.50 (long dashes) adopted in this paper, are indicated.
too and we attributed that to photometric errors. Our final list consists of 607 OB stars.

It was found that the UBV luminosity functions were practically the same as for the samples of blue stars selected according to different colour criteria used by Berkhuijsen and Humphreys (1989).

We applied the classical Q-method (FitzGerald, 1970) in two approaches. In the first one we dereddened all the stars to the zero reddening sequence for the $V$ luminosity class on the UBV two-colour diagram. We linearly extrapolated the dereddened main sequence to $(U-B)_{0}=-2.5$. Thus the stars lying over the reddening curve for the bluest $O$ stars were still left off the sample. Although they left in "normal" stars due to photometrical errors, they posses some reddening information. This way we determined the sum of the reddening in M 31 and in the Milky Way.

The second approach includes a preliminary correction of the colours for the foreground extinction from the Milky Way with a constant amount of $E(B-V)=0.08$ (Burstein and Heiles, 1984). Then the stars were dereddened to the I luminosity


Figure 2 Absorption from the Cepheids in M 31. The dashed line represents the $P-L$ relation in the B-band for the Cepheids in Baade's field IV (open squares). Open circles are 44 stars from Hubble's (1929) sample, closed triangles are three stars from Nedialkov et al. (1996). The horizontal line at $m_{\mathrm{Blim}}=22 \mathrm{mag}$ is the limiting magnitude of the samples used. $E(B-V)=0.5$ indicates the reddening of the heavy obscured Cepheids below the limiting magnitude.
class sequence on the two-colour diagram with a slope of the reddening line 0.50 (Massey, 1995). At the end we excluded all stars which lay either above the reddening curve for the OI star $\left((U-B)_{0}<-1.2\right)$ or down the reddening curve for the B9I star $\left((U-B)_{0}<-0.5\right)$. The final sample consists of 251 stars. Although the flatter reddening line allowed us to include stars which would have laid above the reddening line with a slope of 0.72 , a large percent of stars still had colours bluer than $(U-B)_{0}=-1.2$. Many stars lay between the sequences from I and V luminosity classes and therefore were omitted because of their "negative" reddening, assuming they were main sequence stars. Since it was found that the bulk of our sample consists of $O$ supergiants where reddening is indistinguishable from the reddening of O MS stars on the two-colour diagram, later on we preferred to work with the larger sample of 607 stars in order to have better statistics.

To study the absorption from the Cepheids we chose to use the sample of 44 stars by Hubble (1929) supplemented by three stars from our list (Nedialkov et al., 1996) rather than to use Cepheids in four of Baade's field which have already been


Figure 3 Absorption from the novae in M 31. Dashed line represents the S-shaped MMRD for the Galactic novae and novae in M 31. Open circles are novae from Capacciolli et al. (1989), filled circles are novae discovered by Sharov and Alksnis and triangles are novae from Sharov (1993) with redetermined rates of decline and maximum magnitudes.
studied previously (van Genderen, 1969a; van Genderen, 1969b; van Genderen, 1973). Some preliminary inspections of our compilation list showed that all the Cepheids possessed amplitudes greater than 0.8 mag in the B -band and their great offset from the $\mathrm{P}-\mathrm{L}$ relation are not due to intrinsic scatter (natural width of the P-L relation, different evolutionary stage and therefore different position in the instability strip, differential distance within M 31) and photometrical errors, but rather to absorption.

As a first step we corrected for the absorption the equation for the B-band $\mathrm{P}-\mathrm{L}$ relation in Baade's field IV:

$$
B=23.51-2.25 \log (P)
$$

with an amount of total absorption $A_{B}=0.53$ corresponding to $E(B-V)=0.16$ (Baade and Swope, 1963) rather than $E(B-V)=0.08$ (Burstein and Heiles, 1984). Then we determined the full absorption for each star as a difference between the observed magnitudes and the magnitudes predicted from the $\mathrm{P}-\mathrm{L}$ relation (Figure 2).

Our novae list includes 127 stars with well-defined rates of decline down to 2 magnitudes from the maximum. They come mainly from the compilation of Capacciolli et al. (1989) ( 105 stars) complemented with 22 novae discovered by Sharov and Alksnis (Sharov and Alksnis, 1991; Sharov, 1994; Sharov and Alksnis, 1994a; Sharov and Alksnis, 1994b; Sharov and Alksnis, 1996). We also used our plate archive to determine precisely the rate of decline for 2 novae discovered by Sharov (1993). In cases when it was necessary we converted $m_{\mathrm{pg}}$ to $B$ magnitudes using the transformation equation given by $\operatorname{Arp}$ (1956).

In our notation $M_{B}$ (max) corresponds to the luminosities corrected neither for the absorption from the Galaxy nor for the internal absorption in M 31. We define the absorption in the $B$ pass-band (Figure 3) as the difference between the predicted magnitude from the S-shaped MMRD relation (Figure 12 in Capaccioli et al., 1989) and the observed $M_{B}(\max )$. The intrinsic colour for the novae at the maximum was assumed to be $\left\langle(B-V)_{0}\right\rangle=0.15$ (Cohen, 1985).

We adopted the distance modulus to M 31 of $(m-M)_{0}=24.26 \mathrm{mag}$ (Welch et al., 1986) but preferred to use for scaling purposes a distance of 690 kpc or $(m-M)_{0}=24.19$ mag at which 1 arcmin corresponds to 200 pc .

The sequences on the two-colour diagram for bright supergiants (Ia) and MS stars were taken from Scmidth-Kaler (1982). We used the Galactic reddening low (Johnson and Morgan, 1955): $A_{B} / A_{V}=1.303, A_{U} / A_{V}=1.521$ leading to $E(U-$ $B) / E(B-V)=0.72$ and $A_{V} / E(B-V)=3.3$. The total, true extinction in magnitudes is proportional to the optical depth $A_{\lambda}=1.086 \tau_{\lambda}$. In order to study the radial distribution of the dust we converted the equatorial coordinates of the stars to rectangular $X Y$ coordinates assuming that the centre of M 31 lies at $\alpha_{1950}=$ $0^{h} 40^{m} 0.04^{s}, \delta_{1950}=40^{\circ} 59^{\prime} 43^{\prime \prime}$ (de Vaucouleurs and Leach, 1981). Then $X Y$ were transformed to rectangular coordinates along the major and minor axes assuming a position angle $P=38^{\circ}$ (Baade and Arp, 1964). To obtain the projected distances from the centre we rectified the plane of view with an inclination angle $12.5^{\circ}$ (Simien et al., 1978).

## 3 REDDENING

### 3.1 OB Supergiants

The main concentration of stars on the $(U-B)_{0}-M_{B}$ diagram is near $(-1.20$, -6.75 ) which means that the majority of the stars in our sample are O supergiants. The value of extinction in terms of $E(B-V)$ for each star was derived individually by means of the classical Q-method. This allows us to determine the intrinsic luminosity $M_{B}$ and colour $(U-B)_{0}$. The typical uncertainties were: $\sigma(E(B-V))=0.44$, $\sigma\left((U-B)_{0}\right)=0.46 \mathrm{mag}, \sigma\left(M_{B}\right)=1.45 \mathrm{mag}$ and they are due presumably to large photometrical errors associated mostly with the finite resolution of stellar images that could result in inaccurate colours because of contamination from neighbouring stars.

Table 1. Radial distributions of the mean excess $\langle E(B-V)\rangle$ from OB supergiants calculated by the two approaches described in the text. Bins with less than three stars are omitted

|  | Approach 1 <br> $\left\langle R^{\prime \prime}\right\rangle$ |  |  | $\langle E(B-V)\rangle$ | $\sigma\langle E(B-V)\rangle$ | $N$ |
| ---: | :---: | :---: | :---: | :---: | :---: | ---: |

We divided all the stars in our samples into 500 arcsec-wide bins along the radial distance from the centre (projected onto the disk) in order to study the radial distribution of the excesses. Table 1 represents the radial distribution of the mean excesses of OB supergiants calculated via the two approaches described in the previous section. The mean excess is nearly constant and is consistent within the errors: $0.51 \pm 0.27$ (approach 1) and $0.34 \pm 0.21$ (approach 2). The second mean excess reflects only the internal reddening within the disk of M 31 whereas the first one includes the Galactic reddening toward the Andromeda nebula. Thus an amount of 0.08 in the total difference of 0.17 between two numbers is accounted for. The rest could be attributed mainly to the different fitting methods of the observational data to the unreddened sequences on the colour-colour diagram. It was expected to obtain smaller reddenings when the stars were dereddened to the SG sequence because of its redder intrinsic colours relative to MS. But this difference is indistinguishable for O stars which dominate our sample (Figure 1).

The biggest difference for individual bins occurred in the most distant one from the centre and with the worst statistics. Besides, this bin presumably includes supergiants from Baade's field IV which is known to have unusually small reddening determined from MS stars (Baade and Swope, 1963; Humphreys, 1979; Hodge and Lee, 1988) and equal to our $E(B-V)=0.14$ from the second approach but from three stars only.

Our data yielded a small asymmetry in reddening of the supergiants: those in the eastern half are less reddened than in the western side by $\delta(E(B-V))=0.07$ mag.

Iye and Richter (1985) detected a difference in reddening along the minor axes from globular cluster colours. Later, Haiman et al. (1994b) determined a difference in $E(B-V)$ within associations in the western and the eastern spiral arms of M 31 ( 0.41 and 0.29 respectively). They suggested the higher extinction within the western spiral arm could be responsible for it.

Berkhujsen and Humphreys (1989) discovered a deficit of blue stars on the western side from star counts. They claimed that the difference was real but did not


Figure 4 The total number of neighbouring bins $N$ (top) and correlation coefficient (bottom) of the excesses in neighbouring bins as a function of the bin size.
invoke extinction to account for it. Their conclusion was supported by 100 micron IRAS observations which suggested that the extinction was constant (Walterbos and Schwering, 1987). Efremov et al. (1993) explained this asymmetry with a higher massive star formation rate within the fragment of spiral arm N4 between associations OB29 and OB48.

One of the main goals of this paper was to construct an extinction map from the reddening of the OB stars. The first step was to select a proper resolution (i.e. bin size) because it set the scale on which the values of the individual excesses were averaged. For this purpose we used bins with different widths and calculated the total number of neighbouring bins (neighbours are two bins with common side and each of them contains at least 1 star ) and the correlation coefficient between the mean excesses in the neighbouring bins. Both are shown in Figure 4 as functions of the bin size. Apparently a weak and almost constant correlation between the colour excesses ( $0.3-0.4$ ) exists for bin sizes lower than $60 \operatorname{arcsec}($ about 200 pc ). The maximum correlation of $0.43 \pm 0.07$ is reached at bin size 34 arcsec (about 110 pc ).


Figure 5 Extinction map of M 31. Excesses are averaged within bins with a size of 34 arcsec. The squares on the background indicate extinction with their size increasing for more opaque regions. The isophotes at $B=22,23$ and $24 \mathrm{mag} \operatorname{arcsec}^{-2}$ are indicated.

The small negative values for the correlation coefficient indicating anticorrelation correspond to a bin width of 175 arcsec (about 580 pc ). The maxima at 100,200 and 600 pc are connected with the scales of the stellar associations, aggregates and complexes (Ivanov, 1996). The corresponding extinction map with 34 arcec resolution is given in Figure 5.

We checked what type of population the stars from our sample belong to. It was found that about $3 / 4$ of them are members of the stellar associations as outlined by van den Berg (1964b). The rest could be considered field stars. Only less than $1 \%$ of all stars lay in dust cloud regions and we attributed this to high extinction.

About $80 \%$ (or 484 stars) of our sample lay in spiral arms. We found the same extinction values for $\langle E(B-V)\rangle=0.51$ for spiral arm members and for 123 field stars. The fraction of the stars in the spiral arms is even greater because it is difficult to depict thoroughly the two-armed structure over the real distributions of stars.

### 3.2 Monte Carlo Simulations

The Monte Carlo method allows us to simulate complicated multi-parameter samples of data and is a very powerful tool for both the photometrical and statistical approaches to our problem.

To transform the values of $E(B-V)$ in terms of the full vertical optical thickness of the disk $\tau_{B}$ one has to assume some hypothesis about the dust-to-star distribution. We used one-dimensional simple exponential laws to generate both stellar and dust distributions. Then we calculated the individual excesses of generated stars and compared their distribution with the observational excess distribution.

The two main parameters, namely $\tau_{B}$ and the scale factors ratio $Z_{D} / Z_{S}$, that describe our Monte Carlo simulations were determined for six different galactocentric distances $\langle R\rangle$ from 750 to $3250 \operatorname{arcsec} ; \tau_{B}$ decreased from 11.5 at the inner part of the disk to 2.0 at $0.62 R_{25}$ and $Z_{D} / Z_{S}$ was nearly constant (1.25). An acceptable logarithmic fit was found later for the optical thickness as a function of distance $R$ and led to the conclusion that the radial scale factor of the dust was 4.8 kpc . Finally, we obtained separately values for $Z_{D}=96 \mathrm{pc}$ and $Z_{S}=75 \mathrm{pc}$ adopting a two-armed trailing spiral structure and modeling the distribution of the stars over the body of the disk on a conical surface.

### 3.2.1 Monte Carlo Technique

Recently the Monte Carlo technique was used by Hatano et al. (1997a) to study the observability and spatial distribution of novae in M 31. They assumed a simple Milky Way-like model for the dust distribution in M 31 but with the density of the absorbing matter peaking at 8.8 kpc from the centre where most of the current star formation took place.

We chose to use a simple one-dimensional model to study separately the vertical and radial distribution of the absorbing matter. For that reason our model generates only the $Z$-coordinates for a given number of stars at a fixed galactocentric distance. The model assumes that both the stars and dust are distributed according to exponential laws with powers $-\operatorname{abs}(Z) / Z_{S}$ and $-\mathrm{abs}(Z) / Z_{D}$ respectively where $Z_{S}$ and $Z_{D}$ are the scale factors. We consider the radial distributions later.

Let $\tau_{B}$ be the full optical thickness of the disk in the $B$ passband perpendicular to the plane of the disk at fixed galactocentric distance and $A_{0}$ the corresponding absorption in the $B$ band for a star with coordinate $Z=0$. The light from a star with positive $Z$, i.e. from the near side of a face-on disk, should be dimmed as follows:

$$
A=A_{0} \exp \left(-Z / Z_{D}\right)
$$

and for a star with negative $Z$, i.e. in the opposite side of the disk:

$$
A=A_{0}\left(2-\exp \left(-\operatorname{abs}(Z) / Z_{D}\right)\right)
$$

If $a$ is the mean absorption in $B$ per kpc , then:

$$
1.086 \tau=A=2 A_{0}=\left(2 a Z_{D}\right) / \sin (i)
$$

Other parameters of the model are the total number of generated stars, inclination angle of the M 31 disk, distance modulus, foreground Galactic extinction, and the $A_{U} / A_{V}$ and $A_{B} / A_{V}$ ratios.

Examination of the radial distributions of $(U-B)_{0}$ and $M_{B}$ showed that these properties of OB stars are independent of the distance to the centre of M 31 which confirmed the homogeneity of our sample. Therefore the colours and luminosities of the generated stars were randomly distributed according to the colour and luminosity distributions of our total sample. The generated distributions had no added noise to account for the photometrical errors, because the observed distributions already contained that noise. This approach has the disadvantage that it misses some of the reddest stars that are strongly reddened and went below our observing limit but it is lessened by the fact that we actually compare the distributions of excesses rather than the colours. We need these initial distributions to exclude from our simulation all the synthetic stars which $U B V$ magnitudes out of the ranges of photometry in order to make a realistic comparison of the observed and generated distributions.

The major problems were the mixture of stars with different ages and circumstellar shells around some of the stars. The $U B V$ photometry alone cannot distinguish these cases.

We constructed a two-dimensional grid ( $Z_{D} / Z_{S}, \tau_{B}$ ) with a step 0.25 in $Z_{D} / Z_{S}$ over the interval $0.5-5.0$, and with a step 0.2 for $\tau_{B}$ between 0.0 and 2.0 and a step 0.5 between 2.0 and 30.0. The synthetic distribution depended mainly on these two parameters.

Then we applied the Kolmogorov-Smirnov test to verify the zero-hypothesis according to which the observed and the synthetic distributions of the excesses $W(B-V)$ were derived from different parental distributions and calculated the probability $P$ that the zero-hypothesis is wrong. We adopted the values of $Z_{D} / Z_{S}$ and $\tau_{B}$ which corresponded to the greatest $P$ over our grid. The results for each bin are summarized in Table 2. Because of the poor statistics we failed to obtain good fits for the most distant bins $R>3500 \operatorname{arcsec} \cong 12 \mathrm{kpc}$.

The uncertainties of the determined $Z_{D} / Z_{S}$ and $\tau_{B}$ were equal to the step of the $\operatorname{grid}\left(Z_{D} / Z_{S}, \tau_{B}\right)$ in all bins except No. 5 . Here we have a tail of the distribution of calculated excesses which is hard to approximate with a distribution generated

Table 2. Radial distributions of derived $\tau_{B}$ and $Z_{D} / Z_{S}$, Kolmogorov-Smirnov statistics and the probability $P$ that the null-hypothesis is false

| $\left\langle R^{\prime \prime}\right\rangle$ | $N$ observed | $N$ synthetic | $\tau_{B}$ | $Z_{D} / Z_{S}$ | $K S$ stat | $P$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 750 | 87 | 350 | 11.5 | 1.00 | 0.08 | 0.74 |
| 1250 | 93 | 535 | 7.5 | 1.00 | 0.09 | 0.55 |
| 1750 | 93 | 204 | 6.0 | 1.25 | 0.08 | 0.75 |
| 2250 | 87 | 171 | 5.0 | 1.50 | 0.10 | 0.72 |
| 2750 | 102 | 406 | 2.5 | 1.25 | 0.16 | 0.12 |
| 3250 | 60 | 297 | 2.0 | 1.50 | 0.07 | 0.98 |



Figure 6 Comparison of the true (solid lines) and synthetic (dashed lines) excess distributions in bins No. 1, 5 and 6 as indicated.
from a simple exponential model in the $Z$-direction. Thus the error 1.0 in $\tau_{B}$ is adopted for this bin. Figure 6 presents the distributions in bins No. 1, No. 5 and No. 6 where the extreme values of $\tau_{B}$ and $P$ were obtained.

A comparison of the observed and predicted distributions of stellar excesses for bin No. 1 is shown in Figure 7. The dust-to-star scale height ratio changes the average extinction, shifting the peak of the distribution (top panel). Small $Z_{D} / Z_{S}$ ratios put the absorbing material in a narrow string in the disk plane leaving most of the stars above it and unreddened. $Z_{D} / Z_{S}=1.0$ represents an even mix of dust and stars. Most of the stars are more or less embedded into dust at larger $Z_{D} / Z_{S}$ ratios while $Z_{D} / Z_{S} \rightarrow \infty$ is equivalent to a foreground absorbing sheet. Out best fit is $Z_{D} / Z_{S}=1.0$. Changes in the total optical thickness of the disk at a fixed scale height ratio are reflected mostly by the width of the distribution. A higher thickness produces a tail toward large $\langle E(B-V)\rangle$. Our best fit for this bin is $\tau_{B}=11.5$.


Figure 7 Comparison of observed (filled quadrates) and predicted (solid and dashed lines) distributions of stellar excesses for bin No. 1 for different values of the parameters $Z_{D} / Z_{S}$ and $\tau_{B}$.

### 3.2.2 Monte Carlo Results

Using procedures described in the previous section we obtained that $\tau_{B}$ decreases outwards while the $Z_{D} / Z_{S}$ is nearly constant: $\left\langle Z_{D} / Z_{S}\right\rangle=1.25$. A logarithmic fit of $\tau_{B} / 2$ is presented in Figure 8. It allowed us to determine the radial scale length of the dust: $R_{D}=4.8 \pm 0.4 \mathrm{kpc}$.

So far we determined the ratio of $Z_{D} / Z_{S}$. The next step was to determine $Z_{D}$ and $Z_{S}$ independently of each other. We estimated the average $Z$ coordinate $\langle Z\rangle$ of the observable stars generated from the Monte Carlo technique for each bin for three different values of $Z_{S}=50,100$ and 200 pc and using the ratio $Z_{D} / Z_{S}$ as determined earlier.

The linear fit of $\langle Z\rangle$ versus radius leaded to the conclusion that the stars are located in a protruding surface with $\langle Z\rangle$ greater near the galactic centre and decreasing outward. A conical surface is the simplest approximation. It could be described by the radius $R_{0}$ of the cone base and the angle $\beta$ between the base plane


Figure 8 Fit of the radial distribution of $\log \left(\tau_{B} / 2\right)$ versus radius $R$.
and the surface of the cone. iaking into account the uncertainties of the slope we obtained $R_{0}=8038 \pm 1364$ arcsec or $26.7 \pm 4.5 \mathrm{kpc}$ at the distance of M 31 which was close to the physical size of the disk. We correlated the dependence of $Z_{S}$ on $\beta$ using the slope of the relation $\langle Z\rangle$ - galactocentric radius and obtained a linear dependence $Z_{S}=36.2 \beta \pm 5.1 \mathrm{pc}$. Hence, $Z_{S}$ and $Z_{D}$ could be determined from $\beta$.

To determine $\beta$ we tried to fit the distribution of OB stars on the view plane by the two-armed trailing spiral pattern of Arp (1964) assuming $R_{0}=8038 \operatorname{arcsec}$, $i=12.5^{\circ}$ and varying $\beta$. The arms of M 31 could not be fitted a simple perfect logarithmic spiral pattern with a single $\beta$. Instead, the different fragments of the arms were fitted with different values of $\beta$ (Figure 9). The southern half was more disturbed probably due to the close companionship of M 32 .

We weighted the set of values of $\beta$ according to the length of the approximated spiral pattern in the rectified plane and calculated a mean $\beta=2.1 \pm 1.0 \mathrm{deg}$. It was not affected strongly by $R_{0}$ and the inclination angle $i$. We determined $\left\langle Z_{S}\right\rangle=76 \pm 36 \mathrm{pc}$ and $\left\langle Z_{D}\right\rangle=95 \pm 45 \mathrm{pc}$ assuming $\left\langle Z_{D} / Z_{S}\right\rangle=1.25$.


Figure 9 Best fits of the arms' fragments (solid lines) to a logarithmic two-armed spiral pattern of M 31 (Arp, 1964) modified for the angle $\beta=2.1^{\circ}$ (dashed line). The values of the individual angles $\beta$ in degrees are indicated for each fragment.

In our interpretation the fit of the one-armed leading spiral pattern (Simien et al., 1978) was an artifact of the modification of the spiral arms tracers' distribution caused by the curvature of the plane on which they were located. This yielded a pitch angle of 4.4 deg , unusual for an Sb type galaxy. Nevertheless, a solution with $\beta=1.0 \mathrm{deg}$ is still acceptable at the one sigma level.

Braun (1991) also showed that the spatial distribution of the gas in M 31 is very well described by a global two arm trailing spiral with a pitch angle of 6.7 deg over the galactocentric distances which were studied in this paper.

The average $Z$-coordinates could vary with the galactocentric distance because of the absorption and the curvature of the galactic plane as in the case of M 31 where we had both. Hence the apparent inclination to the line of sight changes. A similar variation of inclination angle with amplitude, 5.1 deg , is reported by Georgiev (1988). Figure 9 confirms the significant variations of the local orientation which is an indication that stars with a different $z$ are projected on the apparent observational plane.

### 3.3 The $N(H) / A_{V}$ Ratio

We consider three zones: inner disk ( $1.7<R<3.4$ ), central zone $(2.5<R<8)$ and H I ring $(8<R<11)$ to study the consistency of the $\mathrm{N}(\mathrm{H}) / A_{V}$ ratio with the ratio of the surface mass density ( $\Sigma, M_{\odot} \mathrm{pc}^{-2}$ )-to- $A_{V}$ (Table 3).

It is impossible to make a direct estimate of the $\langle E(B-V)\rangle / I(\mathrm{CO})$ ratio because of the poor statistics. Indeed, only 2 out of 26 observed point from Loinard et al. (1995) coincide with points on our 34 arcsec resolution extinction map. The mean intensity of the detected CO emission in the spiral arm regions is slightly higher than within the H I hole. A comparison of the coincidence between the regions of CO emission along the major axis with the spiral structure of M 31 showed that the mean filling factors for spiral arms is about 0.7 while the mean suggested by Loinard et al. (1995) for the whole central region is 0.6 . Both these effects would increase

Table 3. Derived $\mathrm{N}(\mathrm{H}) / A_{V}$ ratios in central zone, the H I ring and inner disk

| Zone kpc | $\underset{M_{\odot} p c^{-2}}{\Sigma}$ | $\begin{gathered} N(H) \\ 10^{21} \text { at } \mathrm{cm}^{-2} \end{gathered}$ | $\left\langle\tau_{B}\right\rangle$ | $\begin{gathered} \left\langle A_{V}\right\rangle \\ \mathrm{mag} \end{gathered}$ | Source | $\begin{aligned} & N\left(H I+2 H_{2}\right) /\left\langle A_{V}\right\rangle \\ & 10^{21} \text { at } \mathrm{cm}^{-2} \mathrm{mag}^{-1} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.5-8 | 10 | 1.25 | 7.4 | 6.2 | Lionard et al. (1995) | 0.2 |
| 8-11 | 4.2 | 0.53 | 2.2 | 1.7 | Dame et al. (1993) | 0.3 |
| 1.7-3.4 | 20 | 2.50 | 11.5 | 9.6 | this work | 0.3 |

the mean surface mass density at the centre up to $12 M_{\odot} \mathrm{pc}^{-2}$ and respectively to $\mathrm{N}\left(2 \mathrm{H}_{2}\right) / A_{V}=0.3 \times 10^{21}$ at $\mathrm{cm}^{-2} \mathrm{mag}^{-1}$. In the inner disk zone where $\tau_{B}$ is 11.5 (bin No. 1) the mean filling factor and the mean intensity of the detected CO emission are respectively 1.2 and 1.7 times greater than the same factors for the whole central zone. Thus the surface density in the inner disk increases above $20 M_{\odot} \mathrm{pc}^{-2}$ and yields the similar $\mathrm{N}(\mathrm{H}) / A_{V}=0.27 \times 10^{21}$ ratio.

To determine the $\mathrm{N}(\mathrm{HI}) / E(B-V)$ ratio in the ring $8<R<11 \mathrm{kpc}$, where most of the star formation in M 31 takes place (Magnier et al., 1993) and the hydrogen is predominantly in form of H I (Brinks and Shine, 1984) independently of the above estimations, we superimposed a smoothed version of our extinction map and the H I 21 cm line intensity maps of Unwin (1980a). Our excesses were averaged within $1^{\prime} \times 1^{\prime}$ bins which corresponded to the H I map resolution $\left(0.8^{\prime} \times 1.2^{\prime}\right)$.

Figure 10 shows the average extinction as a function of the H I 21 cm line intensity. Over the first five bins a linear regression was used to determine a slope of $(0.8 \pm 0.2) 10^{-3} \mathrm{~K} \mathrm{~km} \mathrm{~s}^{-1} \mathrm{mag}^{-1}$. The intercept of the regression and $\langle E(B-V)\rangle$ axis is $(0.11 \pm 0.12)$ which includes the expected value of 0.08 (Burstein and Heiles, 1984) for the extinction toward M 31. The H I maps of Unwin (1980a) are given with isolines equally spaced at $600 \mathrm{~K} \mathrm{~km} \mathrm{~s}^{-1}$ and we averaged the values of the H I surface density in our bins which produced intermediate values. For intensities greater than $600 \mathrm{~K} \mathrm{~km} \mathrm{~s}^{-1}$ the extinction reaches saturation which indicates high opacities in the H I zone.

The H I intensity-to-surface number density ratio $\mathrm{N}(\mathrm{H} \mathrm{I}) / \mathrm{I}(\mathrm{H} \mathrm{I})=1.823 \times$ $10^{18}$ at $\mathrm{cm}^{-2} \mathrm{~K}^{-1} \mathrm{~km} \mathrm{~s}^{-1}$ (Unwin, 1980b) yields $\mathrm{N}(\mathrm{H} \mathrm{I}) / A_{V}=(3.4 \pm 1.6) \times$ $10^{20}$ at $\mathrm{cm}^{-2} \mathrm{mag}^{-1}$. Here we added a factor of 2 to account for the fact that $\langle E(B-V)\rangle$ reflects the half-disk thickness. Using the obtained ratios $\mathrm{N}(\mathrm{HI}) / A_{V}=$ $0.34 \times 10^{21}{\text { at } \mathrm{cm}^{-2}}^{\mathrm{mag}}{ }^{-1}$ and $\mathrm{N}(\mathrm{HI}) / \mathrm{I}(\mathrm{HI})=1.823 \times 10^{18}$ we were able to estimate the vertical optical thickness of the disk at the peak of the H I density distribution which is at about $3000 \mathrm{~K} \mathrm{~km} \mathrm{~s}^{-1}=5.5 \times 10^{21}$ at $\mathrm{cm}^{-2}$. We applied the inclination correction ( $i=12.5^{\circ}$ ) and obtained a column density perpendicular to the plane of the disk of $11 \times 10^{20}$ at $\mathrm{cm}^{-2}$, close to the extreme value of $8 \times 10^{20}$ at $\mathrm{cm}^{-2}$ (Sofue and Kato, 1981) for the ridge of one spiral arm. Such densities indicate a full optical thickness at $r=10 \mathrm{kpc}$ from the centre equal to $A_{V}=2-3 \mathrm{mag}$, in good agreement with our Monte Carlo simulations. This confirms the self-consistency of our $\mathrm{N}(\mathrm{H}) / A_{V}$ ratio estimates.

We compared the obtained ratio $\mathrm{N}(\mathrm{H}) / A_{V}=0.3 \times 10^{21}$ at $\mathrm{cm}^{-2} \mathrm{mag}^{-1}$ which was independent of the projected distance from the M 31 centre to the estimates


Figure 10 Linear fit of mean $E(B-V)$ versus H 121 cm intensity data.
collected from the literature. There is an ongoing debate on the value of this ratio. For the solar neighbourhood ( $1-2 \mathrm{kpc}$ ) Bohlin et al. (1978) suggested $\mathrm{N}(\mathrm{H}) / A_{V}=$ $1.9 \times 10^{21} \mathrm{at} \mathrm{cm}^{-2} \mathrm{mag}^{-1}$. Diplas and Savage (1994) found that neutral hydrogen and dust are well correlated with $\mathrm{N}(\mathrm{H} \mathrm{I}) / A_{V}=1.5 \times 10^{21} \mathrm{at} \mathrm{cm}^{-2} \mathrm{mag}^{-1}$ (assuming $A_{V} / E(B-V)=3.3$ ), five times higher than our result. They both employed UV observations of $L_{\alpha}$ toward nearby Milky Way stars. In contrast, Kumar (1979) used M 31 H II regions to obtain $\mathrm{N}(\mathrm{H} \mathrm{I}) / A_{V}=0.5 \times 10^{21}$ at $\mathrm{cm}^{-2} \mathrm{mag}^{-1}$ for $R=4-$ 8 kpc , and $\mathrm{N}(\mathrm{HI}) / A_{V}=1.0 \times 10^{21}$ at $\mathrm{cm}^{-2} \mathrm{mag}^{-1}$ for $R=10-12 \mathrm{kpc}$ in better agreement with our value. There are a number of factors that can account for this discrepancy.
(1) A direct correlation is assumed to exist between the number densities of dust grains and hydrogen atoms. Then the $\mathrm{N}(\mathrm{H}) / A_{V}$ ratio which depends on the value of $A_{V} / E(B-V)$ is 3.1 in the diffuse interstellar medium, and 5.3 in dense cloud regions (Kim et al., 1994). The coefficient $\mathrm{N}(\mathrm{H} \mathrm{I}) / A_{V}=$ $1.9 \times 10^{21}$ at $\mathrm{cm}^{-2} \mathrm{mag}^{-1}$ (Bohlin et al., 1978) is applicable to diffuse clouds
but it is probably reduced (Greenberg and Li , 1995) in dense clouds of the spiral arms where $A_{V} / E(B-V)>3.1$ indicates that the particles are larger. Since the extinction depends both on the number of particles and their area, and since extinction per particle increases with size, the coefficient has to be lower in dense than in diffuse clouds. When diffuse clouds enter the potential minimum at the inner edge of the spiral arm the compression leads to coagulation and growth into giant molecular clouds strongly concentrated in spiral arms. As a result some dissipation occurs during the passage through the arm (Roberts et al., 1990). When the interstellar clouds emerge at the trailing edge of the arm they are dissipated and dispersed and the major fraction of the dust transforms back into diffuse cloud-type. Hence, one can expect to find in the spiral structure a complicated mixture of different ISM phases. The effects of stars and dust geometry were not considered in our $\mathrm{N}(\mathrm{H}) / A_{V}$ estimates. The usefulness of the predicted disk opacities is always compromised by the uncertainty of the actual distribution. Bohlin et al. (1978) looked at stars in the plane of the Milky Way which increased the probability of having more molecular clouds along the line of sight than in M 31 where the line of sight traverses the disk at some angle. This can lead to a decrease of the $\mathrm{N}(\mathrm{H}) / A_{V}$ ratio below $1.9 \times 10^{21}$ at $\mathrm{cm}^{-2} \mathrm{mag}^{-1}$ in the denser clouds in the spiral arms due to the variations of $A_{V} / E(B-V)$ described above.
(2) The environmental conditions in the ISM vary, which leads to uncertainties in the relative fractions of $\mathrm{H}_{2}, \mathrm{HI}$, and H II and the amounts of dust associated with them. Efremov et al. (1993) showed that the great majority of the bluest stars in M 31 are well mixed with H II regions. In particular we have used the same OB stars to derive the extinction and the dust-to-gas ratio in their vicinity changes a lot. For instance, Diplas and Savage (1994) claimed a $17 \%$ decrease of $\mathrm{N}(\mathrm{H}) / A_{V}$ for O stars compared to B stars. Deveruex et al. (1994) demonstrated a striking correspondence between the far infrared and $\mathrm{H}_{\alpha}$ morphology of M 31 indicating that the OB stars were the dominant hydrogen ionization source instead of the general interstellar radiation field as previously thought (Walterbos and Swering, 1987). They concluded that IRAS observations do not reflect the true disks opacities.
(3) The molecular hydrogen-to-CO intensity conversion factor $\mathrm{N}\left(2 \mathrm{H}_{2}\right) / \mathrm{I}(\mathrm{CO})$ is a function of metallicity despite the popular usage of the constant ratio for galaxies with known different metal abundances. Indeed, the conversion factor $\mathrm{N}\left(2 \mathrm{H}_{2}\right) / \mathrm{I}(\mathrm{CO})$ in highly metal abundant systems is 6 to 20 times smaller than in low metallicity ones (Rubin et al., 1991). The higher metallicity is expected to decrease the $\mathrm{N}(\mathrm{H}) / A_{V}$ ratio because the high metal content allows more iron and silicates to be converted into grains. In addition, M 31 has a radial metallicity gradient, although not as well established as in the Milky Way and M 33 (Issa et al., 1990). Consequently, the conversion factor can vary with the distance from the centre.

### 3.4 Cepheids and Novae

We derived the mean excesses from $E(B-V)=A_{B} / 4.3$ and their distributions are shown in Table 4. There is almost no radial gradient. The mean excesses from Cepheids of $0.21 \pm 0.09 \mathrm{mag}$ and from novae of $0.14 \pm 0.10 \mathrm{mag}$ were systematically lower than the mean values obtained from OB stars. We attributed these differences to selection effects - it was hard to obtain the declining rates for the most heavily reddened objects near the limiting magnitude (Figure 2, Figure 3).

Table 4. Radial distributions of the mean excess $\langle E(B-V)\rangle$ from Cepheids and novae calculated on the basis of the P-L and MMRD relations. Bins with less than three stars are omitted

|  |  | Cepheids <br> $\left\langle R^{\prime \prime}\right\rangle$ |  | $\langle E(B-V)\rangle$ | $\sigma\langle E(B-V)\rangle$ | $N$ |
| ---: | :---: | :---: | :---: | :---: | :---: | ---: |

In addition, the values of $E(B-V)$ for Cepheids and the nearest (distance less than 60 arcsec) OB stars were compared for 21 Cepheids located mainly in spiral arm S4. No systematic difference occurred. A similar test is harder to make for novae because they are poorly presented in the disk just opposite to the OB stars which are concentrated there.

## 4 DISCUSSION

The highly variable dust density in the disk of M 31 indicated by the clumpy structure of the dust clouds could be confirmed by the large variance of star to star reddening (Hodge, 1992). Haiman et al. (1994b) considered the reddening variations within associations to be the reason for the observed widening of the MS.

Figure 11 summarizes the results of different authors for the colour excesses of the stars in the disk as a function of the galactocentric distance. Although the discrepancies could be explained by the photometrical errors or different methods used to obtain the values of the mean excesses, there is a clear tendency that the most luminous objects (OB supergiants, H II regions) have greater mean excesses in comparison with less luminous MS stars. Our Monte Carlo simulations demonstrated that in optically thick ( $\tau_{B}>1$ ) regions such as the spiral arms, the OB supergiants should yield greater absorption because due to their higher luminosity they have


Figure 11 Colour excesses as a function of the galactocentric distance. Symbols denote: closed circles - this paper, closed squares - Kumar (1979), open five point star - Ivanov and Golev (1984), $\times$ 's - Kurtev and Ivanov (1984), tripoint stars - Massey et al. (1986), open triangles - Odewan (1987), open diamonds - Hodge and Lee (1988), open circles - Tasheva (1991), crosses - Haiman et al. (1994b), closed triangles - Hunter (1994), open squares - Cananzi (1992):
been observed much deeper into the disk. In the optically thick case the mean excess is not indicative for the total thickness of the disk. This is the main reason why van Genderen (1973) obtained a smaller optical half-thickness ( $\tau_{B} / 2=0.2-$ 0.1 ) based on Cepheids in Baade's four variable-star fields. He directly converted the excesses (e.g. absorption) to optical thickness without a detailed consideration of the vertical dust and star distribution in the disk. This procedure assumed an optically thin disk. We have good agreement in the outher parts (at $R=5760^{\prime \prime}$, Baade field IV) where according to our extrapolated fit we inferred an optically thin disk: $\tau_{B} / 2=0.17$ which is very close to the value of van Genderen (1973): 0.10 .

The mean excesses of novae indicated that we observed only stars with small absorption lying in front of the equatorial plane of the galaxy. A lot of novae progenitors can be expected among the low mass stellar population with vertical
scale length $Z_{S}$ much larger than that of OB supergiants. According to Cowie and Songalia (1986) the local cool gas and dust have a vertical scale length of 135 pc , and the stars of 325 pc (Gilmore and Reid, 1983). Thus, the novae should be less affected by absorption from the dust in the disk.

The logarithmic fit of $\tau_{B} / 2$ as a function of $R$ puts under question the statement about the density peak of the dust at 9 kpc (Hodge, 1992). According to our results, the dust density in M 31 spiral arms is distributed exponentially along the $R$ and $Z$ axes just as the dust density in the Galaxy and with nearly identical values for the radial ( 5 kpc ) and vertical ( 0.1 kpc ) scale factors (Dawson and Johnson, 1994).

The mean $Z_{D} / Z_{S}=1.25 \pm 0.25$ confirmed the assumption of Wainscoat et al. (1992) that the ratio of the vertical scale factors of the dust and the stars should be close to 1 for the early-type stars.

The linear fit shown in Figure 9 allows us to obtain $\tau_{B} / 2$ at the distances along major axis corresponding to different isophotal levels in the $B$ passband (Table 5). Although there were stars distributed out to $\mu_{B}=24 \mathrm{mag} \operatorname{arcsec}^{-2}$, it was possible to carry out Monte Carlo simulations only within $\mu_{B}=23.5 \mathrm{mag} \operatorname{arcsec}{ }^{-2}$ $\left(0.62 R_{25}\right)$ because of the poor statistics in the outermost regions. Extrapolating the exponential fit outward we found that the disk should become optically thin ( $\tau_{B}<1$ ) out of $R>R_{24}=0.78 R_{25}$. Earlier White and Keel (1992) studied pairs of overlapping galaxies and found that the absorption was strongly concentrated in the spiral arms with values of $\tau_{B}=0.3-1.1$ at $0.5-0.85 R_{25}$.

High values of $\tau_{B}$ in the spiral arms within $R=0.78 R_{25}$ seems to confirm Valentijn's (1990) conclusion about high opacities of spiral disks and his remark that optical astronomers may be seeing "only the upper crust of spiral galaxies" within the effective radius. Contrary to Valentijn's (1994) result who insisted Sb spirals were opaque over all their disks, we found that M 31's disk should become optically thin in the outer parts.

The optical depth ( $\tau_{B}=11.5$ ) of the M 31 disk in the spiral arms near the bulge is typical compared to the observed central optical depths of eight spiral galaxies ( $\tau_{B}=2-25$ ) as reported by Phillips et al. (1991) and Evans (1993).

It is possible to extrapolate the exponential fit to the centre although it seems rather unrealistic because according to Simien et al. (1993) there is no evidence that a continuous and strongly absorbing layer exists in the inner bulge of M 31 ( $R<180^{\prime \prime}$ ). The mean value of $\tau_{B}$ within the isophotal level $\mu_{B}=23 \mathrm{mag} \operatorname{arcsec}^{-2}$

Table 5. Values of the half optical thickness of the disk at the different levels in the $B$ passband

| $\mu_{B}, \operatorname{mag} \operatorname{arcsec}^{-2}$ | $R^{\prime \prime}$ | $\tau_{B} / 2$ |
| :---: | :---: | :---: |
| 22 | 1320 | 3.90 |
| 23 | 2940 | 1.26 |
| 24 | 4380 | 0.50 |
| 25 | 5640 | 0.20 |
| 26 | 6780 | 0.01 |

should be between 4 (with a cutoff at $R=500^{\prime \prime}$ ) and 9 (cutoff at $R=0^{\prime \prime}$ ) according to the extrapolation toward the centre.

This result is in very good agreement with a recent paper of Beckman et al. (1996) who used multicolour surface brightness distributions of three nearby faceon spiral galaxies and modelled the distributions of stars and dust to obtain their scale factors and the mean full optical thickness of the disks. They claimed $\tau_{V}=8$ 10 in the spiral arms. Kuchinski and Turndrup (1996) deduced similar values of the optical depth ( $\tau_{V}=4-15$ ) from JHK surface photometry of 15 late-type spirals.

Boselli and Gavazzi (1994) showed that the sandwich model of Disney et al. (1989) applied to a sample of $\mathrm{Sa}-\mathrm{Sc}$ galaxies led to the conclusion that the mean internal absorption (averaged over the whole disk and not relative to a definite isophote) is $\tau_{B}=1.3-2.3$.

The dust causes a variation of the radial scale factor with wavelength (Walterbos and Kennicutt, 1988) and from the $B$ to the $K$ band it decreases by about $30 \%$. We determined the radial scale factor for the dust as $4.8 \pm 0.4 \mathrm{kpc}$ which is noticeably smaller than their scale factor in the B-band ( $6.1 \pm 0.3 \mathrm{kpc}$ ). Therefore optically thin peripheries even in the spiral arm regions would occur when the dust optical depth for a given wavelength decreases enough.

We considered the position in M 31 analogous to the place of the Sun in the Milky Way. It is at $5000^{\prime \prime}=16.7 \mathrm{kpc}$ from the centre of M 31 . The total vertical optical thickness according to our model is $\tau_{B}=0.6$, close to 0.94 for the Galaxy. Mean values for the absorption per unit kpc in the radial direction in the $B$ band at that galactocentric distance are between $1.2 \mathrm{mag} \mathrm{kpc}^{-1}$ for dust density equal to zero for $R<500$ arcsec and $2.6 \mathrm{mag} \mathrm{kpc}^{-1}$ for an extrapolated exponential fit to the centre. In the vertical direction the absorption per unit distance is $3.0 \mathrm{mag} \mathrm{kpc}^{-1}$, close to $1.9 \mathrm{mag} \mathrm{kpc}^{-1}$ for the Galaxy (Hatano et al., 1997b).

Since the dust and gas are related, their scale factors must be similar. We obtain $Z_{D}=95 \pm 45 \mathrm{pc}$ in good agreement with the scale height of the gas $Z_{G}=120 \mathrm{pc}$ from Brinks and Shane (1984).

## 5 CONCLUSIONS

We can draw the following conclusions:
(1) The disk of M 31 in the spiral arm regions is optically thick $\left(\tau_{B}>1\right)$ within $\mu_{B}=24 \mathrm{mag} \operatorname{arcsec}^{-2}=0.78 R_{25}$. The radial dependence of the opacity modifies the distribution of visible OB stars increasing their $\langle Z\rangle$ coordinates closer to the centre. Thus the plane of the galaxy appears curved and can be roughly approximated by a conical surface.
(2) Our Monte Carlo simulations show that the total optical thickness of the disk in spiral arm regions is $\tau_{B}=11.5$ near the bulge and decreases to $\tau_{B}=2.0$ in the outer parts $\left(R / R_{25} \geq 0.62\right)$ which is the limit of application of this technique. The dust distribution in M 31 resembles that in the Galaxy with
a radial scale factor $R_{D}=4.8 \pm 0.5 \mathrm{kpc}$ and vertical scale factor $Z_{D}=$ $95 \pm 41 \mathrm{kpc}$. The vertical scale factor for OB supergiants is $Z_{S}=76 \pm 35 \mathrm{pc}$ with a mean value of the relative scale factor of $Z_{D} / Z_{S}=1.25$.
(3) The mean colour excesses are: $E(B-V)=0.51 \pm 0.27$ from the brightest OB stars, $E(B-V)=0.21 \pm 0.09$ from Cepheids and $E(B-V)=0.14 \pm 0.10$ from novae. They are not an indicator of the total optical thickness of an optically thick disk. Instead, the distributions of the excesses of $O B$ supergiants could be successfully approximated with the synthetic distributions from Monte Carlo simulations. The later distributions depend on two main parameters the full vertical optical thickness of the disk $\tau_{B}$ and the relative vertical scale height of the dust and stars $Z_{D} / Z_{S}$. The differences of the above mentioned excesses are due mainly to the different luminosities of the probing objects and the limiting magnitudes of the samples used.
(4) The correlations between the colour excesses found for different bin sizes correspond to the scales of the stellar associations (maximum at $34 \operatorname{arcsec}=110 \mathrm{pc}$ ), aggregates (a moderate correlation for bin sizes lower than $60 \operatorname{arcsec}=200 \mathrm{pc}$ ) and complexes (small negative correlation at bin size 175 arcsec $=580 \mathrm{pc}$ ). Hence, we confirm the result of Haiman et al. (1994b) that extinction seems to vary randomly from one association to another, and from group to group. The coincidence between the scales of these maxima with the scales of the stellar associations, aggregates and complexes is an independent indication of the existence of these structures in M 31.
(5) A comparison of our extinction estimates with the H I column density yields a $\mathrm{N}(\mathrm{H}) / A_{V}$ ratio of $\mathrm{N}(\mathrm{H}) / A_{V}=0.3 \times 10^{21} \mathrm{at} \mathrm{cm}^{-2} \mathrm{mag}^{-1}$. This is somewhat lower than UV observations of Ly absorption toward nearby stars due to geometrical effects and environmental variations in the ISM.

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