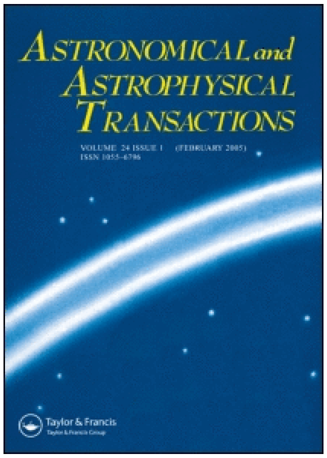

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THE QUANTUM EARLY UNIVERSE

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From the viewpoint of the modern cosmological concepts, the de Sitter vacuum is believed to be the inital stage of evolution of the Universe [1, 2]. The de Sitter vacuum decays into expanding matter called the Friedmann world where we live. There arises a question: what was before the de Sitter stage? The Universe is considered to be born from this vacuum by tunnelling through a certain potential barrier from the pre-de-Sitter domain to the de Sitter one [3, 4, 5, 6, 7, 8, 9].

The problem is formulated as follows. From the Einstein equations for a homogeneous isotropic universe [8, 9]

$$\frac{\dot{a}^2}{2} - \frac{4\pi G\varepsilon}{3c^2}a^2 = -\frac{kc^2}{2}, k = 0, \pm 1 \tag{1}$$

$$\ddot{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3p)a \tag{2}$$

for the equation of state

$$p = \alpha\varepsilon \tag{3}$$

we obtain for any k

$$\dot{\varepsilon}a + 3(1+\alpha)\dot{a}\varepsilon = 0, \tag{4}$$

which gives

$$\varepsilon = \varepsilon_{\rm pl}(a/l_{\rm pl})^{-3(1+\alpha)}, \tag{5}$$

where $\alpha = -1$, $\varepsilon = \varepsilon_{\rm pl}$ corresponds to the de Sitter vacuum.

The de Sitter vacuum has Planckian parameters. Really, the cosmological constant

$$\Lambda = 3H^2/c^2 = 8\pi G\varepsilon/c^4 \tag{6}$$

expressed in terms of the black-body radiation energy density at the de Sitter horizon ($R = c/H$)

$$\varepsilon = 4\sigma\Theta^4/c, \tag{7}$$

where $\sigma = \pi^2/60\hbar^3c^2$, with the effective temperature [10]

$$\Theta = \hbar h/2\pi, \tag{8}$$

allows the Hubble constant to be easily calculated as follows:

$$H = 3\sqrt{10\pi}/t_{\rm pl}. \tag{9}$$

The Planckian parameters are

$$t_{\rm pl} = \sqrt{\hbar G/c^5},\ l_{\rm pl} = \sqrt{\hbar G/c^3},\ m_{\rm pl} = \sqrt{\hbar c/G},\ \varepsilon_{\rm pl} = c^7/\hbar G^2. \tag{10}$$

Equation (1) is representable in the form [8]

$$(da/d\eta)^2 = H^2a^4/c^2 - ka^2, \tag{11}$$

where η is a conformal time defined as

$$c\,dt = a\,d\eta. \tag{12}$$

The Lagrangian is

$$L = (da/d\eta)^2 + H^2a^4/c^2 - ka^2. \tag{13}$$

The generalized momentum is given by the formula

$$P = \partial L/\partial(da/d\eta) = da/d\eta. \tag{14}$$

Replacing the generalized momentum by the corresponding quantum operator

$$\hat{P} = (1/i)l_{\rm pl}^2(d/da), \tag{15}$$

we reduce (11) to the Wheeler–DeWitt Equation (6)

$$d^2\psi/da^2 - V(a)\psi = 0, \tag{16}$$

where

$$V(a) = l_{pl}^{-4}(ka^2 - H^2a^4/c^2) \tag{17}$$

is the potential derived by A. Vilenkin for $\varepsilon = \text{const}$, which is in agreement with the Higgs potential describing spontaneous symmetry breaking [9]

$$W(\varphi) = -a\varphi^2 + b\varphi^4, \tag{18}$$

where $a > 0$, $b > 0$ is a scalar field. $V(a) = -W(\varphi)$ for $\varphi \sim a$ (a field inside the ball).

Generalize the Vilenkin potential by considering the Universe energy density to be a superpositon of energy densities of various kinds of matter including vacuum-like ones ($\alpha < 0$). Supposing coexistence of various kinds of matter at the Planckian densities, we have

$$H^2 = (8\pi G\varepsilon_{\rm pl}/3c^2)\sum_{n=0}^{6} B_n(l_{\rm pl}/a)^n, \tag{19}$$

where $n = 3(1+\alpha)$, B_n are constants, $(8\pi G\varepsilon_{\rm pl}/3c^2) = (c/l_{\rm pl})^2$.

The set of equations of state included in (19) satisfies the condition of weak energy dominance and of the velocity of sound being less than or equal to that of light in vacuum [11]:

$$|p| \leq \varepsilon. \tag{20}$$

Namely, $n = 0(\alpha = -1)$: vacuum; $n = 1(\alpha = -2/3)$: domain walls; $n = 2(\alpha = -1/3)$: strings; $n = 3(\alpha = 0)$: dust; $n = 4(\alpha = 1/3)$: relativistic gas; $n = 5(\alpha = 2/3)$: bosons and fermions; $n = 6(\alpha = 1)$: ultrastiff matter.

Separating a term independent of the scale factor in the potential, we reduce the Wheeler–DeWitt equation to the Schrödinger one in an effective flat space, which permits the conventional quantum-mechanical procedure to be used:

$$-(\hbar^2/2m_{\rm pl})d^2\psi/da^2 + [U(a) - E]\psi = 0. \tag{21}$$

The identification of (16) and (21) occurs if

$$V(\gamma) = (2m_{\rm pl}/\hbar^2)[U(\gamma) - E], \tag{22}$$

where

$$\begin{aligned} U(\gamma) &= (m_{\rm pl}c^2/2)(k\gamma^2 - \gamma^4 - B_1\gamma^3 - B_2\gamma^2 - B_3\gamma - B_5\gamma^{-1} - B_6\gamma^{-2}), \\ E &= m_{\rm pl}c^2B_4/2, \quad \gamma = a/l_{\rm pl}. \end{aligned} \tag{23}$$

For $B_1 = B_3 = B_5 = B_6 = 0$, the potential (23) has extrema: a maximum at $\gamma = \sqrt{(k - B_2/2}$ and a minimum at $\gamma = 0$. Its nulls are $\gamma = 0$ and $\gamma = \sqrt{k - B_2}$. Whence it follows that the condition of the scale factor being real is

$$k - B_2 > 0. \tag{24}$$

Near the minimum $\gamma = 0$, the energy spectrum of the Universe is described by the formula for a harmonic oscillator [12]:

$$E_N = \hbar\omega(N + 1/2), \tag{25}$$

where $N = 0, 1, 2, \ldots,$

$$\omega = c\sqrt{k - B_2}/2l_{\rm pl}. \tag{26}$$

The wave function is

$$\psi(a) = (k - B_2)^{1/8}/(2\pi)^{1/4}\sqrt{2^N N! l_{\rm pl}}\exp(-\sqrt{k - B_2}a^2/2l_{\rm pl}^2)H_N(a/l_{\rm pl}), \tag{27}$$

where H_N is a Hermite polynominal. The condition of level existence is

$$E_N < U_{\max}, \tag{28}$$

where

$$U_{\max} = m_{\rm pl}c^2(k - B_2)^2/8, \tag{29}$$

takes the form

$$(k - B_2)^{3/2} > 4(N + 1/2). \tag{30}$$

At $B_5 \neq 0$, $B_6 \neq 0$ for small γ, one may take account only of terms with negative powers of γ. In this case the Schrödinger equation reads:

$$d^2\psi/d\gamma^2 + (B_5\gamma^{-1} + B_6\gamma^{-2} + 2E/m_{\rm pl}c^2)\psi = 0. \tag{31}$$

Its solution is the psi function [12]:

$$\psi = c\rho^{s+1}e^{-\rho/2}F(-p, 2s+2, \rho), \tag{32}$$

where F is a degenerate hypergeometric function,

$$\rho = 2\gamma\sqrt{-1E/m_{\rm pl}c^2},\ B_5/2\sqrt{-2E/m_{\rm pl}c^2} = n,\ n-s-1 = p = 0,1,2,\dots,$$
$$B_5 > 0,\ s = -1/2 + \sqrt{1/4 - B_6}.$$

For $B_6 \leq 1/4$ there exist discrete levels (there occurs repulsion for $B_6 < 0$ and attraction for $0 \leq B_6 \leq 1/4$ at small γ). The energy spectrum of the Universe is of the form

$$E_p = -B_5^2 m_{\rm pl}c^2[8(p + 1/2 + \sqrt{1/4 - B_6})^2]^{-1}. \tag{33}$$

For $|B_6| \gg p$ $B_6 < 0$ we have

$$E_p \approx [1/8B_6 - (p+1/2)(4B_6\sqrt{-B_6})^{-1}]B_5^2 m_{\rm pl}c^2 \tag{34}$$

Near the potential minimum at $\gamma = -2B_6/B_5$, the spectrum is of the oscillator form. For $B_5^2/8|B_6| \ll 1$, the potential well is not so deep to create planckeons. For $|B_6| \ll p$, ($B_6 >< 0$) the spectrum is hydrogen-like.

For very small γ, Equation (31) reduces to

$$\gamma^2 d^2\psi/d\gamma^2 + B_6\psi = 0. \tag{35}$$

Its solution is given by the formula

$$\psi = \sqrt{\gamma}\begin{cases} C_1\cos(n\ln\gamma) + C_2\sin(b\ln\gamma),\ b^2 = B_6 - 1/4 > 0; \\ C_1\gamma^b + C_2\gamma^{-b},\ b^2 = 1/4 - B_6 > 0; \\ C_1 + C_2\ln\gamma,\ B_6 = 1/4. \end{cases} \tag{36}$$

For $B_6 > 1/4$ there occurs a fall to the field centre, which corresponds to $E_0 = -\infty$ [12].

The transmission factor for the Universe tunnelling through a barrier from the pre-de-Sitter domain to the de Sitter one calculated quasiclassically ($a \gg l_{\rm pl}$) for $B_1 = B_3 = B_5 = B_6 = 0$ is

$$D = \exp\left[-2/\hbar\left|\int_{a_1}^{a_2}\sqrt{2m_{\rm pl}(E-U)}\,da\right|\right]. \tag{37}$$

Calculating the integral for $E = 0$, $a_1 = 0$, $a_2 = l_{\rm pl}\sqrt{k - B_2}$, we have

$$D = e^{-(2/3)(k-B_2)^{3/2}},\ (k - B_2)^{3/2} \gg 1. \tag{38}$$

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References

1. Gliner, E. B. (1965) *Zhurn. Eksper. i Teor. Fiz.* **49**, 542;
(1970) *DAN SSSR* **192**, 771;
Gliner, E. B., Dymnikova, I. G. (1975) *Pis'ma v Astron. Zhurn.* **1**, No. 5, 7;
Gurevich, L.E. (1975) *Ap. & Sp. Sci.* **38**, 67.
2. Guth, A. (1981) *Phys. Rev. D* **23**, 347.
3. Tryon, E. P. (1973) *Nature* (London) **246**, 396.
4. Fomin, P. I. (1975) *DAN Ukr. SSR* **9A**, 931.
5. Atkatz, D., Pagels, H. (1982) *Phys. Rev.* **25D**, 2065.
6. Vilenkin, A. (1984) *Phys. Rev.* **30D**, 509;
(1985) *Nucl. Phys.* **252B**, 141.
7. Hartle, J. B., Hawking, S. W. (1983) *Phys. Rev.* **28D**, 2960.
8. Dolgov, A. D., Zel'dovich, Ya. B., Sazhin, M. V. (1988) *Cosmology of the Early Universe*, Moscow University Press, Moscow.
9. Linde, A. D. (1990) *Elementary Particle Physics and Inflationary Cosmology*, Moscow, Nauka.
10. Beciu (1985) *Studii Si Cercetări de Fizica* **35**, 17.
11. Grib, A. A., Mamaev, S. G., Mostepanenko, V. M. (1980) *Quqntum Effects in Intensive External Fields*, Moscow, Atomizdat
12. Landau, L. D., Lifshitz, E. M. (1963) *Quantum Mechanics. non-Relativistic Theory*, Moscow, Fizmatgiz.