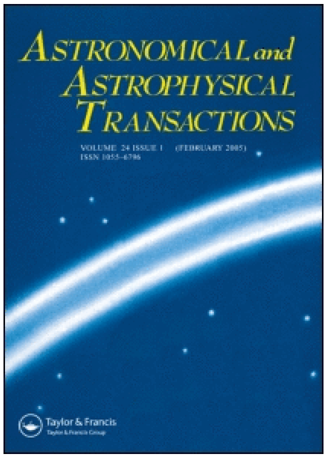

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ANALYSIS OF TROPOSPHERIC PATHLENGTH FLUCTUATIONS USING GEOSTATIONARY SATELLITE OBSERVATIONS

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We present results of the measurements of the fluctuations in the arrival angle of 4 GHz radio waves from a geostationary satellite. Observations were performed using interferometer of the 145 m baseline. The measurements consist of 123 observation sessions, each of 10 minutes or longer, randomly selected in terms of the time of day.

Relationships between the measured temporal structure function of fluctuations and the temporal and spatial structure functions of the pathlength through the troposphere are obtained.

Our results are in a good agreement with the turbulent model for the pathlength structure function.

Using the experimental data, we estimate parameters of the temporal structure function of the pathlength through the troposphere (the power-law exponent and structure coefficient), the value of the spatial structure function and the average velocity of motion of the irregularity, which allows to relate spatial and temporal parameters to each other. The distribution functions of these quantities are found.

KEY WORDS Troposphere, pathlength fluctuations, structure function, angle of arrival, satellite, interferometer.

1. INTRODUCTION

The phase fluctuations arising when radio waves pass through the troposphere are a natural source of errors for the earth-based radio astronomical systems. These fluctuations limit the real attainable accuracy of measuring co-ordinates with long-baseline interferometers. At millimeter and shorter waves, such fluctuations restrict the possibilities of single-dish radio telescopes.

Considering the irregularity in the troposphere to be a result of turbulent stirring, it is possible to apply the statistical turbulence theory (Tatarskii, 1964) to derivation of the spatial and temporal structure functions of the pathlength through the troposphere for wide spatial and temporal domains (Stotskii, 1973; Stotskii, 1972). The resulting turbulent model is in agreement with the available observation data—see Figure 1 (Stotskii, 1972; Baars, 1967; Waters, 1967; Basart *et al.*, 1970; Milley *et al.*, 1970; Lipovka and Stotskii, 1972; Armstrong and Sramek, 1982).

The aim of the present paper is to obtain more detailed statistical characteristics and quantitative estimations of the structure function parameters from measurements of the fluctuations of the arrival angle of radio waves emitted by the geostationary satellite.

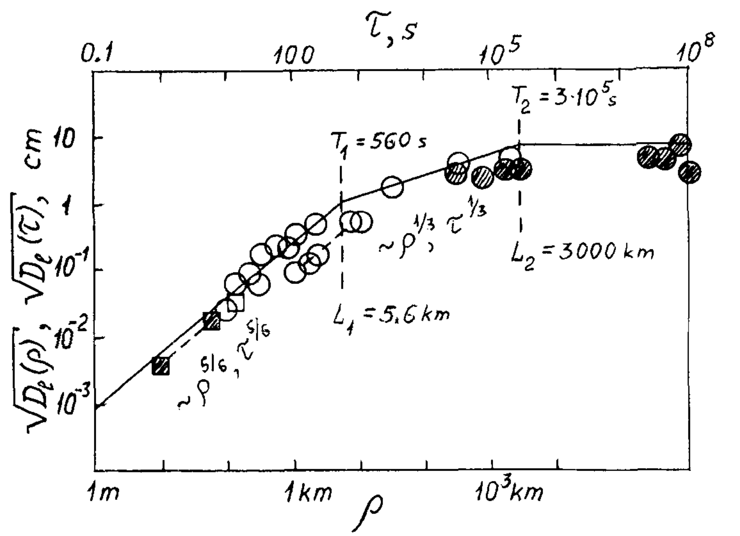

Figure 1 The turbulent model of the spatial and temporal structure functions of the pathlength through the Earth troposphere (the zenith direction). Experimental data: ○, □—for the spatial structure function, ◍, ▨—for the temporal structure function, □, ▨—the values obtained in this work.

The geostationary satellite is a convenient signal source because of its little motion relative to the observer and high intensity of its emission.

2. THE METHOD OF MEASUREMENTS

Measurements were made with horizontal interferometer, which had the baseline $d = 145$ m. The interferometer consisted of two elements of the main multi-element ring reflector of the RATAN-600 radio telescope (Korolkov and Pariiskii, 1979). The radio telescope is situated in the North Caucasus near Zelenchukskaya at the height of 1000 m above the sea level.

As a signal source, we used the geostationary TV satellite Horizon-13 operating in a frequency range of 4 GHz. The nominal longitude of its position is 53°E. The elevation angle of the satellite at the radio telescope location is $h = 38°$, azimuth 344°.

We measured the horizontal fluctuations in the apparent position of the satellite. To do this, the interferometer pattern was fixed in such a position that the satellite would be just on a slope of an interferometer beam, at its middle (linear) part. Since the observed fluctuations angular amplitude was smaller than the half-width of the interference lobe, variations of the interferometer output signal were nearly proportional to the horizontal angular displacement of the satellite.

Calibration was performed by displacing the horn feed horizontally by a certain distance (10 mm). Since the geometry of the interferometer is known, the horn feed displacement Δx can be easily recalculated into the angular displacement of

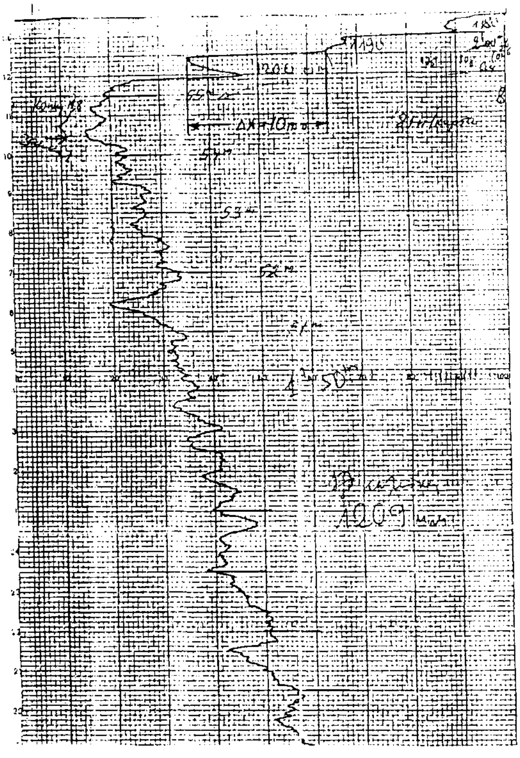

Figure 2 An example of recording of the fluctuations of the radio wave angle of arrival and calibration steps.

the pattern $\Delta\theta$ or into the change of the difference between the pathlengths in the interferometer branches ΔL (for $\Delta x = 10$ mm we have $\Delta\theta = 7.43''$ and $\Delta L = 5.23$ mm).

An example of the record of the fluctuations of radio wave angle of arrival and the calibration are given in Figure 2.

Preliminary observations have shown that during a day the satellite described a near-elliptic path around the observation point (Figure 3). For each element of the interferometer, the beam is extended in the horizontal direction, strongly exceeding in the horizontal width the satellite displacement range and being below the latter along the vertical direction. This is why between the observation sessions we corrected angular positions of the interferometer elements in order to reach a maximum of the received signal. During the observations, the interferometer elements remained fixed.

First preliminary observations were started in 1981 (Stotskii and Stotskaya, 1984). A basic series of measurements was accomplished in February 1986. It consisted of 123 observation sessions in a 5-day period. Observation sessions had very different durations ranging from 10 minutes to 3 hours depending on the period when the satellite was within the linear part of the interferometer beam. As for the time of day, the distribution of measurement sessions was random.

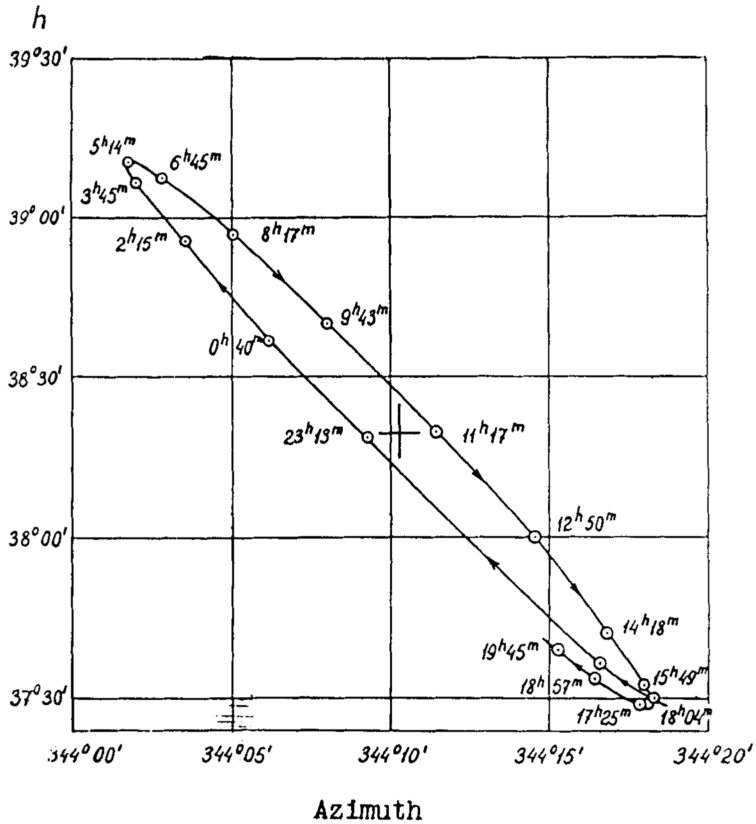

Figure 3 Changes in the satellite position.

The total observations time amounts to 46 hours. The signal was recorded every 1.28 s and thus the whole set of observed data contains 129370 points.

An initial processing of the records consisted in separation of the fluctuations in the apparent position of the satellite caused by atmospheric effects from the satellite intrinsic motions about the average position. At this stage, the fluctuation magnitude was also scaled in accordance with the calibration data.

The satellite intrinsic azimuthal motion was approximated by a harmonic function of the one-day period. Parameters of this function (the amplitude and phase) were calculated using the least squares method from the available measurements (with short data series, a linear approximation was used). The curves of intrinsic satellite motion, obtained in such a way, were then subtracted from the data. The remaining fluctuation part was considered to be a record of unknown values of the radio wave angle of arrival fluctuations, caused by fluctuations in the atmospheric refraction index.

Fluctuations in the angle of arrival $\Delta\theta$ measured at the elevation angle h have been used for calculation of a more universal characteristic of the troposphere irregularities, the fluctuations in the difference between the pathlengths through the troposphere in the zenith direction, Δl, for a two paths whose separation equals the interferometer baseline distance d. When the elevation angle is not very small, these two quantities are connected by the following relation:

$$\Delta l = \Delta \theta d \sqrt{\sin h}. \tag{1}$$

As a basic characteristic of the angle of arrival fluctuations, it is convenient to consider the temporal structure function $D_\theta(\tau)$. As follows from Eq. (1) it is related to the temporal structure function of the zenith pathlength differences $D_{\Delta l}(\tau)$ as follows:

$$D_{\Delta l}(\tau) = D_\theta(\tau) d^2 \sin h. \tag{2}$$

The function $D_{\Delta l}(\tau)$ is, in turn, related to the structure functions of the pathlength through the troposphere, the temporal $D_l(\tau)$, and spatial ones, $D_l(\rho)$. This relationship is discussed below.

Below we present our results concerning $D_\theta(\tau)$ as well as $D_{\Delta l}(\tau)$, $D_l(\tau)$ and $D_l(\rho)$.

3. THEORY

Let us find how the temporal structure function $D_{\Delta l}(\tau)$ of the difference between the pathlengths of two parallel paths in the troposphere, separated by the distance d, is related to the temporal structure function $D_l(\tau)$ of the pathlength through the troposphere. Here $D_l(\tau)$ is assumed to be a power-law function,

$$D_l(\tau) = C_{lt}^2 \tau^\mu. \tag{3}$$

The corresponding spectrum also represents a power-law (Tatarskii, 1964):

$$W_l(\omega) = C_{lt}^2 A(\mu) \, |\omega|^{-(1+\mu)}, \tag{4}$$

where

$$A(\mu) = (1/2\pi)\Gamma(1+\mu)\sin(\pi\mu/2). \tag{5}$$

In a certain sense, an interferometer can be described as a filter characterized by the following, transfer function in the near-field zone:

$$W_i(\omega) = 4\sin^2(\omega\tau_0/2), \tag{6}$$

where $\tau_0 = d/v$ and v is the average velocity of irregularity motions.

The spectrum of fluctuations in the pathlength difference $W_{\Delta l}(\omega)$ can be considered as a result of transformation of the spectrum (4) by the filter with the frequency response (6). Thus we have

$$W_{\Delta l}(\omega) = 4W_l(\omega)\sin^2(\omega\tau_0/2). \tag{7}$$

Knowing the spectrum of fluctuations, one can easily calculate their structure function:

$$D_{\Delta l}(\tau) = 2\int_{-\infty}^{+\infty} (1 - \cos\omega\tau) W_{\Delta l}(\omega)\, d\omega.$$

Substituting (7) and (4) into this expression we obtain

$$D_{\Delta l}(\tau) = 32C_{lt}^2 A(\mu)\int_0^\infty \sin^2(\omega\tau/2)\sin^2(\omega\tau_0/2)\omega^{-(1+\mu)}\,d\omega. \tag{8}$$

It is convenient to introduce the following dimensionless function:

$$\Delta(\xi) = (\xi/2)^\mu \int_0^\infty \sin^2 x \sin^2(x/\xi)\, x^{-(1+\mu)}\,dx, \tag{9}$$

where $\xi = \tau/\tau_0$.

Let us consider asymptotic forms of the function $\Delta(\xi)$ for small and large values of ξ. When τ is small ($\xi \to 0$), the factor $\sin^2(x/\xi)$ in the integral is a rapidly oscillating function as compared to other factors and, therefore, it can be replaced by its mean value which is 1/2. Then

$$\Delta(\xi) = 1/2(\xi/2)^\mu \int_0^\infty \sin^2 x\, x^{-(1+\mu)}\,dx. \tag{10}$$

To evaluate this integral, we employ the following relation [7]:

$$\int_0^\infty \sin^2 ax\, x^{-(1-\gamma)}\,dx = \frac{\Gamma(\gamma)\cos(\gamma\pi/2)}{2^{\gamma+1}a^\gamma}, \qquad (a > 0,\ -2 < \operatorname{Re}\gamma < 0).$$

The result

$$\Delta(\xi) \underset{\xi\to 0}{=} M(\xi)\xi^\mu,$$

where

$$M(\mu) = \Gamma(2-\mu)\cos(\pi\mu/2)/4(1-\mu).$$

For large τ (or $\xi \to \infty$) we perform in Eq. (9) transformation to the variable $y = x/\xi$. In this case the role of a rapidly oscillating function plays $\sin^2 \xi y$. Substituting for the latter its mean value 1/2 we come again to an integral of the same type (10) and the result has the form

$$\Delta(\xi) \underset{\xi\to\infty}{=} M(\mu).$$

Taking into account the asymptotic expressions obtained, a suitable form for the structure function (8) is

$$D_{\Delta l}(\tau) = 32C_{lt}^2 A(\mu)\tau_0^\mu F(\xi, \mu),$$

where $F(\xi, \mu) = \Delta(\xi)/M(\mu)$ is the dimensionless normalized structure function $D_{\Delta l}(\tau)$.

One can easily see that $A(\mu)\ M(\mu) = 1/16$. Thus, we finally obtain the following:

$$D_{\Delta l}(\tau) = 2C_{lt}^2 \tau_0^\mu F(\tau/\tau_0, \mu), \tag{11}$$

or, introducing the structure coefficient $C_{\Delta l}^2 = 2C_{lt}^2$, we have

$$D_{\Delta l}(\tau) = C_{\Delta l}^2 \tau_0^\mu F(\tau/\tau_0, \mu). \tag{12}$$

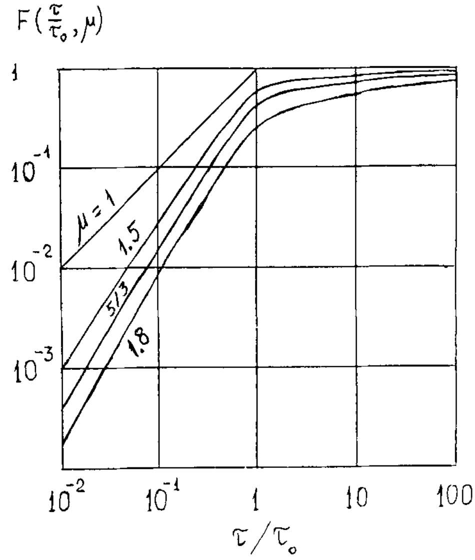

Figure 4 The dimensionless normalized structure function $F(\tau/\tau_0, \mu)$.

The form of the function $F(\tau/\tau_0)$ for $0.01 \leq \tau_0 \leq 100$, and various values of μ, is illustrated in Figure 4. For $\mu = 1$, this function degenerates into a piecewise-linear one.

For small and large values of τ/τ_0 we can write:

$$D_{\Delta l}(\tau) = \begin{cases} 2C_{lt}^2\tau^{\mu}, & \text{for } \tau \ll \tau_0, \\ 2C_{lt}^2\tau_0^{\mu}, & \text{for } \tau \gg \tau_0. \end{cases} \tag{13}$$

In terms of the structure coefficient $C_{\Delta l}^2$ and the dispersion $\sigma_{\Delta l}^2 = C_{lt}^2\tau_0^{\mu}$, this takes the form

$$D_{\Delta l}(\tau) = \begin{cases} C_{\Delta l}^2\tau^{\mu} & \text{for } \tau \ll \tau_0 \\ 2\sigma_{\Delta l}^2 & \text{for } \tau \gg \tau_0. \end{cases} \tag{14}$$

Comparing these expressions and the initial structure function of the tropospheric pathlength (3), one can see that for short time intervals ($\tau \ll \tau_0$) the structure function $D_{\Delta l}(\tau)$ is twice the structure function $D_l(\tau)$, whereas for the large intervals ($\tau \gg \tau_0$) saturation occurs.

This dependence has a clear physical interpretation. At low values of τ the main contribution to the pathlength fluctuations structure function comes from the small-scale inhomogeneities of the refraction index (at the scales below the spacing between the paths) and, therefore, they are statistically independent for the neighboring paths. Therefore, the resulting total fluctuations are determined by the sum of two independent random processes, and this makes the structure function doubled.

As τ increases, at $\tau \simeq \tau_0$ the large-scale inhomogeneities of the refraction index begin to play a prevailing role. These large-scale inhomogeneities, however, are the same for both paths and have no effect on the fluctuations of the difference of the pathlengths. This is why for still larger values of τ there is no enhancement in the fluctuations, or the structure function saturates.

Thus, the small-scale portion of the temporal structure function $D_{\Delta l}(\tau)$ can be used to infer parameters of the temporal structure function of the pathlength through the troposphere, $D_l(\tau)$.

The large-scale segment of $D_{\Delta l}(\tau)$ evidently approaches the doubled dispersion of the difference between the path lengths spaced at the distance d. If we assume that the turbulence is isotropic and the principle of frozen turbulence holds, then this dispersion gives a value of the spatial structure function of the tropospheric pathlength $D_l(\rho)$ for $\rho = d$:

$$\sigma_{\Delta l}^2 = D_l(d). \tag{15}$$

Various asymptotic forms for the structure functions are shown in Figure 5.

Parameter τ_0 has a simple physical and geometric measuring: on one hand, this is the average time during which the tropospheric irregularities travel the distance equal to the separation of the observed rays (i.e. the base of the interferometer d); on the other hand, this is the abscissa of the point where intersect the asymptotic low- and large-scale branches of the structure functions $D_{\Delta l}(\tau)$. As follows from (14),

$$\tau_0 = (2\sigma_{\Delta l}^2 / C_{\Delta l}^2)^{1/\mu}.$$

From the experimentally determined value of τ_0 one can find the average value of the irregularity advection velocity $v = d/\tau_0$. This parameter is very important since the principle of frozen turbulence then enables to connect the temporary and spatial characteristics of the fluctuations.

From the above reasoning and the way the transformations have been made to derive the differential structure function $D_{\Delta l}(\tau)$, it follows that the asymptotic relations

$$D_{\Delta l}(\tau) \to 2D_l(\tau) \qquad \text{for } \tau \ll \tau_0$$

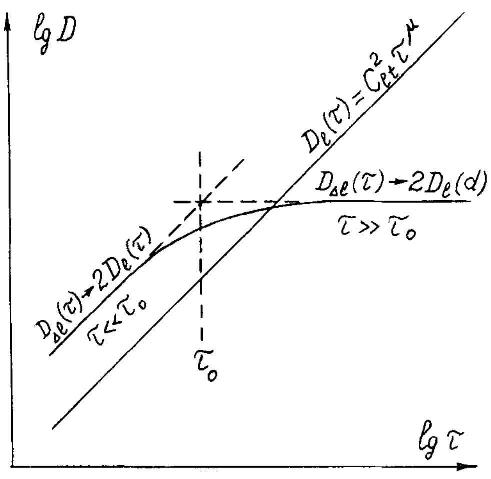

Figure 5 Relation between the measured structure function $D_{\Delta l}(\tau)$ and the structure functions of the pathlength through the troposphere, $D_l(\tau)$ and $D_l(d)$.

and

$$D_{\Delta l}(\tau) \to 2D_l(d) \qquad \text{for } \tau \gg \tau_0$$

are valid also for arbitrary form of the structure function $D_l(\tau)$, rather than only for a power-law function. In this case within an intermediate range ($\tau \simeq \tau_0$) the differential structure function $D_{\Delta l}(\tau)$ naturally differs in its form from $F(\tau/\tau_0, \mu)$ and depends on the particular form of the function $D_l(\tau)$.

Thus, measuring fluctuations of the radio wave angle of arrival provides the way not only to access characteristics of these fluctuations themselves, but also to estimate the following parameters of the turbulent model of the pathlength fluctuations through the troposphere:

(i) the exponent of the structure function, μ;
(ii) the structure coefficient C_{lt} of the temporal structure function;
(iii) the magnitude of the spatial structure function, $D_l(d)$;
(iv) the average velocity of irregularity motions in the troposphere, v.

4. RESULTS

For each session of observations, we calculated the temporal structure function $D_\theta(\tau)$ of the arrival angle of fluctuations. In most cases these structure functions look much alike and can be easily fitted by the dependence of the form (12):

$$D_\theta(\tau) = C_\theta^2 \tau_0^\mu F(\tau/\tau_0, \mu). \tag{16}$$

Within the range of time intervals $\tau < 10$ s, these functions increase as a power law with the exponent μ close to 5/3, but this growth becomes slower for longer intervals and for $\tau > 1$ min saturation takes place, $D_\theta(\tau) \to 2\sigma_\theta^2$. Examples of the structure function obtained for individual sessions are given in Figure 6.

Basing on the complete data set recorded in all 123 sessions we obtained the average structure function shown in Figure 7. Approximating the result using Eq. (16) we obtain the following values for the parameters of the arrival angle fluctuations:

$$\mu = 1.707, \qquad C_\theta = 0.115'' s^{-\mu/2}, \qquad \sigma_\theta = 0.80'', \qquad \tau_0 = 14.5\ \text{s}.$$

The corresponding parameters of the temporal structure function of the pathlength fluctuations through the troposphere in the zenith direction are as follows:

$$\mu = 1.707, \qquad C_{lt} = 0.045\ \text{mm} \cdot \text{s}^{-\mu/2}.$$

The magnitude of the spatial structure function for the base of $d = 145$ m is obtained to be

$$[D_l(145\ \text{m})]^{-1/2} = 0.44\ \text{mm}.$$

The average velocity of the irregularity motions is given by

$$v = 10.0\ \text{m/s}.$$

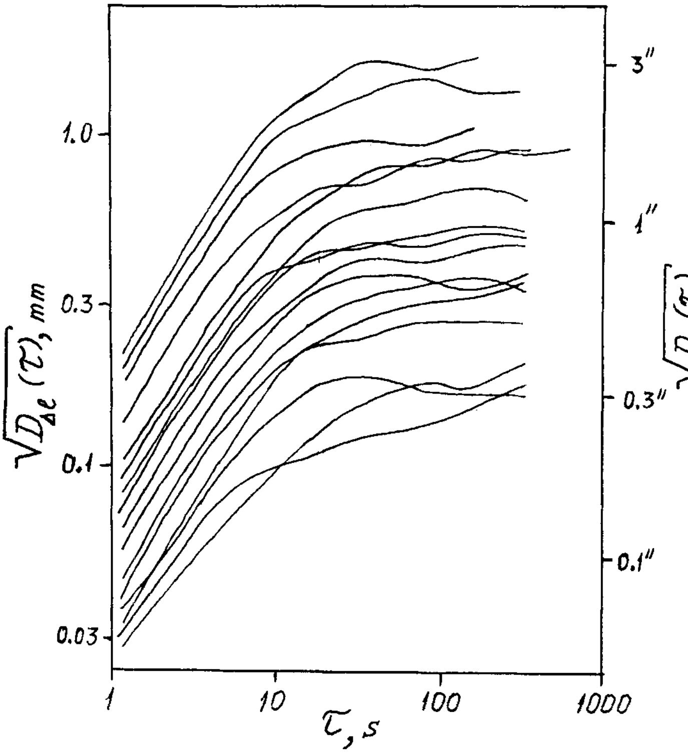

Figure 6 Examples of experimental structure functions for different sessions of observations.

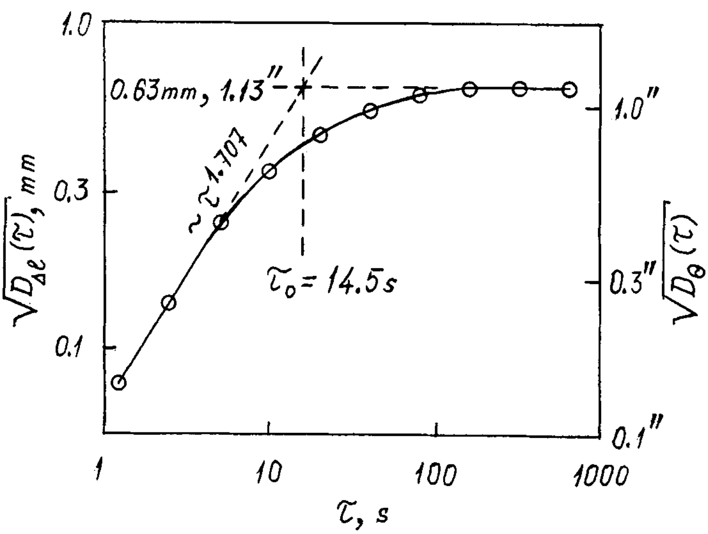

Figure 7 The averaged experimental structure function.

These estimates are in a good agreement with the turbulent model of the structure function discussed above (see Figure 1) which has following parameters:

$$\mu = 1.667, \qquad C_{lt} = 0.054\,\mathrm{mm}\cdot\mathrm{s}^{-\mu/2}, \qquad [D_l(145\,\mathrm{m})]^{-1/2} = 0.50\,\mathrm{mm}.$$

To reach a more detailed statistical description of the fluctuations, we note that the structure function approaches the form (11) already within a relatively short interval of observation. Thus, the small-scale power-law segment of the structure function acquires stable values μ and C_θ or C_{lt} even for recorded time series of duration of several seconds. To estimate σ_θ or $\sigma_{\Delta l}$, an observation session of several τ_0 is required.

Thus, one can consider the structure function in the form (11) with the parameters μ, C_{lt}, $\sigma_{\Delta l}$ and τ_0 varying in time. In order to estimate these variations, the slide average values were calculated for each parameter, with the averaging interval $T \gg \tau_0$, and the distribution functions of this values were obtained.

It turned out to be enough to make averaging within a 5-minute interval. Further increase of T does not affect the results essentially. Instead of τ_0, we finally consider a more universal quantity, $v = d/\tau_0$.

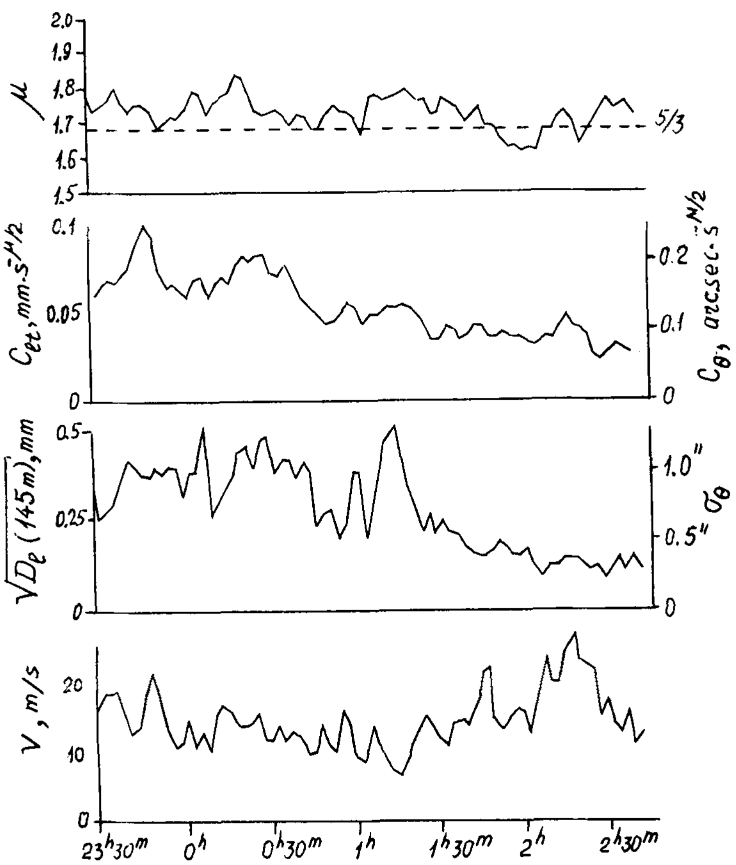

Figure 8 Variations of the structure function parameters.

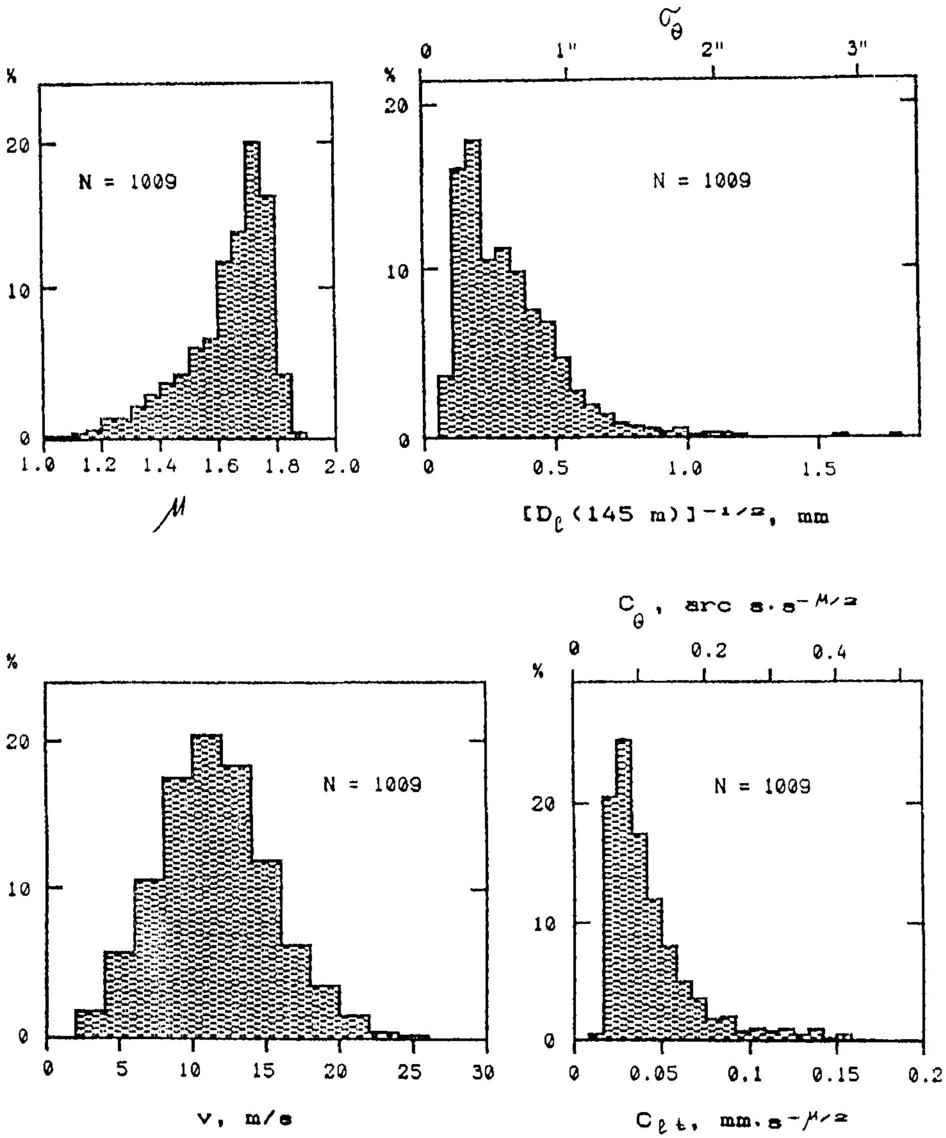

Figure 9 The probability distributions of the structure function parameters.

Variations of the structure function parameters averaged over the interval $T = 5$ minutes are exemplified in Figure 8 and their distribution functions are shown in Figure 9.

The value of μ turned out to be the most stable of all the parameters. It should be noted that estimates of this parameter are most reliable in this experiment because it is independent of the calibration errors and variations in the receiver gain. The distribution function of μ is strongly asymmetric with a gradual increase and a sudden drop, which explains why the most probable value of 1.725 is somewhat higher than both the theoretical one 5/3 and the estimate 1.707 obtained from the averaged structure function.

The remaining parameters display by far stronger time variations. The structure coefficient C_{lt} and the magnitude of the spatial structure function $D_l(145\,\mathrm{m})$ also have asymmetric distributions but with a rapid increase at the lower values and gradual decrease at the larger values. These distributions are close to the lognormal one which is common for parameters of the fluctuations in the atmosphere. The distribution of the irregularity motions velocity v is close to the normal one.

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