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VORTICES IN THE STELLAR GALACTIC DISK: KINETIC APPROACH

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The possibility of the existence of stationary vortex-line waves in the stellar galactic disk is considered within the framework of collisionless Boltzmann equation. A family of vortex-like solutions is constructed for the case of a rigidly rotating disk. It is shown that the existence of stationary vortex-like perturbations is impossible in differentially rotating disk with a linear rotation law. In particular, this is the case for flat rotation curves.

KEY WORDS Galaxy, stellar disk, vortices.

1. INTRODUCTION

Recently, it has been recognized that vortex-like wave perturbations can exist in gaseous subsystems of disk galaxies (Korchagin and Petviashvili, 1985; Korchagin *et al.*, 1988; Fridman, 1988). These vortices are similar to those in planetary atmospheres and in rotating shallow water (Nezlin, 1984). There is some observational data confirming the existence of such vortices (Zasov and Kyazumov, 1981; Afanas'ev and Rassokhin, 1982; Afanas'ev *et al.*, 1990). These observations are also related to gaseous components of the disks. By analogy with spiral-like wave perturbations existing in both gaseous and stellar subsystems of galactic disks, it is natural to consider vortex perturbations in stellar subsystems as well. Korchagin and Ryabtsev (1991) demonstrated the possibility of existence of solitary vortices in stellar disks within the framework of collisionless fluid dynamics with isotropic pressure tensor (Marochnik, 1966). This approach is valid only for rigidly or weakly differentially rotating disks under a set of severe restrictions on the velocity and wavelength of perturbations.

Here we consider localized perturbations in stellar disks in the framework of collisionless Boltzmann equation. It is shown that stationary solitary vortices can exist in uniformly rotating stellar disks, thus confirming the result of Korchagin and Ryabtsev (1991). The existence of such perturbations is shown to be forbidden in differentially rotating disks in important case of a linear rotation law. Thus, the “no-go” theorem proved by Antonov and Zheleznyak (1989) is generalized for stellar disks.

2. KINETIC EQUATION FOR LOCATIZED PERTURBATIONS

We start with the Boltzmann equation in cylindric coordinates:

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \frac{v_\phi}{r} \frac{\partial f}{\partial \phi} + \left(-\frac{v_r v_\phi}{r} + \frac{1}{r} \frac{\partial U}{\partial \phi} \right) \frac{\partial f}{\partial v_\phi} + \left(\frac{v_\phi^2}{r} + \frac{\partial U}{\partial r} \right) \frac{\partial f}{\partial v_r} = 0. \quad (1)$$

Let $\Omega(r)$ be the angular velocity of disk rotation and r_0 the distance from the center. We introduce peculiar velocities as

$$\begin{aligned} c_r &= v_r, \\ c_\phi &= v_\phi - r\Omega(r), \end{aligned} \quad (2)$$

and use the reference frame rotating at angular velocity $\Omega_0 = \Omega(r_0)$. Taking into account that the unperturbed gravity potential is given by

$$U_0 = -r\Omega^2(r), \quad (3)$$

we obtain

$$\begin{aligned} \frac{\partial f}{\partial t} + V_0(r) \frac{1}{r} \frac{\partial f}{\partial \phi} + c_r \frac{\partial f}{\partial r} + c_\phi \frac{1}{r} \frac{\partial f}{\partial \phi} \\ - \left(c_r \frac{\kappa^2}{2\Omega} - \frac{1}{r} \frac{\partial U'}{\partial \phi} \right) \frac{\partial f}{\partial c_\phi} + \left(2\Omega c_\phi + \frac{\partial U'}{\partial r} \right) \frac{\partial f}{\partial c_r} = 0, \end{aligned} \quad (4)$$

where $V_0(r) = (\Omega(r) - \Omega_0)r$ is the residual velocity in the rotating frame and $U' = U - U_0$ is the perturbation of the gravity potential.

In what follows we shall consider localized perturbations in the vicinity of a point O' , at the distance r_0 from the galactic center. Therefore, we introduce a local Cartesian frame centered at O' with the x -axis directed along the radius and the y -axis in the direction of galactic rotation. We shall consider perturbations neglecting selfgravity, i.e., $U' = 0$. Indeed, though selfgravity can be important in the dynamics of vortices, it cannot dominate in the formation of their structure because the vortices are waves in stellar disks rather than gravitationally bounded groups of stars, i.e., stellar clusters. The latter cannot be described in terms of one-particle kinetic equation (1), where the phase density f refers to the galactic disk. Finally, equation for localized perturbation in the local Cartesian frame reads

$$\frac{\partial f}{\partial t} + V_0(x) \frac{\partial f}{\partial y} + c_x \frac{\partial f}{\partial x} + c_y \frac{\partial f}{\partial y} - c_x(2\Omega_0 + V'_0) \frac{\partial f}{\partial c_y} + 2\Omega c_y \frac{\partial f}{\partial c_x} = 0. \quad (5)$$

The terms of order $1/r_0$ are neglected in Eq. (5). Equation (5) is our starting point for the investigations of vortex solutions in stellar disks.

3. VORTEX SOLUTION FOR A UNIFORMLY ROTATING DISK

In the uniformly rotating disk, we have $V_0 = 0$. Let us search for stationary axially symmetric solutions of Eq. (5) in this case. It is convenient to introduce local

polar coordinates as

$$x = \rho \cos \theta; \quad y = \rho \sin \theta.$$

In terms of this coordinates, stationary kinetic equation reads

$$c_\rho \frac{\partial f}{\partial \rho} - \left(\frac{c_\rho c_\theta}{\rho} + 2\Omega c_\rho \right) \frac{\partial f}{\partial c_\theta} + \left(\frac{c_\theta^2}{\rho} + 2\Omega c_\theta \right) \frac{\partial f}{\partial c_\rho} = 0. \quad (6)$$

The stationary phase density f is a function of two independent integrals of the system

$$\frac{d\rho}{c_\rho} = - \frac{dc_\theta}{\frac{c_\rho c_\theta}{\rho} + 2\Omega c_\rho} = \frac{dc_\rho}{\frac{c_\theta^2}{\rho} + 2\Omega c_\theta}. \quad (7)$$

The two integrals can be easily written out. These are the specific energy,

$$E = \frac{1}{2}(c_\rho^2 + c_\theta^2), \quad (8)$$

and the specific angular momentum,

$$I = c_\theta \rho + \Delta \rho^2. \quad (9)$$

Let σ be the velocity dispersion of stars. The localized solution must approach asymptotically the Shwartzschild law when ρ increases. Therefore, it must have the form

$$f = \frac{N}{2\pi\sigma} \exp\left(-\frac{E}{\sigma^2}\right) + \delta f(E, I), \quad (10)$$

where $\delta f(E, I) \rightarrow 0$ for $I \rightarrow \infty$. In Eq. (10), N stands for the unperturbed number density of stars. The shape of the perturbations remains indetermined in the stationary theory, similarly to the case of the fluid dynamical approach (Korchagin and Ryabtsev 1991). A soliton-like, exponentially decreasing vortex solution can be written as follows:

$$f = \frac{N}{2\pi\sigma^2} \exp\left(-\frac{E}{\sigma^2}\right) \left[1 + \delta_0 \exp\left(-\frac{I}{\Omega L^2}\right) \right] \quad (11)$$

In Eq. (11), the dimensionless parameter δ_0 determines the magnitude of the perturbation; L determines its spatial scale. The corresponding perturbation of the stellar number density has the form

$$\delta N = \int d^2c \delta f = \delta_0 N \exp\left[-\frac{\rho^2}{L^2} \left(1 - \frac{\sigma^2}{2\Omega^2 L^2}\right)\right] \quad (12)$$

Rotational velocity in the vortex is given by

$$v_{\text{rot}} = \frac{1}{N} \int d^2c c_\theta f = -\frac{\sigma^2 \rho}{\Omega L^2} \delta_0 \exp\left[-\frac{\rho^2}{L^2} \left(1 - \frac{\sigma^2}{2\Omega^2 L^2}\right)\right] \quad (13)$$

As can be seen from Eq. (13), the rotation velocity is nonvanishing due to the dependence of the distribution function on the angular momentum integral I . The

solution is localized if

$$L > \frac{\sigma}{2^{1/2}\Omega}. \quad (14)$$

The half-width of the vortex is

$$L_v = L \left[2 \left(1 - \frac{\sigma^2}{2\Omega^2 L^2} \right) \right]^{-1/2}. \quad (15)$$

For $L = \sigma/\Omega$, the vortex is most compact. In this case $L_v = \sigma/\Omega$ and the maximum rotational velocity is $\sigma\delta_0 e^{-1/2}$. Thus, for typical values of the dispersion and velocity, say, $\sigma = 50$ km/s and $\Omega = 50$ km/s kpc, the vortex size can be a few kiloparsecs and the rotational velocity is large as several tens of kilometers per second.

4. PROOF OF THE ABSENCE OF VORTEX SOLUTIONS IN DIFFERENTIALLY ROTATING DISK WITH A LINEAR ROTATION LAW

The linear law of differential rotation is a good approximation for extended parts of rotation curves in many galaxies. In particular, rotation curves are flat in outer parts of galactic disks. Therefore, it is important to examine whether solitary vortices can occur for such rotation curves.

In the case of a linear rotation law, $V_0(x) = V'x$ and $V' = \text{const}$. Consider stationary solutions of Eq. (5) with allowance for a possible drift along the y -axis, i.e., $\partial/\partial t = u(\partial/\partial y)$, where u is the drift velocity. In general, the solution is a function of three integrals of the system

$$\frac{dy}{u + V_0 + c_y} = \frac{dx}{c_x} = \frac{dc_x}{2\Omega_0 c_y} = -\frac{dc_y}{(2\Omega_0 + V'_0)c_x}. \quad (16)$$

Linking the second and the fourth terms of the proportion (16), we obtain the following:

$$J = c_y + V_0(x) + 2\Omega_0 x. \quad (17)$$

Another integral is obtained from linking the third and the fourth terms in Eq. (16). It reads

$$E = \frac{1}{2} \left(\frac{\kappa^2}{4\Omega^2} c_x^2 + c_y^2 \right). \quad (18)$$

One more integral can be written out explicitly in the case of a linear rotation law. It has the form

$$K = y - \frac{2\Omega}{\kappa^2} c_x + \frac{\text{sign}(c_x)}{\kappa} \arcsin \left(\frac{c_x}{(2E)^{1/2}} \right) \left[u + \left(1 - \frac{4\Omega^2}{\kappa^2} \right) \left(c_y + x \frac{\kappa^2}{2\Omega} \right) \right]. \quad (19)$$

However, the phase density f should be independent of K , because K is a

multi-valued function (Ogorodnikov, 1958). Hence, f depends only on J and E . It is obvious that such dependence rules out vortex-like solutions because it implies that the velocity component along the x -axis is zero. Indeed,

$$\int d^2c c_x f(J, E) = 0 \quad (20)$$

because J is independent of c_x and E is an even function of c_x .

Note that K becomes single-valued in a uniformly rotating disk if $u = 0$:

$$K = y - \frac{c_x}{2\Omega}. \quad (21)$$

In this case the angular momentum integral I , which leads to the existence of a vortex solution, can be obtained from J , E and K as

$$I = \frac{J^2 + (2\Omega K)^2 - 2E}{4\Omega} \quad (22)$$

Thus, we have rederived the result of Section 3.

5. CONCLUDING REMARKS

We have demonstrated the existence of stationary solitary vortices in a uniformly rotating stellar disk and the absence of such perturbations in the disks with a linear rotation law. The existence of stationary vortices is closely related to the existence of a sufficiently rich set of single-valued integrals of the kinetic equation. The set of integrals must be sufficient for the construction of the stationary phase density with nonzero mean circular flow. This is the case for the axially symmetric kinetic equation for a uniformly rotating disk. Differential rotation breaks down this symmetry and reduces the number of single-valued integrals. This fact has been demonstrated by explicit calculations for a linear rotation law.

Thus, we have confirmed the hypothesis (Korchagin and Ryabtsev, 1991) that rotation laws close to the uniform one are most favorable for the existence of solitary vortices.

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