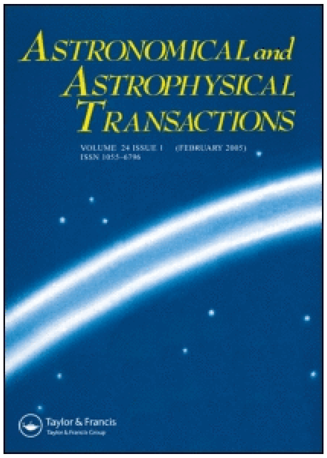

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Families of integrable and stochastic trajectories in the **N**-body problem

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FAMILIES OF INTEGRABLE AND STOCHASTIC TRAJECTORIES IN THE N-BODY PROBLEM†

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One can find, in the phase space and the parameter space, sufficiently large region where the N-body problem (for any N) is integrable in the classical sense. These bodies have large relative velocities and the close approaches are absent. On the other hand, the families of trajectories with regular approaches are constructed. These families are similar to a stochastic process even for small N and naturally are non-integrable. Intermediate cases are also considered.

KEY WORDS N-body problem, integrability, stochastic motions

The classical N-body problem has only ten well-known global integrals. The well-known results of Poincare and Bruns set limits on the integrability of the problem. But the nonintegrability is not a universal characteristic of the motions too. The properties of the trajectories vary rather strongly in different phase space regions. One must use specific technique to investigate the motion in each individual region.

In particular, zones with a very simple structure of the motions exist. In the phase space and the parameter space, one can find a large region where the N-body problem (for any N) is integrable in the classical sense [1]. These bodies have large relative velocities and the close approaches are absent. The integrals can be derived using the exact N-body problem solutions, constructed by means of the Picard iterations applied to the differential equations of motion. The zero-order approximation is a linear motion with a constant velocity. The convergence of the iterations is based on the contraction operators technique and sufficiently rapid decreasing of perturbations along the zero-order approximation.

If the close approaches are accepted under condition of high kinetic energy, one can separate the trajectories with exchange, escape and capture in the three-body problem. To prove the existence of the explicit solutions with properties under consideration, the same technique of Picard iterations may be used, but the time axis divides into three parts [2].

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The motion near collisions at moderate energies is sufficiently more complicated. Let the planets move in frameworks of the KAM-theory, a zero-mass body travels near the orbits with collisions. The planetary masses are small, and a convenient approximation is the point-like action sphere method. At the collision, the planetocentric velocity vector rotates by an arbitrary angle. The contraction of the action sphere to a point formally leads to an indeterministic (stochastic) description of motion. The approximation of the family of trajectories in this case is a stochastic process described by means of the tree-like Markov process. Branches are collision orbits, nodes are collisions [3, 4]. The motions under consideration are unstable, an identification of trajectories by means of the initial conditions and numerical integration is very difficult. To consider the perturbations one must use special methods.

The results of investigation of the tree properties is the evidence for the presence of trajectory passing near every Keplerian elliptic orbit intersecting any noncircular planetary orbit. One may reach almost arbitrary elliptic orbit using only gravitation forces, without fuel. The properties of the motion retain under simplifications made in operation. In the small neighborhood of the stochastic approximation one can find an explicit solution of the equations of motion.

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DISCUSSION

Zhelesnyak: What is the ratio of kinetic energy to potential one in the N-body system when this system could be integrated?
Sokolov: The theorems about the N-body problem integrability contain a condition for the smallest ratio of potential and kinetic energies of pairs (points). Numerical estimations give 10^{-3} usually enough.
Fesenko: Some of your statements were based on the intuition (as you have pointed out). What do you mean?
Sokolov: My statements are based on mathematical theorems. Informal, intuitive ideas were used for the construction of these theorems. They are convenient to clarify the meaning of the results too.