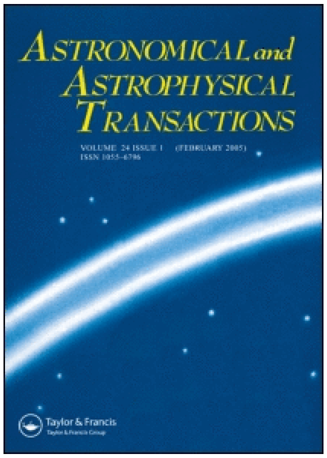

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

On the motion of the seventh satellite of saturn

N. O. Kirsanov [a]
[a] Institute for Theoretical Astronomy, Russian Academy of Sciences, St. Petersburg, Russia

Online Publication Date: 01 May 1995
To cite this Article: Kirsanov, N. O. (1995) 'On the motion of the seventh satellite of saturn', Astronomical & Astrophysical Transactions, 7:4, 255 - 257
To link to this article: DOI: 10.1080/10556799508203269
URL: http://dx.doi.org/10.1080/10556799508203269

Astronomical and Astrophysical Transactions, 1995,
Vol. 7, pp. 255–257
Reprints available directly from the publisher.
Photocopying permitted by license only

ON THE MOTION OF THE SEVENTH SATELLITE OF SATURN†

N. O. KIRSANOV

Institute for Theoretical Astronomy, Russian Academy of Sciences, St.Petersburg, Russia

(Received December 25, 1993)

A computer algebra system is used to develop a first-order theory of the motion of Hyperion.

KEY WORDS Theory of motion, computer algebra system, Saturn's satellites

The motion of Hyperion involves some difficulties due to the perturbing action of Titan [5]. These difficulties arise especially due to the commensurability between the mean motions of both satellites. This is the cause due to which it is necessary to construct a numerical theory of Hyperion's motion.

To this end a technique of analytical computation of the Newcomb polynomials in operator form based on a computer algebra system has been developed [1]. Since Newcmb's polynomials are developed in terms of the differential operator $D = \alpha\partial/\partial\alpha$ (where α is the ratio of the semi-major axes of the bodies) we cannot use the traditional programming languages, for we cannot deal with a differential operator as an object of the language. Hence, the programs written earlier computed only the coefficients at the proper powers of D, but the user had to do the very powering (i.e. the superposition of the differentiations) [2]. The technique gives easily the results of the action of Newcomb's polynomials [3, 4] on the Laplace coefficients in the analytical as well as numerical form and it can be used both for the inner and outer position of the perturbing body orbit.

Letting r and r' be the radius-vectors of the satellites, f and f' the distances from the common node to the satellites, V the angle between the satellites centered on Saturn's center, γ the angle of the mutual inclinations of the orbits and $\sigma = \sin\gamma/2$, we can write the following for the perturbation functions:

$$R = (r^2 - 2rr'\cos V + r'^2)^{-1/2} - r/r'^2 \cos V$$

$$R' = (r^2 - 2rr'\cos V + r'^2)^{-1/2} - r'/r^2 \cos V,$$

†Proceedings of the Conference held in Kosalma

where the letters with prime mean Hyperion and without is, mean Titan. According to [4], we can write the first terms of the expansion:

$$\begin{aligned}
a'[\Delta^{-1}]_0 &= 1/2(P_0^0 + e^2P_0^2 + e'^2P_{00}^{02} + e^4P_0^4)\cos V_i \\
&+ (ee'P_{-11}^{11} + e^3e'P_{-11}^{31} + ee'^3P_{-11}^{13})\cos(V_i - M + M') \\
&+ e^2e'^2P_{-22}^{22}\cos(V - 2M + 2M') + e^2e'P_{2-1}^{2\,1}\cos(V_i + 2M - M') \\
&+ (eP_1^1 + e^3P_1^3 + ee'^2P_{10}^{12})\cos(V_i + M) \\
&+ (e'P_{01}^{01} + e^2e'P_{01}^{21} + e'^3P_{01}^{03})\cos(V_i + M') \\
&+ (e^3e'P_{3-1}^{3\,1})\cos(V_i + 3M - M') \\
&+ (e^2P_2^2 + e^4P_2^4 + e^2e'^2P_{20}^{22})\cos(V_i + 2M) \\
&+ (ee'P_{11}^{11} + e^3e'P_{11}^{31} + ee'^3P_{11}^{13})\cos(V_i + M + M') \\
&+ e^3P_3^3\cos(V_i + 3M) + e^2e'P_{21}^{21}\cos(V_i + 2M + M'),
\end{aligned}$$

where $P_{mm'}^{nn'} = \prod_{mm'}^{nn'} a'A_i$, $\prod_{mm'}^{nn'} = \prod_m^n \prod_{0m'}^{0n'}$, $\prod_{0m'}^{0n'} = \prod_{m'}^{n'}(-s, -D-1)$, where $\prod_m^n(s, D)$ is Newcomb's polynomial, defined as follows:

$$\left(\frac{r}{a}\right)^D \exp\left(\sqrt{-1}sv\right) =$$

$$\sum_{-\infty<m<\infty} \quad \sum_{n-|m|=0(\mathrm{mod}2),n\geq m} \quad \prod_m^n(s,\, D)e^n \exp\left[\sqrt{-1}(s+m)M\right].$$

where a, e, r, v, M are respectively the semi-major axis, eccentricity, radius-vector, true and mean anomalies,

$$\begin{aligned}
a'A_i &= c_1^{(i)} - \frac{1}{2}\sigma^2(c_3^{(i-1)} + c_3^{(i+1)}) + \frac{3}{8}\sigma^4(c_5^{(i-2)} + 4c_5^{(i)} + c_5^{(i+2)}), \\
a'B_i &= \frac{1}{2}c_3^{(i)} - \frac{3}{4}\sigma^2(c_5^{(i-1)} + c_5^{(i+1)}), \\
a'C_i &= \frac{3}{8}c_5^{(i)}, \quad c_n^{(i)} = a^{(n-1)/2}b_n^{(i)},
\end{aligned}$$

with $b_n^{(i)}$ the Laplace coefficients: $b_n^{(i)} = 2\frac{(n/2)i}{(1)i}\ \alpha^i\ F(n/2,\, n/2+i,\, i+1;\, \alpha^2)$, $F(a,\, b,\, c;\, z) = \sum_{i=0}^{\infty} \frac{(a)i(b)i}{(c)i(1)i}\ z_i$ is the hypergeometric function, and $(\xi)i = \prod_{j=0}^{i-1}(\xi + j)$.

The technique has been used for computing the perturbation function of Hyperion from Titan and an asteroid from Jupiter for several commensurabilities. To test the technique, the equations representing the conditions of the existence of the Schwarzschild-type periodic solutions of the restricted three-body problem has been constructed and solved [1]. Then the technique has been used for computing the perturbation function of Hyperion from Titan and the derivatives of the latter with respect to the Kepler orbital elements in the form of trigonometric series. The first terms of the expansion are given below:

$$a'R/fm \;=\; 1.3298 + 0.34216\cos Pc - 0.012924\cos Ps + 0.05123\cos 9Psh$$

$$+ \quad 0.084434\cos 8Psh + 0.12975\cos 7Psh + 0.19148\cos 6Psh$$
$$+ \quad 0.27600\cos 5Psh + 0.39334\cos 4Psh + 0.56114\cos 3Psh$$
$$+ \quad 0.81648\cos 2Psh - 0.19286\cos Psh$$

where $Pc = 4M' - 3M + 3\tau' - 3\tau$ is the critical term, $Psh = M' - M + \tau' - \tau$ is the short-periodic term, and $Ps = \tau' - \tau$ is the secular term. Newcomb restricted himself to $a'R/fm = 1.3297 + 0.34002\cos Pc$. These results allow to develop a first-order analytical theory for the motion of Hyperion by the proper integration of different terms in the Lagrange equations for osculating elements. We are going to use the orbit given by this theory as an intermediate one for numerical computations based on the Encke method.

References

Kirsanov, N. O. (1992) On the calculation of Newcomb polynomials using computer algebra system. *Prep. ITA RAS* No. 27, 21.

Krassinsky, G. A. (1970) Calculation of the Newcomb polynomials on a digital computer. *Bull. ITA.* **XII**, No. 6 (139), 474–477.

Newcomb, S. (1891) Development of the Perturbative Function and its Derivatives in sines and consines of multiples of Eccentric Anomalies and in powers of the eccentricities and inclinations. *Astronomical Papers.* **III**, pt. I.

Orlov, B. A. (1936) Development of the perturbative function by the Newcomb's method. *Tr. AO LGU.*.**VI**, 82—125.

Woltjer, J. (1928) The motion of Hyperion. *Annalen van de Sterrewacht te Leiden.* Deel XVI, derde stuck.