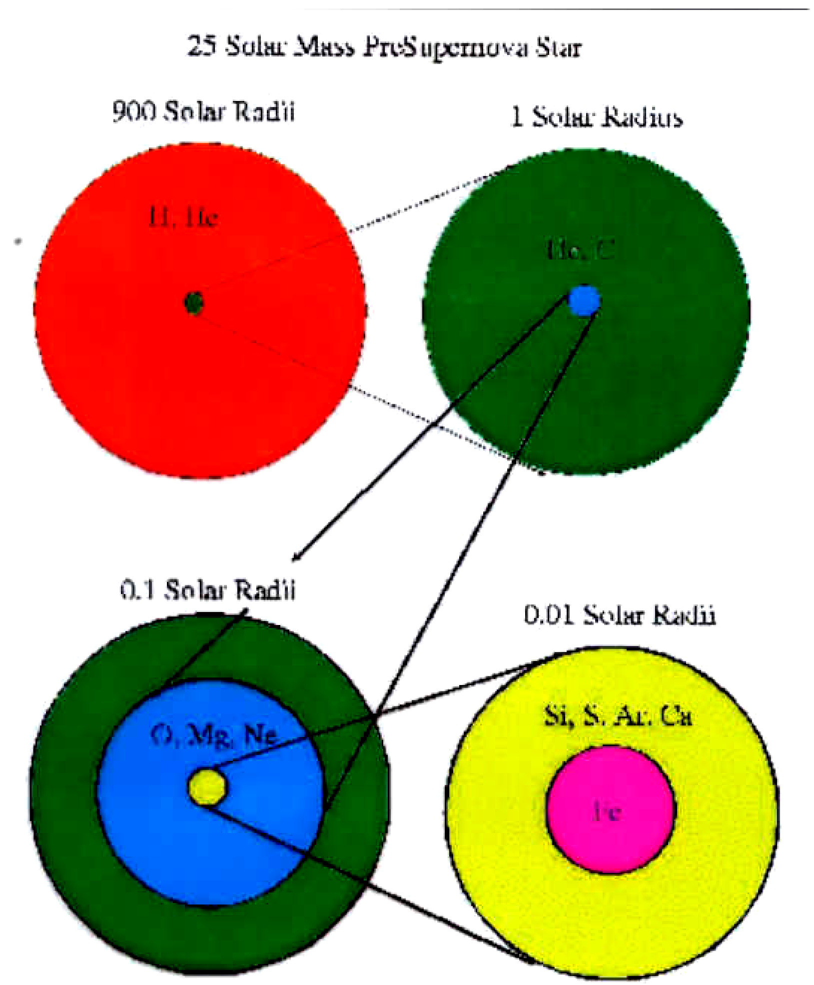
Lecture 5. Loss of stability in the stellar core. Photodesintegration of iron. Pair creation. Neutronization of matter and neutrino losses. Core collapse.

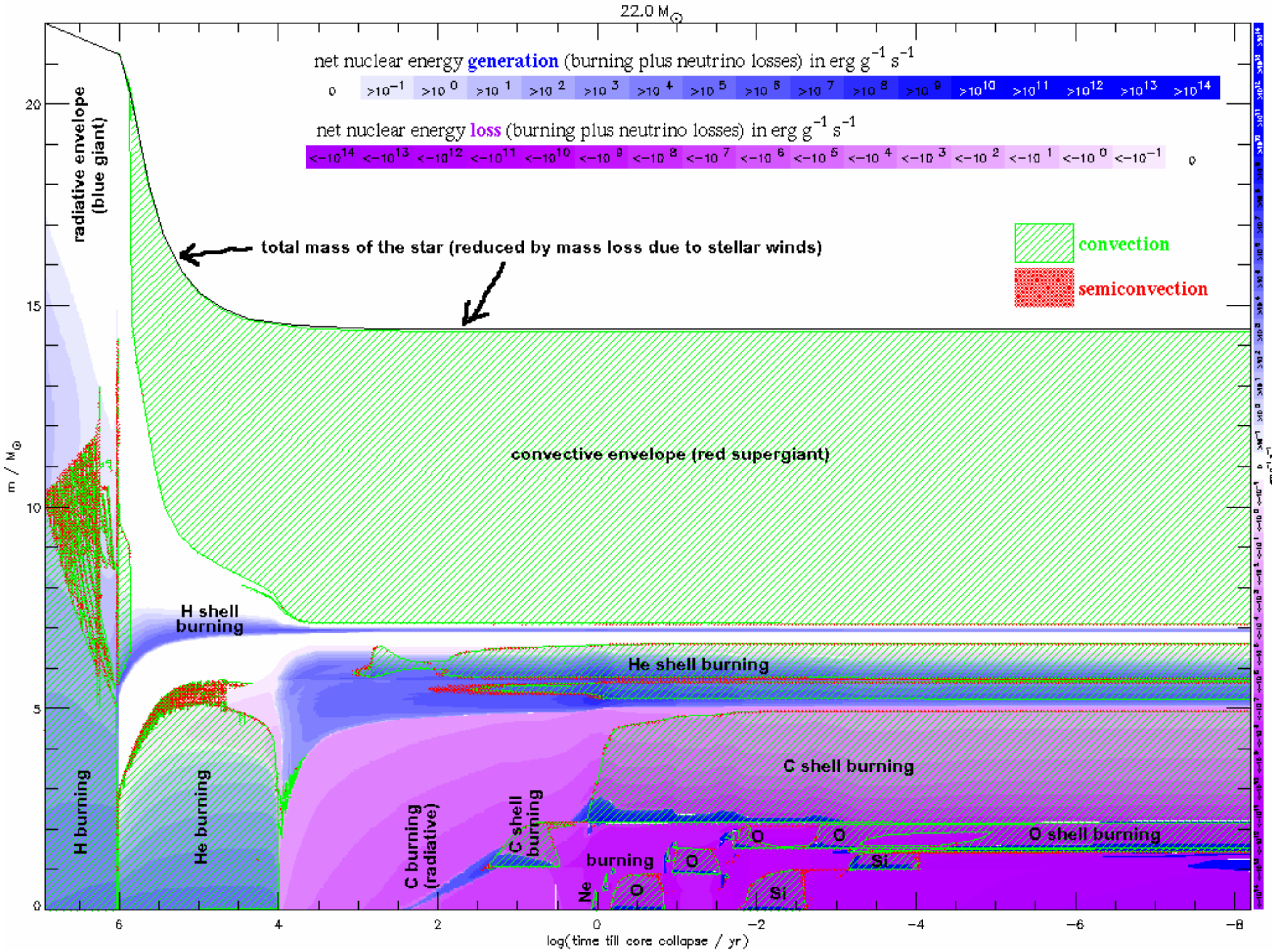
Shell structure of pre-SN star



Nuclear burning @ various masses

Burning stages		20 $M_\odot$ Star		200 $M_\odot$ Star	
Fuel	Main Product	T (10^9 K)	Time (yr)	T (10^9 K)	Time (yr)
H	He	0.02	10^7	0.1	2×10^6
He	O, C	0.2	10^6	0.3	2×10^5
C	Ne, Mg	0.8	10^3	1.2	10
Ne	O, Mg	1.5	3	2.5	3×10^{-6}
O	Si, S	2.0	0.8	3.0	2×10^{-6}
Si	Fe	3.5	0.02	4.5	3×10^{-7}

Particular example: Woosley, Heger, Langer models – the "Kepler code" (www.supersci.org)



Summary of Advanced Nuclear Burning Stages of Massive Stars (taken from Woosley, 2005)

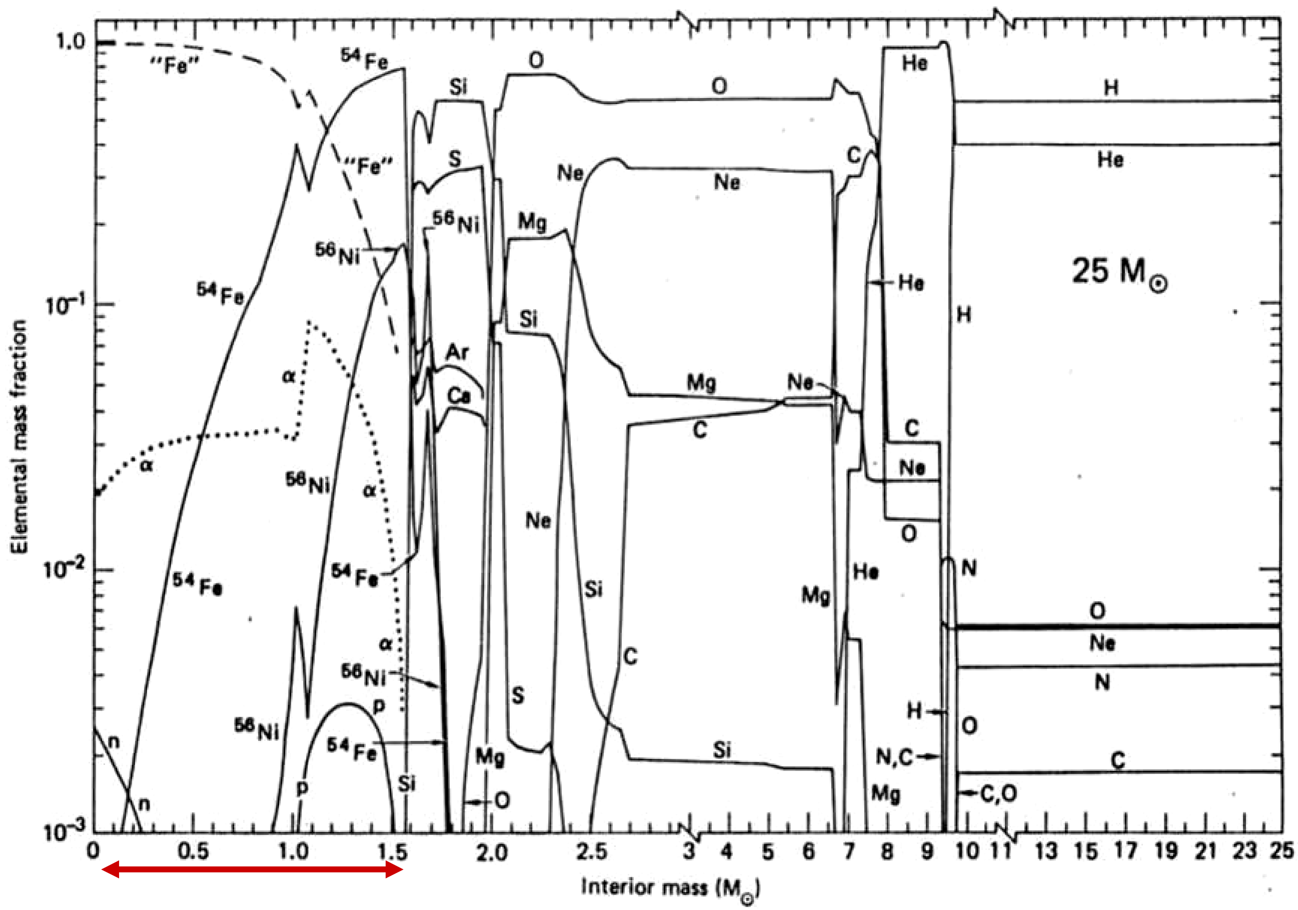
Because of the importance of neutrino losses, stellar evolution after helium burning is qualitatively different. Once the central temperature exceeds $\sim 5 \times 10^8$ K, neutrino losses from pair annihilation dominate the energy budget. Radiative diffusion and convection remain important to the star's structure and appearance, but it is neutrino losses that, globally, balance the power generated by gravitational contraction and nuclear reactions. Indeed, the advanced burning stages of a massive star can be envisioned overall as the neutrino-mediated Kelvin-Helmholtz contraction of a carbon-oxygen core, punctuated by occasional delays when the burning of a nuclear fuel provides enough energy to balance neutrino losses. Burning can go on simultaneously in the center of the star and in multiple shells, and the structure and composition can become quite complex. Owing to the extreme temperature sensitivity of the nuclear reactions however, each burning stage occurs at a nearly unique value of temperature and density.

Nucleosynthesis in these late stages is characterized by a great variety of nuclear reactions made possible by the higher temperature, the proliferation of trace elements from previous burning stages, and the fact that some of the key reactions, like carbon and oxygen fusion, liberate free neutrons, protons, and α-particles. It is impossible to keep track of all these nuclear transmutations using closed analytic expressions and one must resort to "nuclear reaction networks", coupled linearized arrays of differential rate equations, to solve for the evolution of the composition. As we shall see, these late burning stages, both before and during the explosion of massive stars, account for the synthesis of most of the heavy elements between atomic mass 16 and 88, as well as the p-process, and probably the r-process.

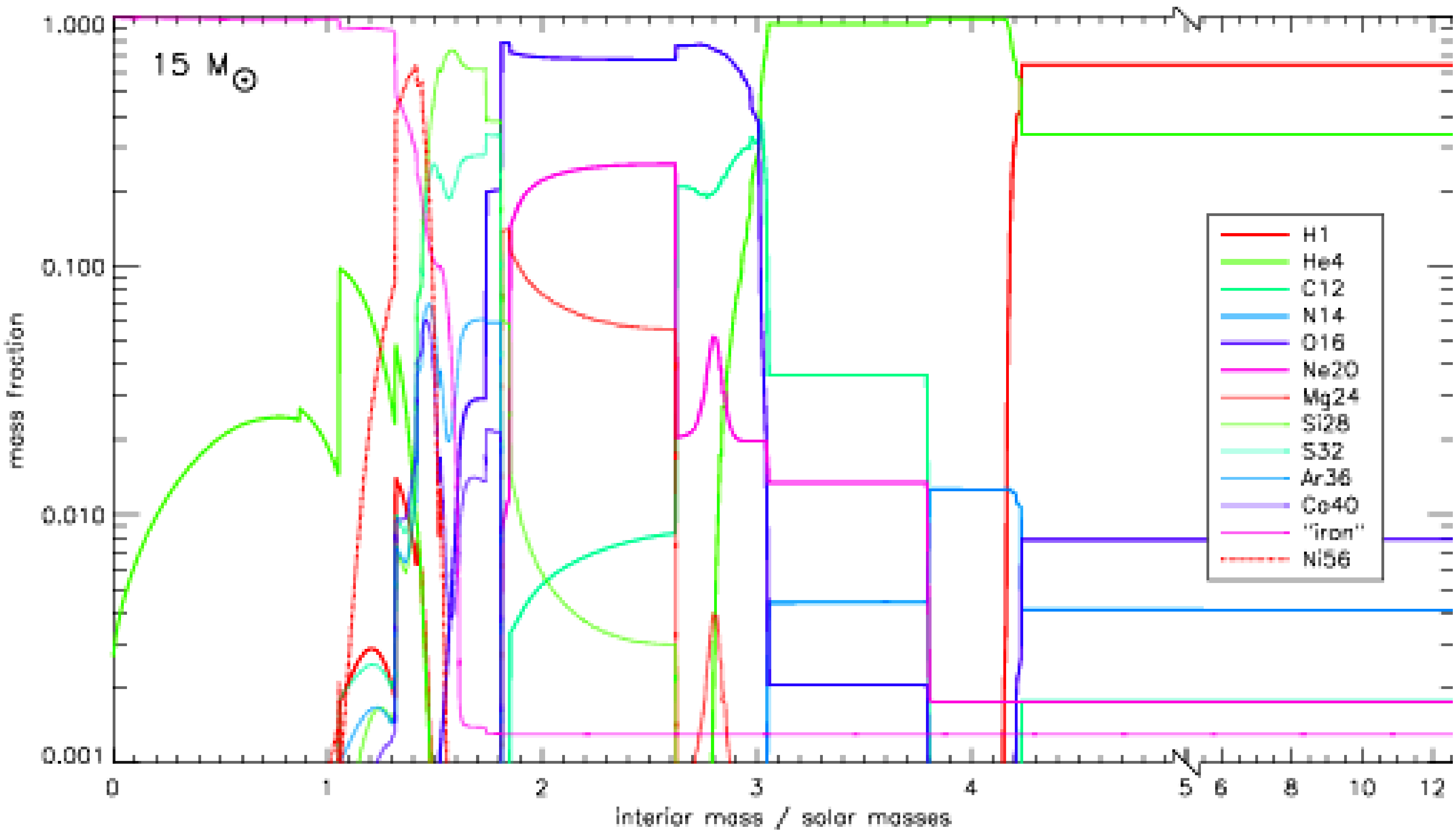
Except for a range of transition masses around 8 - 11 $M_\odot$, each massive star ignites successive burning stage at its center using the ashes of the previous stage as fuel for the next. Four distinct burning stages follow helium burning, characterized by their principal fuel - carbon, neon, oxygen, and silicon. Only two of these - carbon burning and oxygen burning - occur by binary fusion reactions. The other two require the partial photodisintegration of the fuel by thermal photons.

Because the late stages transpire so quickly, the surface evolution fails to keep pace and "freezes out". If the star is a red supergiant, then the Kelvin Helmholtz time scale for its hydrogen envelope is approximately 10,000 thousand years. Once carbon burning has started, the luminosity and effective emission temperature do not change until the star explodes. Wolf-Rayet stars, the pro-

Chemical structure of pre-Sn (after Woosley)

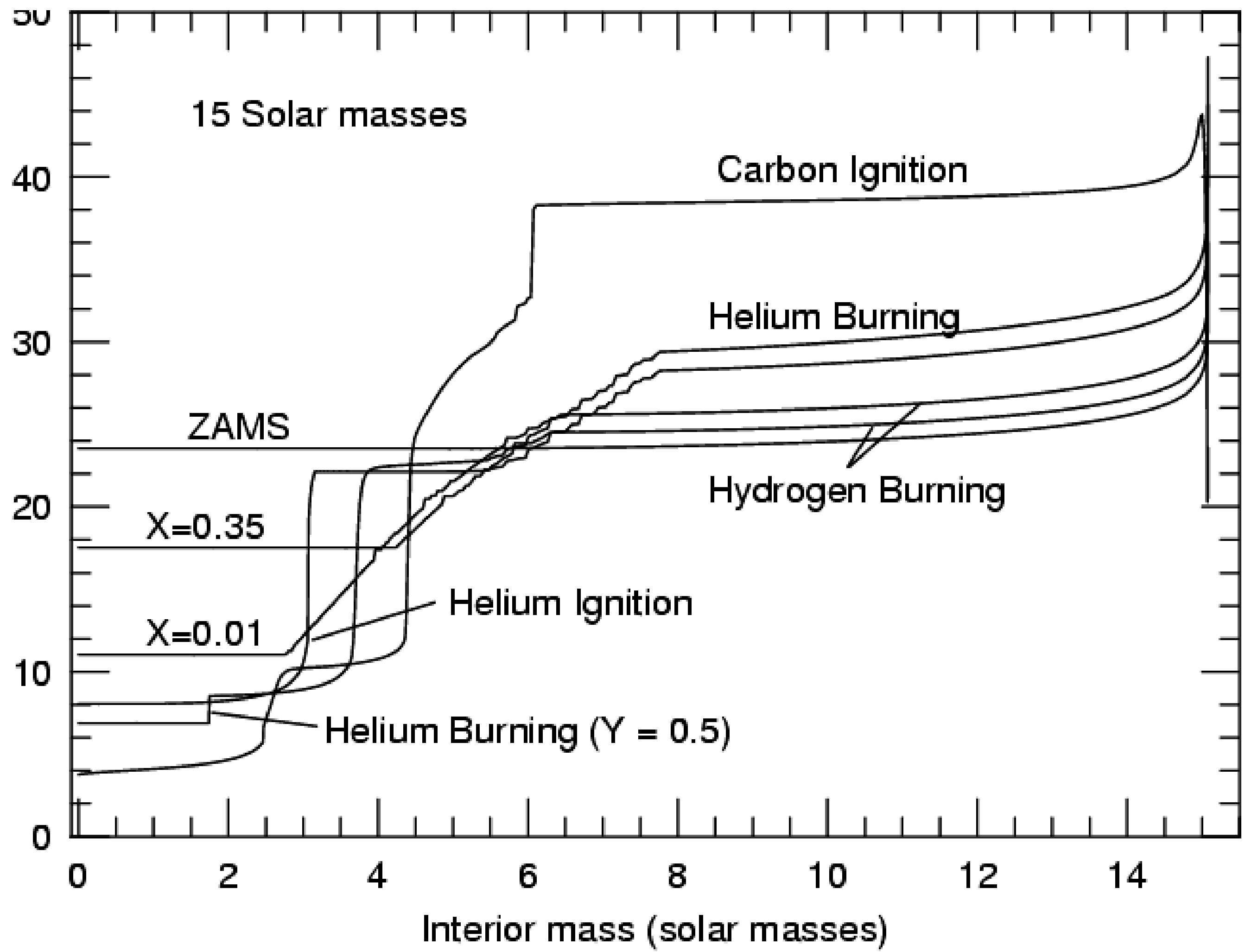


Iron core

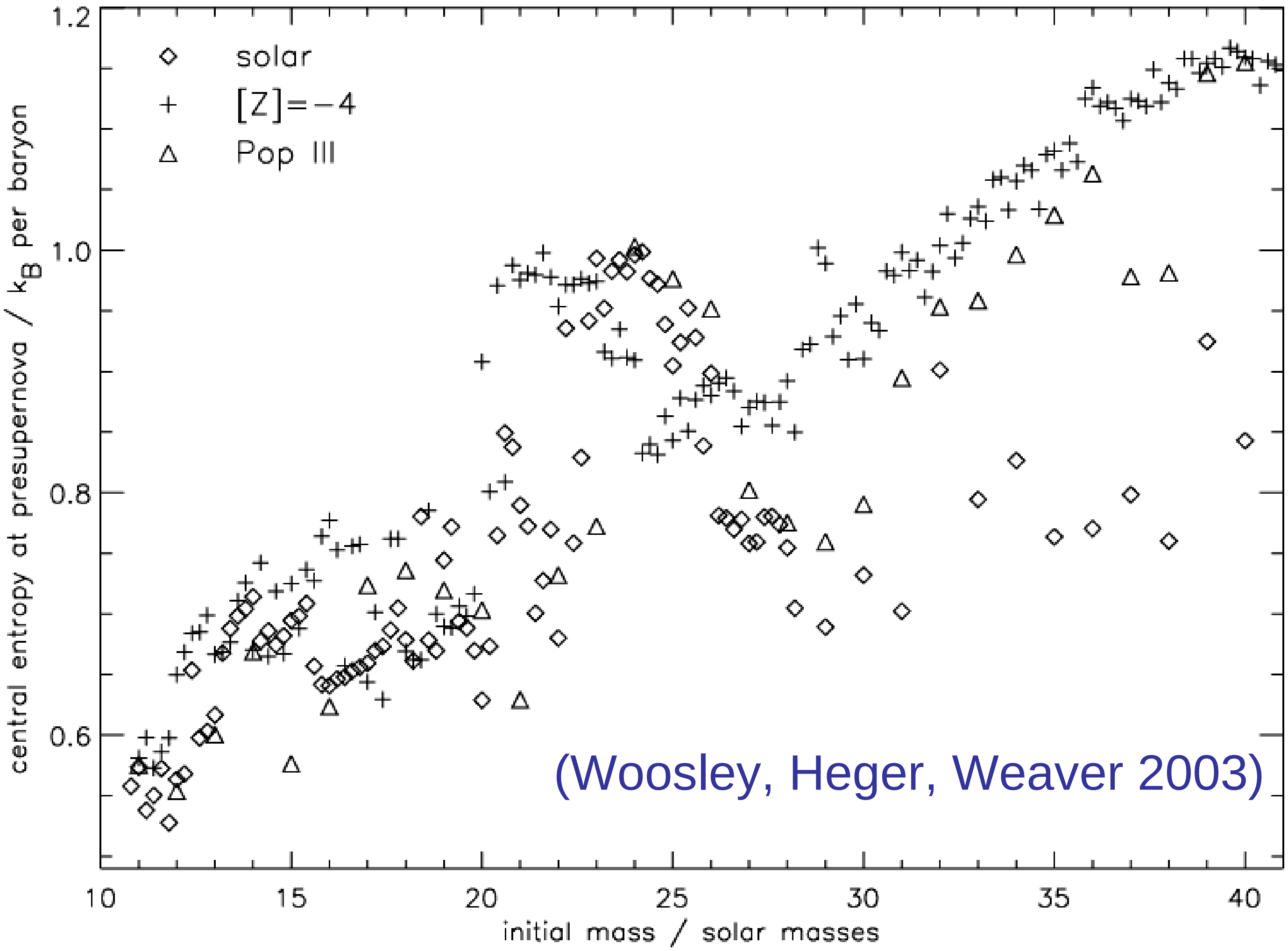


15 $M_\odot$
1.000
0.100
0.010
0.001
mass fraction
0
1
2
3
4
5
6
8
10
12
interior mass / solar masses
H1
He4
C12
N14
O16
Ne20
Mg24
Si28
S32
Ar36
Ca40
"iron"
Ni56

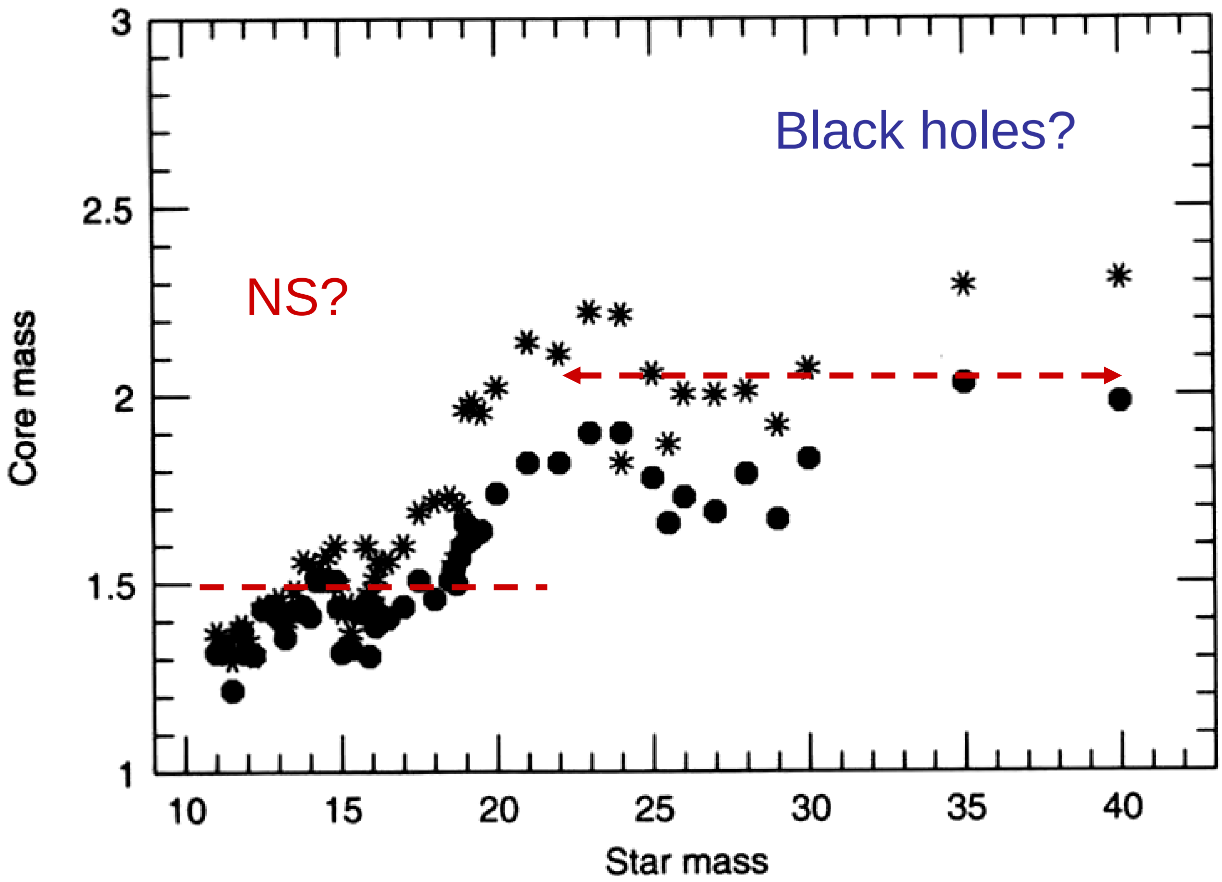
Entropy distribution ($s=S/k_B$) per nucleon determines TD properties of matter in the core



Central entropy prior the collapse



Iron core masses



Timmes, Woosley, Weaver 1995

After Si shell burning, mass of iron core ~1.3 $M_\odot$, entropy per baryon s~ 0.7. Neutrino losses further reduce s. As the density and temperature rise, the star seeks a new nuclear fuel to burn, but instead meets a phase transition in which α-particles are favored over bound nuclei (photodesintegration). This reduces the compressibility of the matter (reduces the adiabatic index Γ!). Electron captures by nuclei (neutronization) becomes important thus reducing lepton parameter $Y_e=n_e/n_b$ and hence the pressure. At higher densities pair production also reduces the pressure. And do not forget general relativity effects – pressure has "weight"! The star starts to collapse on thermal time scale and accelerates to dynamical time scale.

Physical reasons for collapse: I. Photodesintegration of iron (Hoyle, Fowler)

At T~0.5 MeV: $\gamma + {}^{56}Fe \rightarrow {}^{52}Cr + \alpha$ begins. Less bound nuclei are quickly dissolved down to helium. So

$$\boxed{{}^{56}Fe \rightarrow 13\alpha + 4n}$$

(*Note*: neutrons are not really "free", they are bound to lighter nuclei!)

Energy required for photodissociation:

$$\boxed{\chi = (13\mathrm{m}_\alpha + 4m_n - m_{Fe})c^2 \simeq 120\ \mathrm{MeV}}$$

(Note: photodissociation starts at $\mathrm{T} \ll \chi$ -- for the same reason as in the Saha eq. for equilibrium ionization -- huge statistical weight of particles in the dissolved (Saha -- ionized) state !):

$$\frac{n_\alpha^{13} n^4_{\mathrm{n}}}{n_{Fe}} \sim T^{24} e^{-\chi/T}$$

The work of pressure is done to dissolve nuclei $\Rightarrow$ **compressibility decreases**.

At at higher even α-particles dissolve into neutrons.

BUT: Due to low entropy nuclei do not desintegrate completely (H.Bethe) and survive in the collapse down to the formation of neutron star!

General feature: compressibility decreases for any dissociation (e.g. ionization, dissociation of nuclei....). Explanation:

Before dissociation: Ideal gas of particles with atomic weight A_i (=26 for iron) (OK for iron nuclei in the pre-SN core):

$$\boxed{S=\frac{N_A k_B}{A_i}\ln\left(\frac{T^{3/2}}{n_i}\right)+const=\frac{3}{2}\frac{N_A k_B}{A_i}\ln\left(\frac{P}{n_i^{5/3}}\right)+const}$$

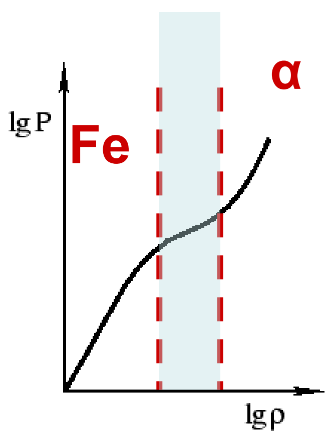
so for pressure: $P \propto n^{5/3}$, *i.e.* $\Gamma = 5/3$. Through density: $\rho = m_i n_i$:

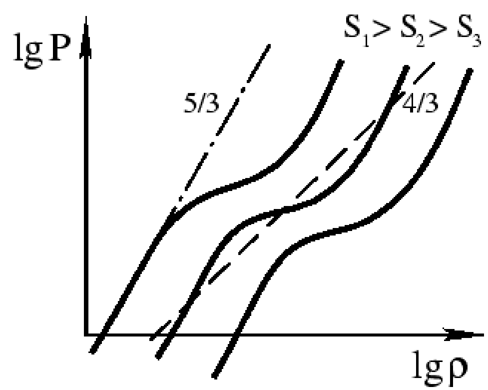
$$\boxed{P_{before} \sim e^{\frac{2A_i S}{3N_A k_B}}\rho^{5/3}}$$

After adiabatic compression with S=const $Fe \rightarrow \alpha$ with $A_\alpha = 4$,

$P_{after} \sim e^{\frac{2A_\alpha S}{3N_A k_B}}\rho^{5/3}$, *i.e.* $\Gamma = 5/3$ again, but the coefficient is smaller.

In the region of dissociation must be $\Gamma<5/3$!





And do not forget that entropy decreases due to neutrino losses!

Ia. Pair creation

For very massive stars radiation dominates in the pressure (problem 1 in exercises 2!). For radiation:

$$S_r = \frac{4}{3} a_r \frac{T^3}{\rho} \quad \text{(here } a_r = \frac{4\sigma_B}{c} \text{ is the radiation constant).}$$

$$P = \frac{1}{3} a_r T^4 = \frac{a_r}{3}\left(\frac{3S\rho}{4a_r}\right)^{4/3}, \text{ so } \Gamma = \frac{d\ln P}{d\ln \rho} = \frac{4}{3}.$$

Very massive radiation-dominated stars are on the verge of the mechanical stability!

At high temperatures in non-degenerate gas $T \gg m_e c^2$ electron-positron pairs are produced copiously,

with $n_- - n_+ \sim e^{-\frac{kT}{2m_e c^2}}$, $n \sim T^3$ and energy density $\varepsilon_\pm \rho \approx \frac{7}{4} a_r T^4$.

Radiation + pairs: $\rho\varepsilon = a_r T^4 + \frac{7}{4} a_r T^4 = \frac{11}{4} a_r T^4$

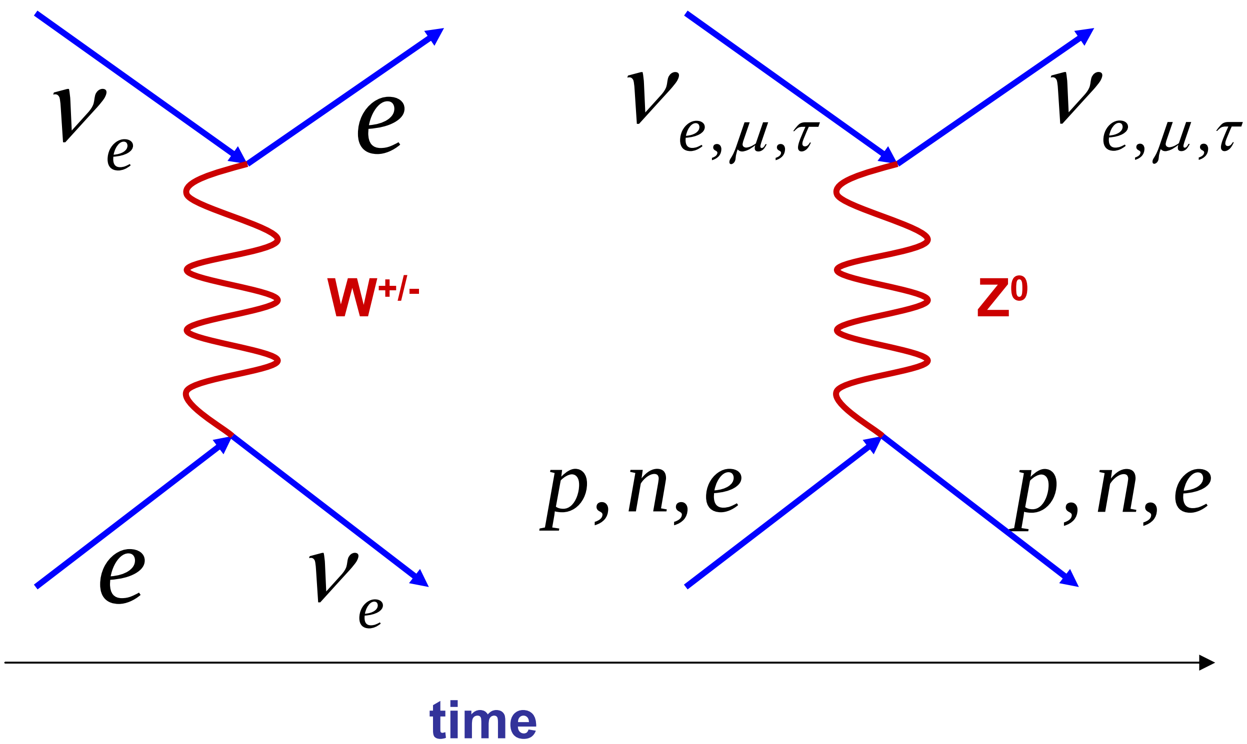
Pressure: $P = \frac{1}{3}\rho\varepsilon = \frac{11}{12} a_r T^4$

Entropy per gram: $S = \frac{11}{3} a_r \frac{T^3}{\rho} \Rightarrow P = \frac{11}{12} a_r T^4 = \frac{11 a_r}{12}\left(\frac{3S\rho}{11 a_r}\right)^{4/3}$

After pair creation $P \propto \rho^{4/3}$ again, but coefficient is smaller!
So Γ must be smaller 4/3 during pair creation. The work of pressure is done to create new particles, so compressibility decreases.
That is why there are no supermassive stars!

Physical reasons for collapse: II.Neutronization of matter

Weak interactions



M_W~80 GeV
M_Z~91 GeV
(CERN, 1983)

$$x \sim \frac{\hbar}{M_W c} \simeq 10^{-15}\,\mathrm{cm}$$

Rich variety of processes: with $W^{\pm}$ ("charge currents):

$n \to p + e^{-} + \tilde{\nu}_e$ (β^{-} n decay, $t_{1/2}$=890 s)

(actually $n \to p + W^{-}, W^{-} \to e^{-} + \tilde{\nu}_e$)

$p + e^{-} \to n + \nu_e$ (neutronization)

$p + \tilde{n} \to e^{+} + \nu_e$ (annihilation)

For muons:

$\mu^{-} \to e^{-} + \tilde{\nu}_e + \nu_\mu, \quad \mu^{+} \to e^{+} + \nu_e + \tilde{\nu}_\mu, \ (t_{1/2} = 2.2 \cdot 10^{-6}\mathrm{s})$

$\mu^{-} + p \to n + \nu_\mu$, etc.

More processes through Z^0 ("neutral currents"):

$e^{-} + \nu_e \rightleftharpoons e^{-} + \nu_e$

$e^{-} + e^{+} \rightleftharpoons \nu_e + \tilde{\nu}_e$

$e^{-} + (A,Z) \rightleftharpoons e^{-} + (A,Z) + \nu_e + \tilde{\nu}_e$ (cannot go on free e)

All these processes are described by Fermi constant

$G_F \approx 1.41 \times 10^{-49} \mathrm{erg} \cdot \mathrm{cm}^3 = 1.16 \times 10^{-5} \mathrm{GeV}^{-2} \ (\hbar = c = 1)$

and are characterized by the crossection

$$\sigma \approx 10^{-44} \mathrm{cm}^2 (E / 1\mathrm{MeV})^2$$

In stars, neutrinos fly away and carry energy.

URCA processes (Gamow, Schöenberg 1940):

$$\boxed{\begin{aligned}&(A,Z)+e^- \xrightarrow{E_e>\Delta} (A,Z-1)+\nu_e\\ &(A,Z-1)\to(A,Z)+e^-+\tilde{\nu}_e\end{aligned}}$$

(A,Z) – stable, (A,Z-1) - unstable

do not change the total number of nuclei: $n_{A,Z}+n_{A,Z-1}$

Neutrinos escape and carry energy away from stellar interiors.

At high T, most nuclei are dissociated,

so on free nulcleos

$$e^- + p \to n+\nu,\ \Delta > 0.78\text{MeV} = (m_n - m_p)c^2,$$

$$n \to p + e^- + \tilde{\nu}_e$$

When $T \geq 0.8$ MeV, a lot of e^+e^- pairs appears, and

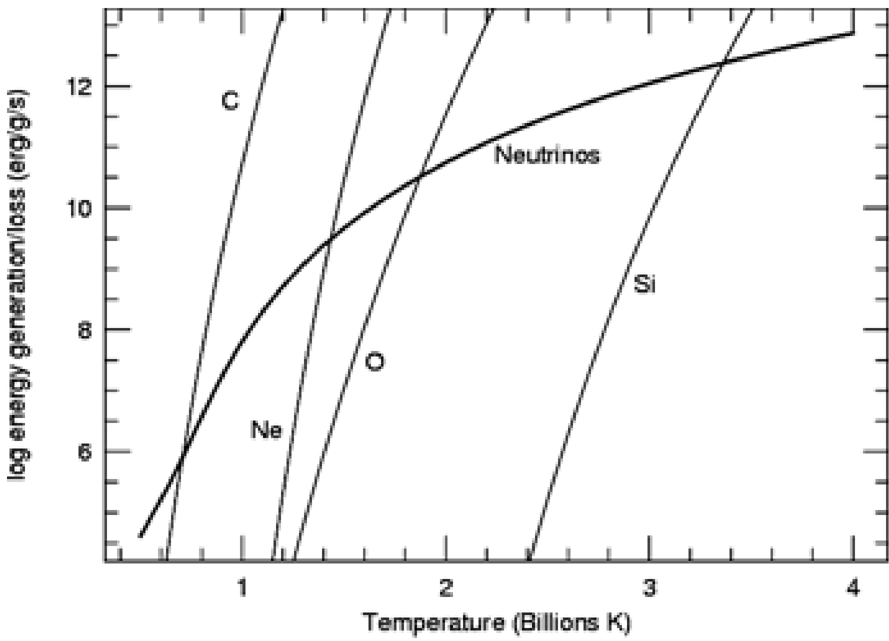
$$\boxed{e^+ + (A,Z\text{-}1) \to (A,Z) + \tilde{\nu}_e.}$$

Energy losses at $T \gg m_e c^2$:

$$\rho\varepsilon^- \sim n\sigma c E_\nu \propto T^3\cdot E^2 \cdot E_\nu \propto T^6\ .$$

At T>3 MeV: $e^+, e^- \to (\nu\tilde{\nu})_{e,\mu,\tau}$, so

$$\rho\varepsilon^- \sim n_+ n_- \sigma c E_\nu \propto T^3\cdot T^3\cdot T^2\cdot T \propto T^9.$$



At high densities: **neutronization** $p + e \rightarrow n + \nu$

can go even at zero T!

Energy threshold is just Fermi energy of electron, which (at T=0) is a function of density: $\Delta > 0.8$ MeV.

When p is bound in a nicleus, Δ strongly varies:

$$^{3}He + e^{-} \rightarrow {}^{3}H + \nu, \quad \Delta = 18 \text{ keV } (\rho > 10^{4} \text{ g/cm}^{3})$$

$$^{4}He + e^{-} \rightarrow {}^{4}H + n + \nu, \quad \Delta = 20 \text{ MeV } (\rho > 10^{11} \text{g/cm}^{3})$$

$$^{56}Fe + e^{-} \rightarrow {}^{56}Mn + \nu, \quad \Delta = 4 \text{ MeV } (\rho > 10^{11} \text{g/cm}^{3})$$

Due to neutronization, number of electrons Y_e *decreases*, which is equivalent to phase transition of the first kind $\Rightarrow$ brake on the logP-logρ diagram.

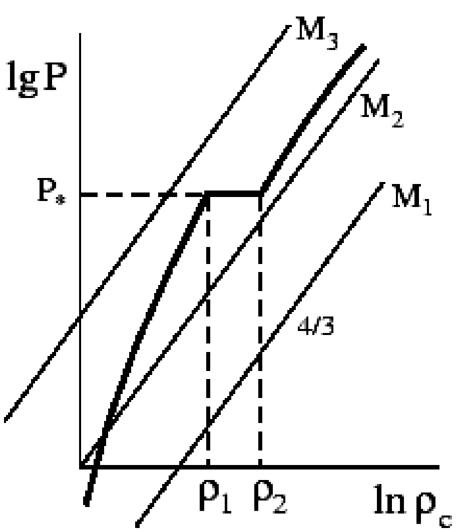
Remember: from hydrostatic equilibrium

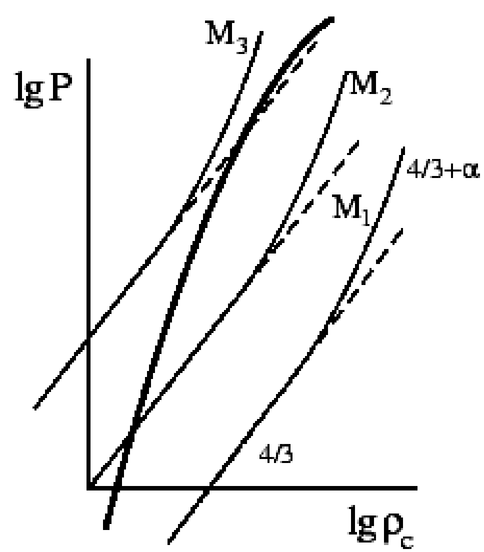
$$\boxed{\frac{P_c}{\rho_c} \sim \frac{GM}{R} \sim GM^{2/3}\rho^{1/3} \Rightarrow P_c \sim GM^{2/3}\rho_c^{4/3}}$$

$EOS: P_c = K\rho_c^{\Gamma}$, $\Gamma \to 4/3$ for relativistic degenerate fermions at high densities. So mechanical equilibrium is lost when

$$\boxed{\mathrm{M} \to \mathrm{M}_{\mathrm{Ch}} = (K/G)^{3/2} \approx 5.83 M_{\odot} Y_e^2 \quad \downarrow \text{ when } \mathrm{Y_e} \downarrow}$$

Any phase transition of 1st kind brings the core to instability as the Chandrasekhar limit decreases.





Effects of GR :

$$F(r) \approx \frac{GMm}{r(r-r_g)},\ r_g = \frac{2GM}{c^2}$$

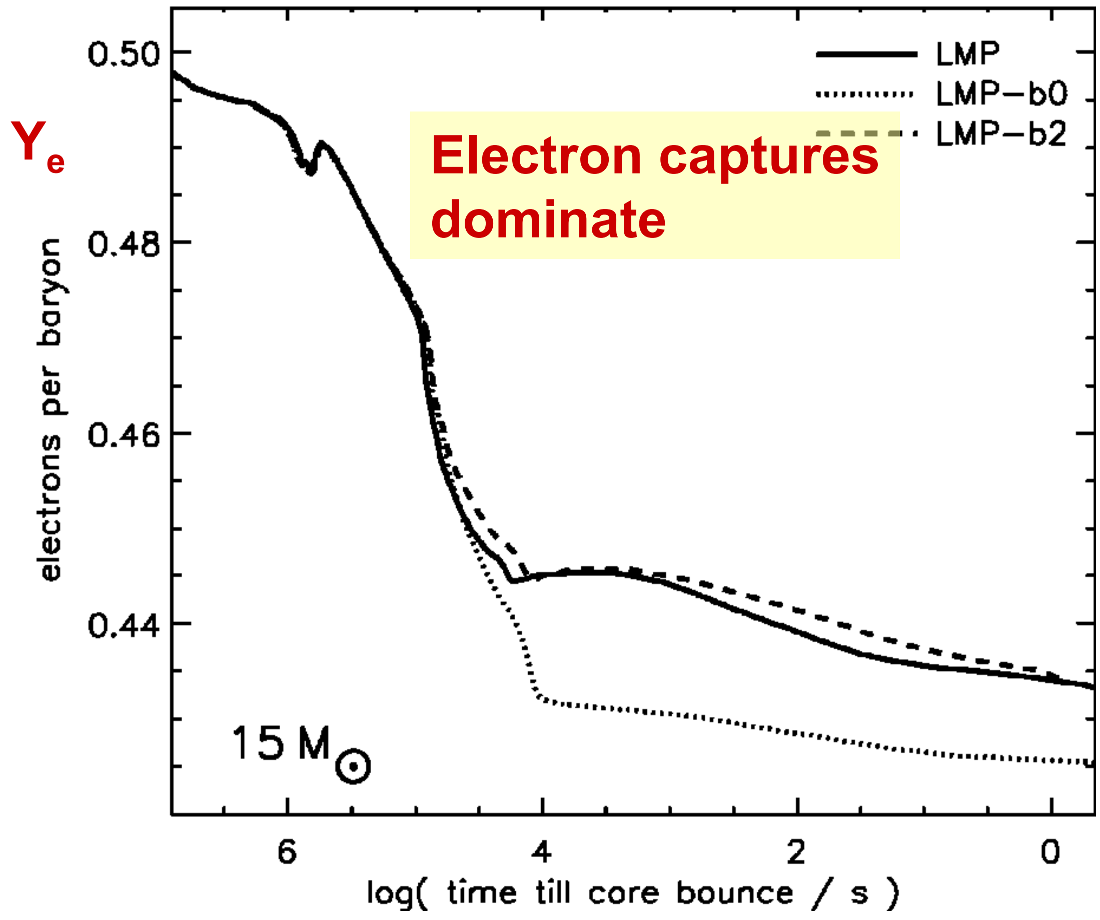
changes hydrostatic equilibrium:

$$P_c \propto \rho_c^{4/3+\alpha},\ \alpha > 0 \Rightarrow$$

the instability is reached at **a finite density**!

And $\mathrm{M_{Ch}}$ *decreases.*

Calculations by Woosley, Heger and Weaver (2002)



What happens?

As the density rises, so does the pressure (it never decreases at the middle), but so does gravity. The rise in pressure is not enough to maintain hydrostatic equilibrium, i.e., $\Gamma < 4/3$. The collapse accelerates.

Photodesintegration also decreases s_e because at constant total entropy (the collapse is almost adiabatic), s_i increases since 14 particles have more statistical weight than one nucleus. The increase in s_i comes at the expense of s_e.

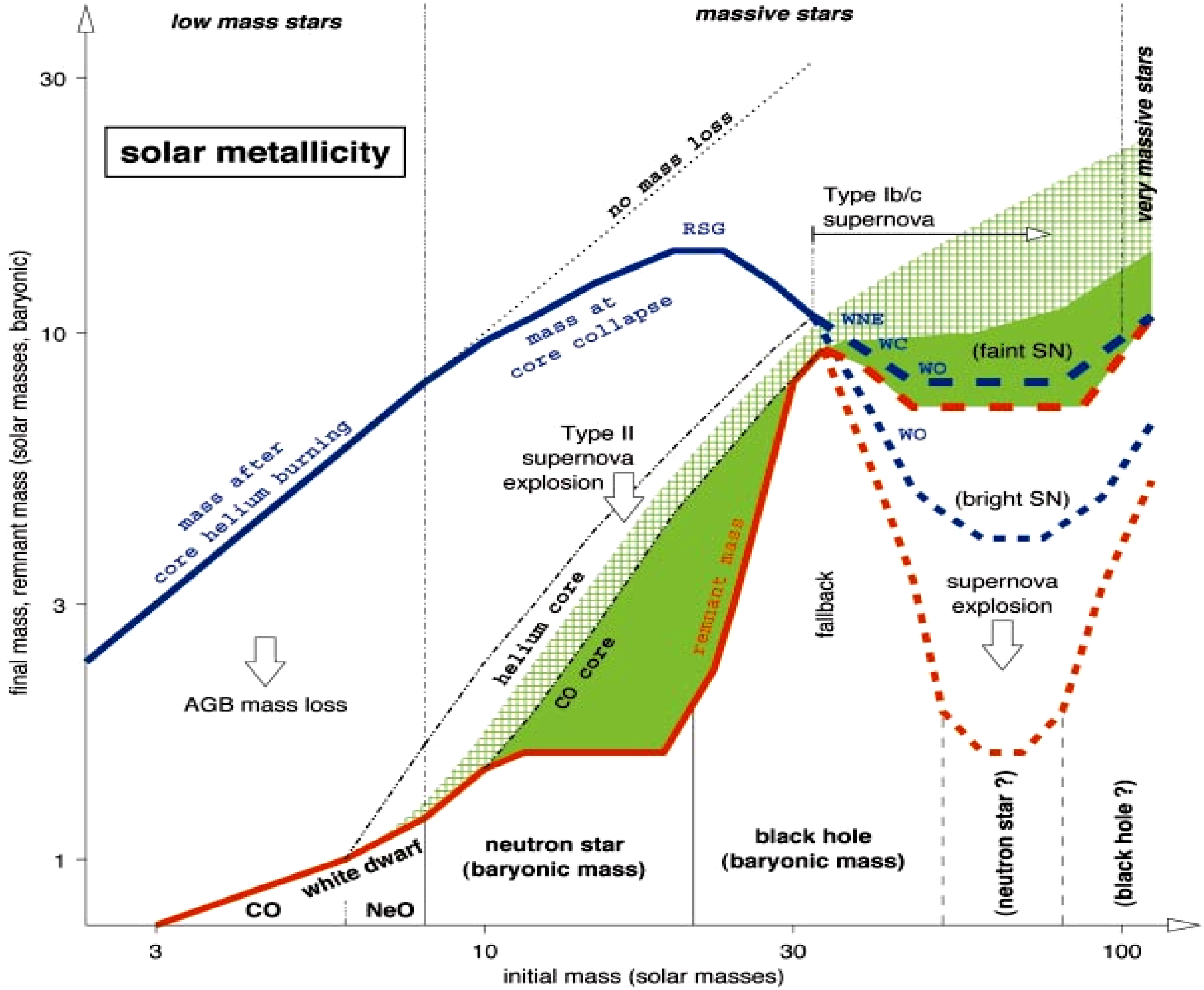
But the star does not a) photodisintegrate to neutrons and protons; then b) capture electrons on free protons; and c) collapse to nuclear density as a free neutron gas as some texts naively describe.

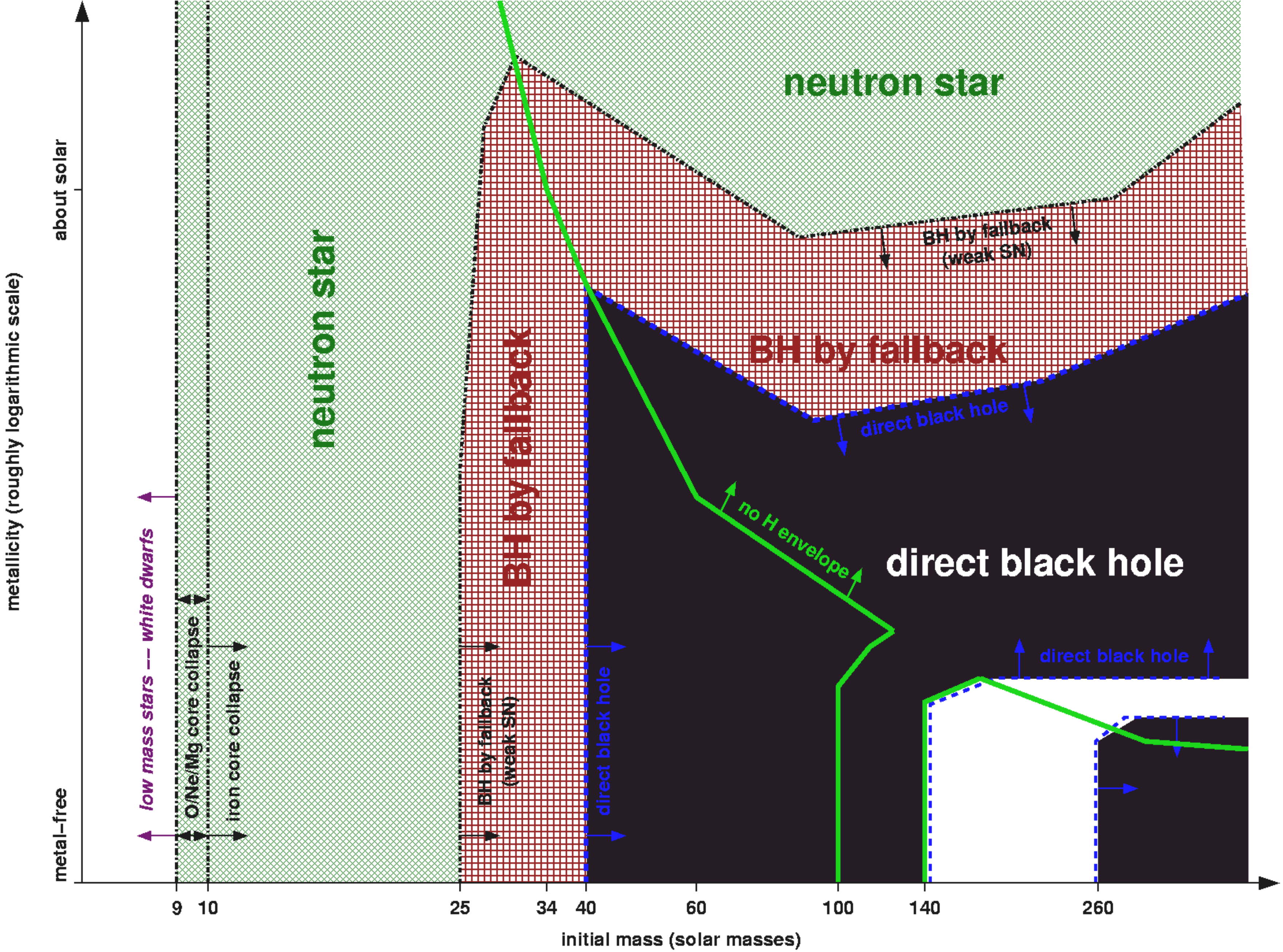
Bound nuclei persist, then finally touch and melt into one gigantic nucleus with ~10^{57} nucleons – the neutron star.

Y_e declines to about 0.37 before the core becomes opaque to neutrinos. (Y_e for an old cold neutron star is about 0.05; Y_e for the neutron star that bounces when a supernova occurs is about 0.29).

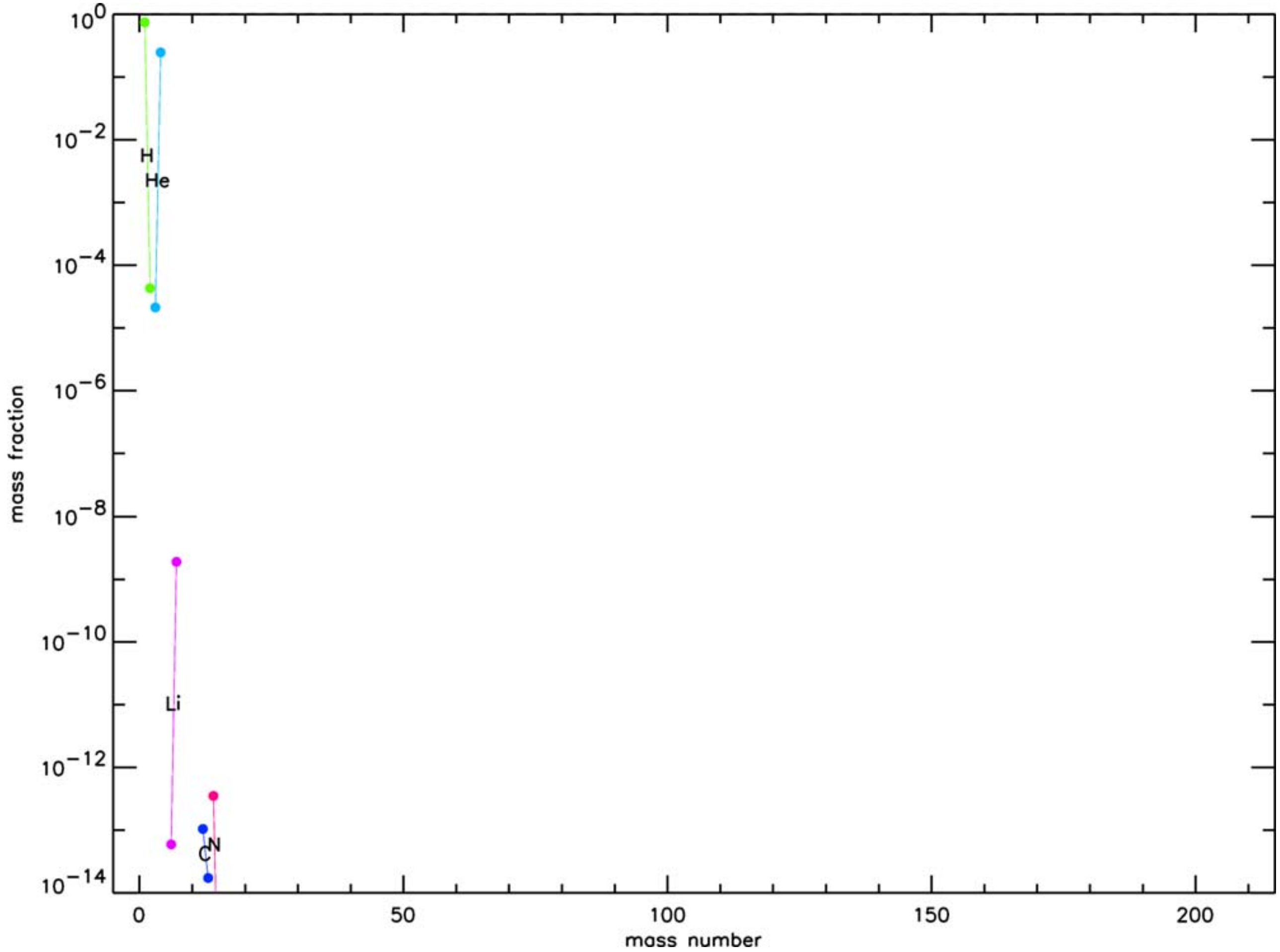
The effects of a) exceeding the Chandrasekhar mass, b) photodisintegration and c) electron capture operate together, not independently.

Stellar evolution remnants (from Woosley, Heger, Weaver 2002)

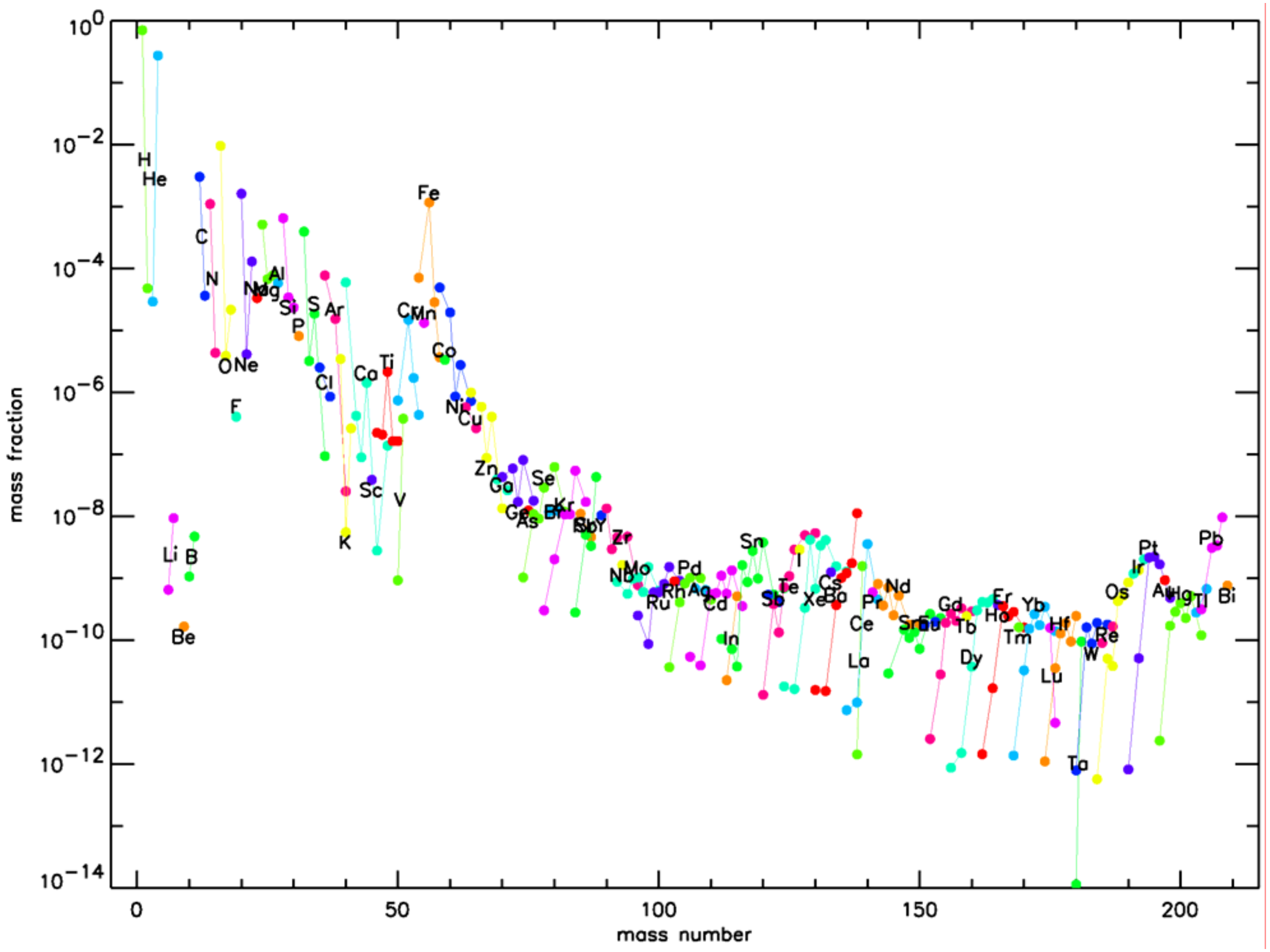


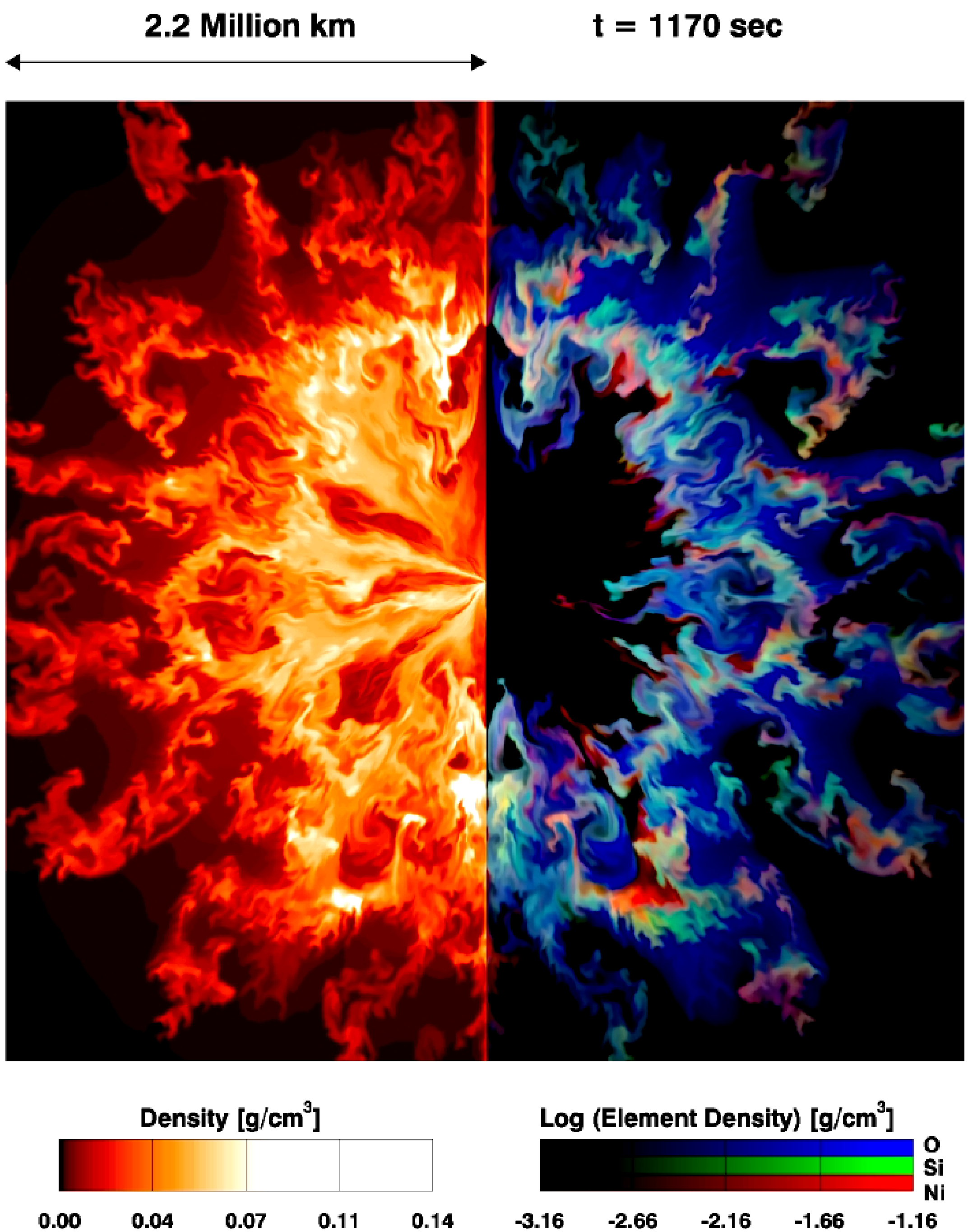

neutron star
neutron star
BH by fallback (weak SN)
BH by fallback
BH by fallback
direct black hole
no H envelope
direct black hole
direct black hole
direct black hole
BH by fallback (weak SN)
low mass stars -- white dwarfs
O/Ne/Mg core collapse
iron core collapse
metallicity (roughly logarithmic scale)
about solar
metal-free
9
10
25
34
40
60
100
140
260
initial mass (solar masses)

Primordial nucleosynthesis (B.Fields)



Today (Solar abundance pattern)





Mixing

early during the supernova explosion may allow materiel from the bottom of exploding star come out
-- even if most of the core falls back to form a black hole.

(Kifonidis et al. 2000)