# Lecture 3. Helium burning (3α-process). Degeneracy of matter in stellar interiors. Evolution of stars with $M<8\ M_{\odot}$. Red giants. AGB stars. Formation of planetary nebulae.

# Off the main sequence.Core He burning

- After core hydrogen exhaustion: core contracts, central density and temperature rise. Star goes off the main sequence toward red giant branch as conditions for H burning in shell layers appear
- helium burning (~10% of $t_H$): starts at $T \sim 2\text{-}3 \times 10^8$K, $\rho \sim 10^3\text{-}10^4$g/ccm
- No stable nuclei with A=5 and A=8 ➔ first stable reaction $3\ ^4He \rightarrow\ ^{12}C$

I. $^4He + ^4He \leftrightarrow ^8Be$ - 92 keV

II. $^8Be + ^4He \leftrightarrow ^{12}C^*$ - 0.29 MeV

III. 1/2500 cases: $^{12}C^* \rightarrow ^{12}C$ + 7.65 MeV ($e^+e^-$)

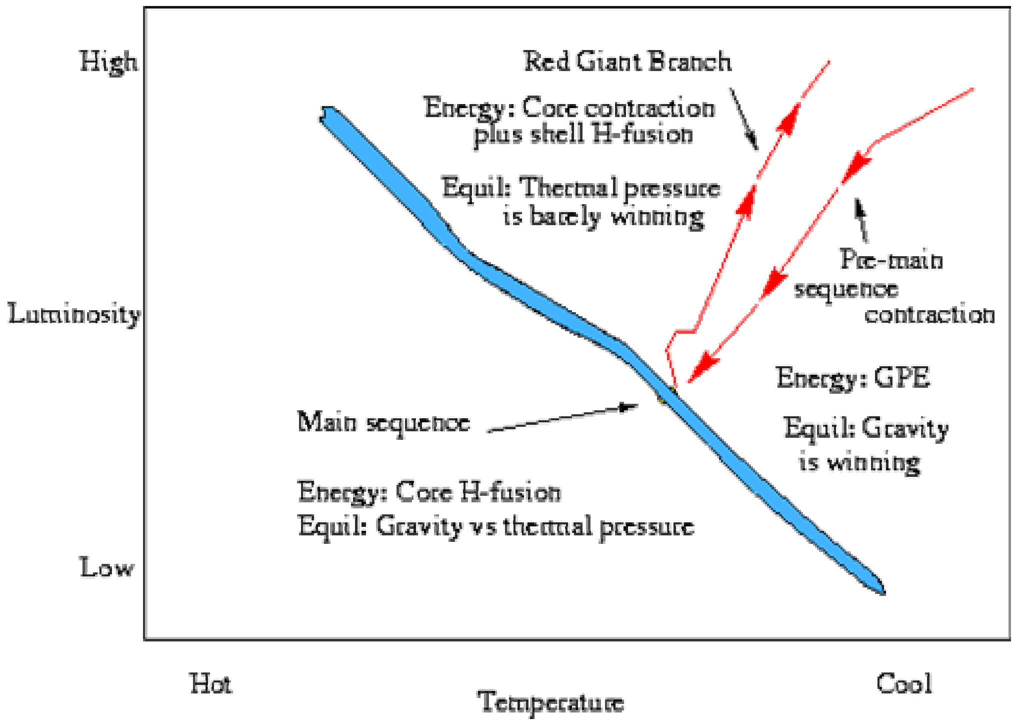
- *Problem:* $(3m_\alpha - m_{C12})c^2 = 7.28$ MeV. If $^{12}C^*$ has no appropriate energy level, the reaction proceeds extremely slow and no other element after Helium would form in the Universe :(((.Fortunately, it has (predicted by F. Hoyle and found by W. Fowler) ➔ resonance exists!
- Energy generation per gram $\varepsilon \sim \rho^2$ (3 particles must meet)
- Immediately followed by: $^4He + ^{12}C \rightarrow ^{16}O$ + 7.16 MeV

Result: $M < 10\ M_\odot$ - (degenerate) Carbon cores with admixture $^{16}O$

$10 < M < 30\ M_\odot$ - CO-cores

$M > 30\ M_\odot$ - Oxygen cores

# Off the main sequence: qualitative



# Off the main sequence: more quantitative

- Central density increases → degeneracy of matter becomes important (especially in low mass stars as on the main sequence $\rho \sim 1/M^2$ ). Indeed, from virial theorem for $\Gamma=5/3$

$$Q=3/2\, \mathcal{R} \langle T \rangle M = -\, 1/2\, U \sim GM^{5/3}\rho^{1/3},\ E=Q+U \sim -GM^{5/3}\rho^{1/3}$$

  Star emits radiation so $\Delta E<0$ → density must increase
- Principal consequence of increasing density – Coulomb corrections + degeneracy effects to Maxwell-Boltzmann ideal gas EOS. It is the physical reason for divergence of evolution of stellar cores with different mass after the main sequence.
- If $M_{core} < M_{Ch} \sim 1.4\, M_\odot$ (i.e. $M<8\text{-}10\, M_\odot$), degeneracy wins;
- If $M_{core} > M_{Ch}$ (i.e. $M>8\text{-}10\, M_\odot$), nuclear evolution proceeds under non-degenerate conditions up to the formation of mostly bound iron group nuclei

# Intermezzo 1: Degenerate Fermi gases

W.Pauli
(1900-1958)

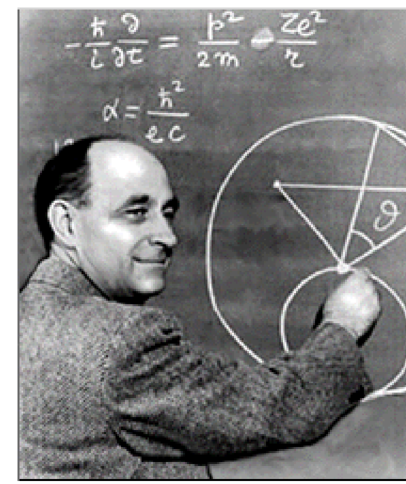
Pauli exclusion principle (1925): 2 electrons in atom cannot have identical quantum numbers – explains Periodic Table of elements! More generally: for any 2 fermions (half-integer spin: e, n, ν…). Follows from

(i) particles identity in quantum mechanics and

(ii) odd parity of wave functions of fermions

Degeneracy of fermions in matter at high densities or low temperatures:

*Only 2 electrons with opposite spins can occupy one phase space cell*



E.Fermi
(1901-1954)

$$2\frac{d^3p\,d^3x}{(2\pi\hbar)^3}=\frac{dN}{dVdp},\quad \int_0^{p_F}\frac{dN}{dVdp}dp=n=\frac{8\pi}{3}\frac{p_F^3}{(2\pi\hbar)^3},\quad p_F=\pi\hbar\left(\frac{3n}{\pi}\right)^{1/3},$$

$$\varepsilon_F=\sqrt{p_F^2c^2+m^2c^4}$$

$$Degeneracy\ in\ ideal\ gas:\ \ \varepsilon\sim kT\le\varepsilon_F,\ T_{dgnr}\sim 1eV\times\left(\frac{\rho}{10^3(g/cm^3)}\right)^{2/3}$$

# Intermezzo 2: Chandrasekhar limit (1930)
## (Nobel Prize in physics 1983, with W.Fowler)

$$\Delta x \Delta p \geq \hbar \quad (Heisenberg\ uncert.,\ 1927)$$

$$for\ electrons:\ p_{\lim} = p_F \sim \hbar / \Delta x = \hbar n_e^{1/3},$$

$$n_e = \frac{\rho}{m_p} Y_e,\ Y_e \equiv \frac{n_e}{n_b} = (1\ \ for\ H,\ 0.5\ \ for\ \ He,\ \frac{26}{56}\ \ for\ \ Fe)$$

$$Energy:\ \varepsilon_e \sim \varepsilon_F \simeq \frac{p_F^2}{2m_e}(non-rel.)... \simeq p_F c\,(ultrarel.)$$

$$Non-rel.\ pressure\ :P_e \sim n_e \varepsilon_e = \frac{\hbar^2}{2m_e} n_e^{5/3} \propto \rho^{5/3}$$

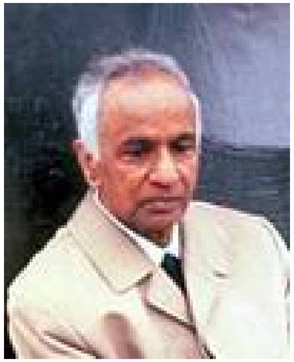
$$Ultrarelativistic\ \ gas:$$

$$P_e \sim p_F c n_e \approx \hbar c n_e^{4/3} = K_{rel} \rho^{4/3},\ \ K_{rel} = \frac{\hbar c}{m_p^{4/3}} Y_e^{4/3}$$

$$Chandrasekhar\ \ mass:$$

$$M_{Ch} = \left(\frac{K_{rel}}{G}\right)^{3/2} = \left(\frac{\hbar c}{m_p^{4/3}} \frac{Y_e^{4/3}}{G}\right)^{3/2} = N_{Ch} m_p Y_e^2 \approx 5.85 M_{\odot} Y_e^2 \approx 1.4 M_{\odot} (He, CO...)$$

$$N_{Ch} = \left(\frac{m_{Pl}}{m_p}\right)^3 \approx 10^{57},\ m_{Pl} = \sqrt{\frac{\hbar c}{G}} \approx 10^{19} GeV\ - Plankian\ mass$$



**S.Chandrasekhar (1910-1995)**

# Why evolution of massive stars is different from evolution of low mass star and which mass determines the boundary

$$EOS: ideal + Fermi\ non-rel.\quad P \approx \rho \Re T + K_{nr}\rho^{5/3}$$

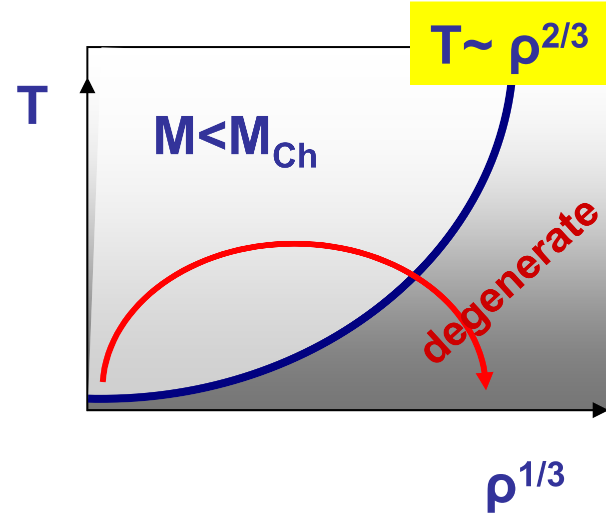
$$Hydrostatic\ eq.:\ P_c/\rho_c \sim GM/R = GM^{2/3}\rho_c^{1/3}$$

$$Central\ temperature:\ \Re T_c = GM^{2/3}\rho_c^{1/3} - K_{nr}\rho_c^{2/3}$$

$\Rightarrow$ *at high densities* $T_c$ *starts to decrease*

*and* $T \to 0$ *(white dwarfs!) as* $\Gamma \to 4/3$

*for any* $M \le M_{Ch}$.

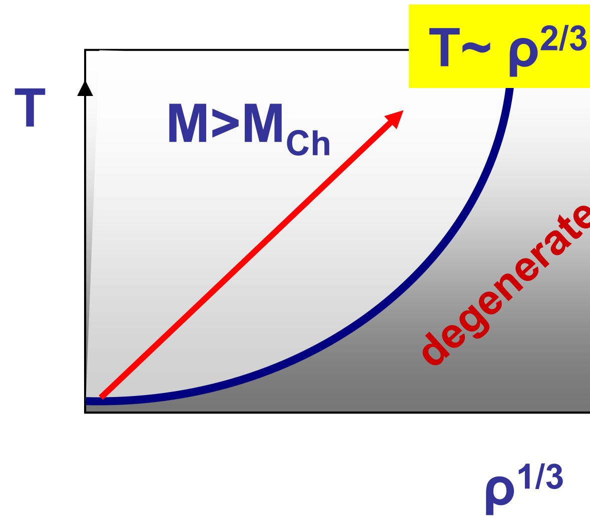



$$Fermi\ ultrarelativistic:\ P \approx \rho \Re T + K_{rel}\rho^{4/3}$$

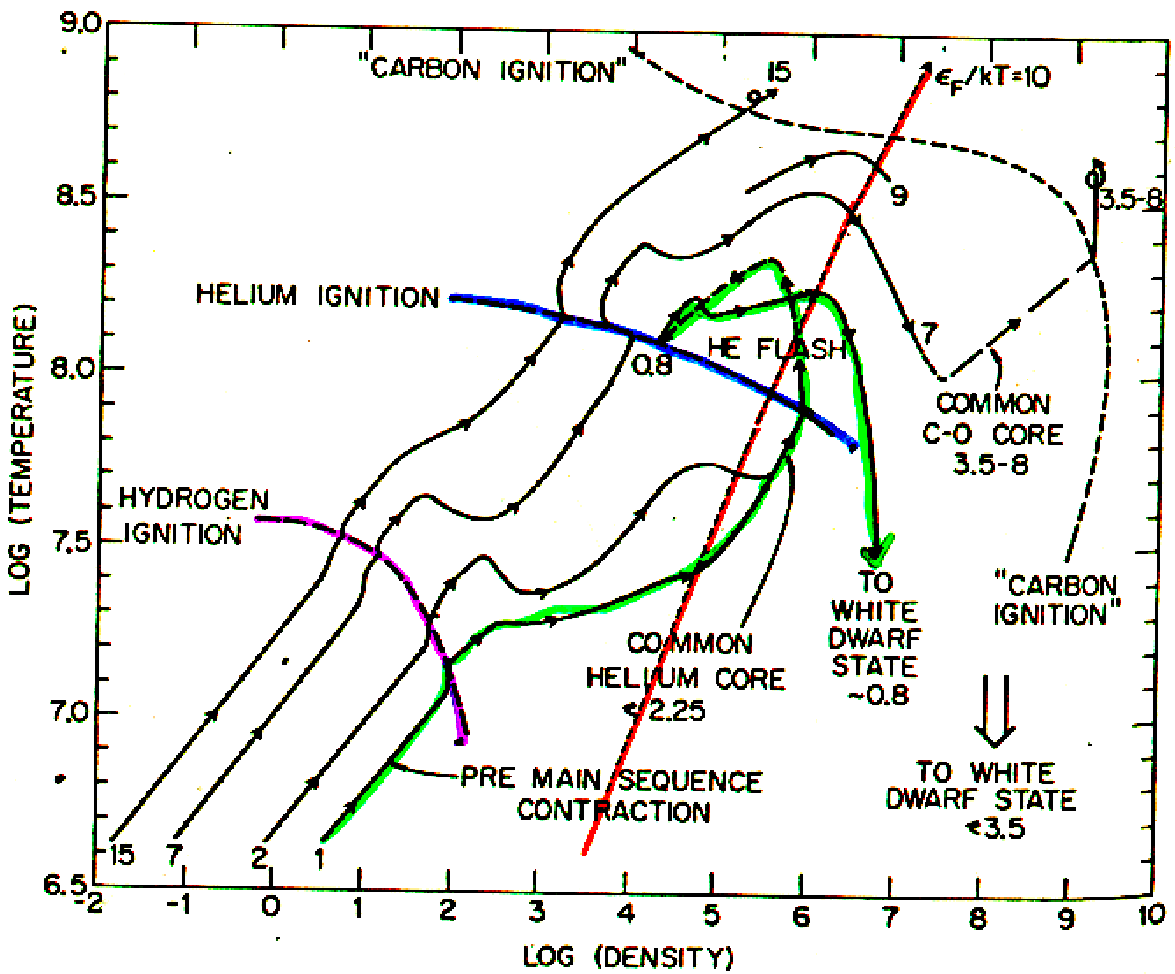
$$\Re T_c = GM^{2/3}\rho_c^{1/3} - K_{rel}\rho_c^{1/3} \Rightarrow$$

*if* $M > M_{Ch} = (K_{rel}/G)^{3/2}$ *temperature always*

*increases* $T_c \propto \rho_c^{1/3}$ *(case for massive stars)*

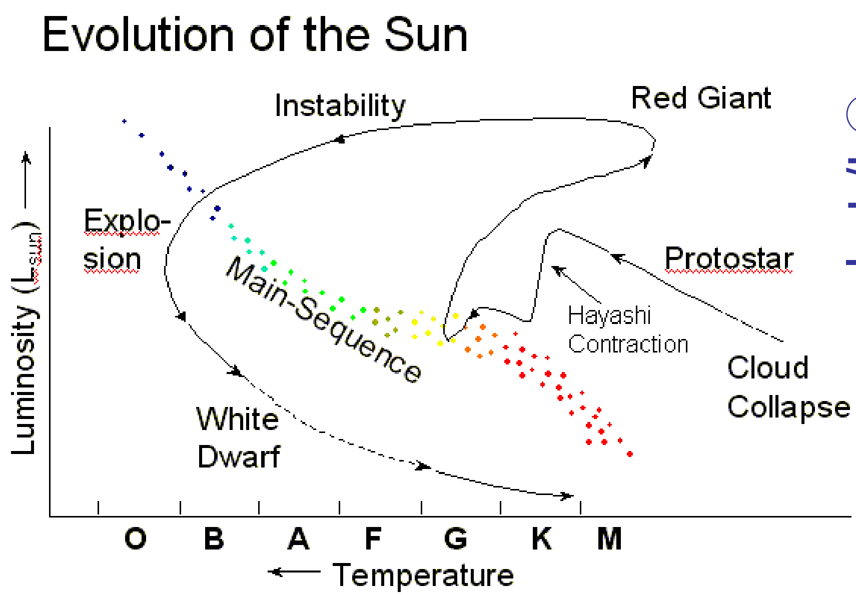


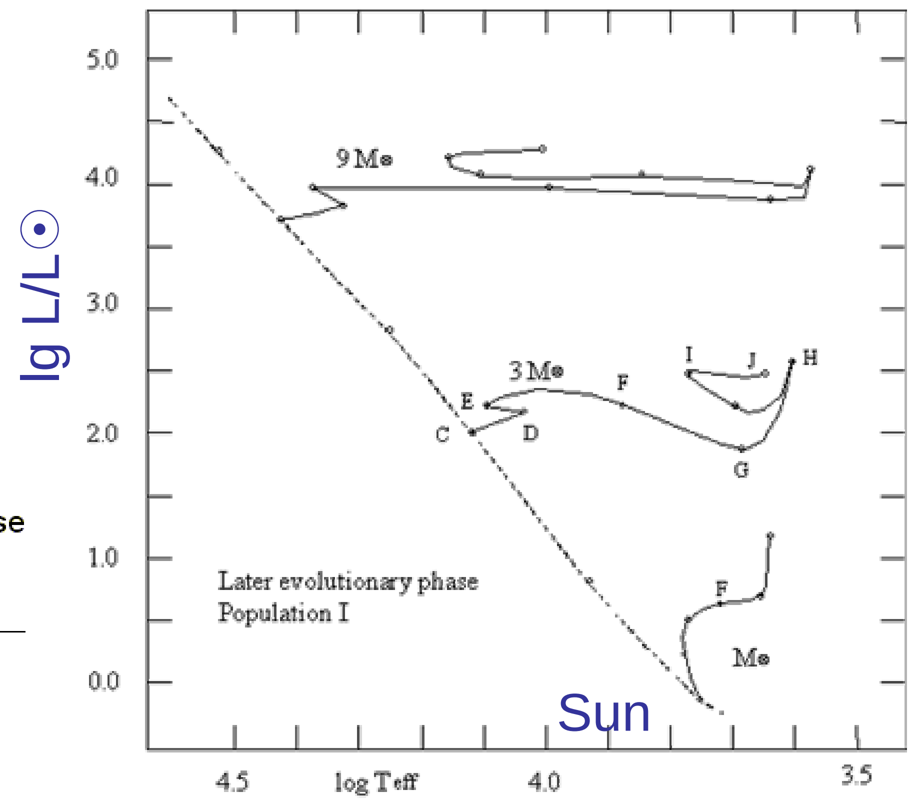
# Physical diagram of stellar evolution: ρ-T plane



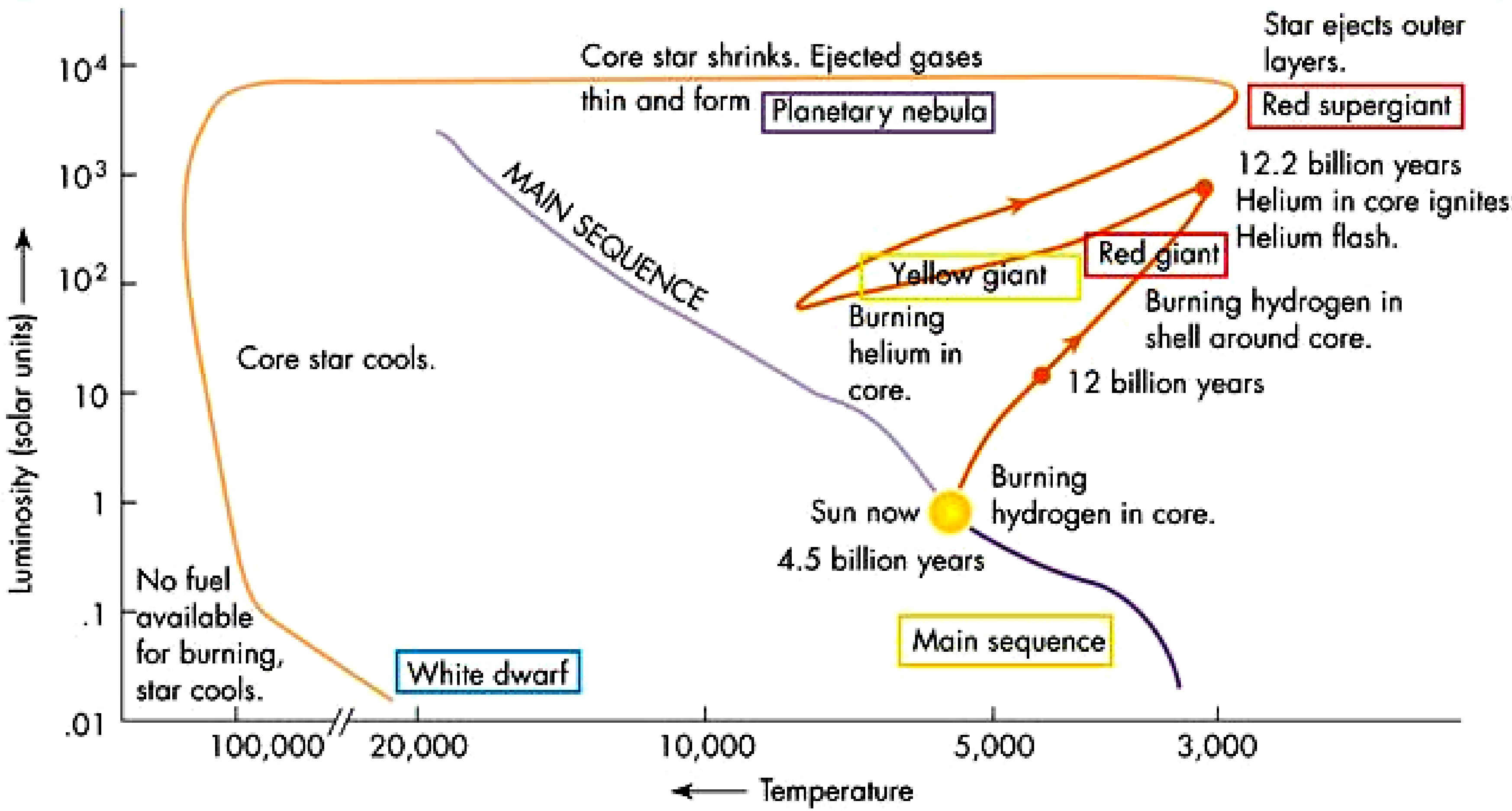

Tracks in the ρ-T plane traced out by the centers of stars of various masses

# Astronomical diagram for stellar evolution: HR diagram

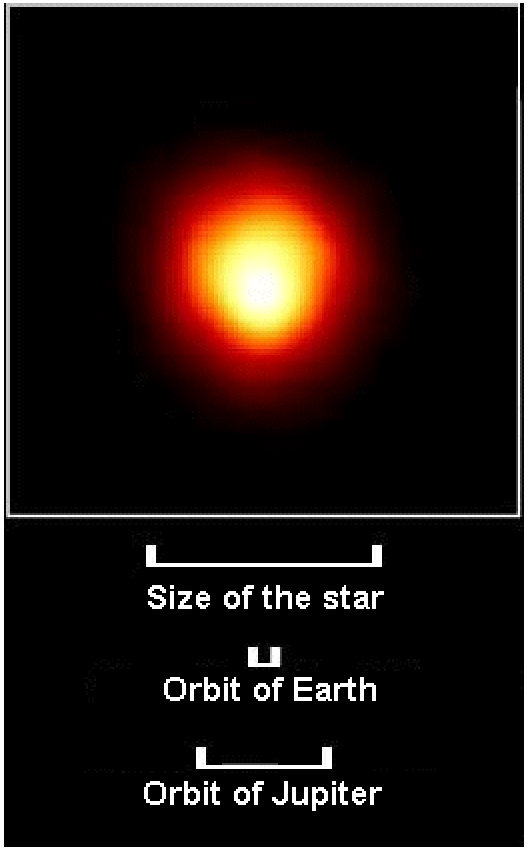


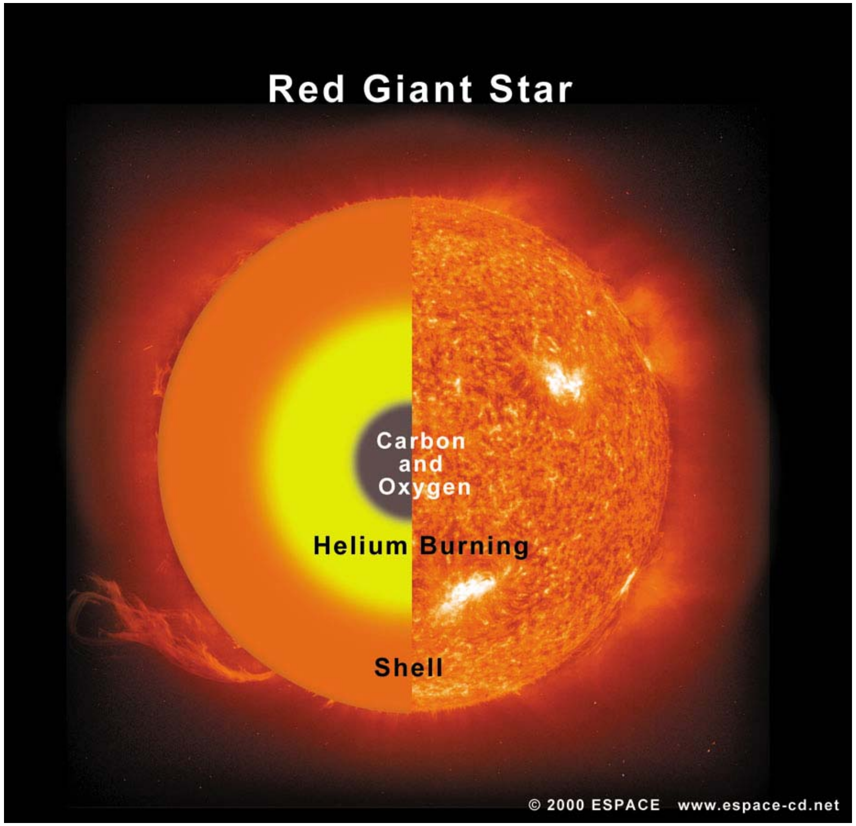



| | | | | | |
|---|---|---|---|---|---|
| | ~9 billion yrs | ~1 billion yrs | ~100 million yrs | ~10,000 yrs | |
| Time spent as | Main sequence → | Red giant → | Yellow giant → | Planetary nebula → | White dwarf |
| Sun's age | 4.5 billion yrs (now) | 12.2 billion yrs | 12.3 billion yrs | 12.3305 billion yrs | 12.3306 billion yrs |



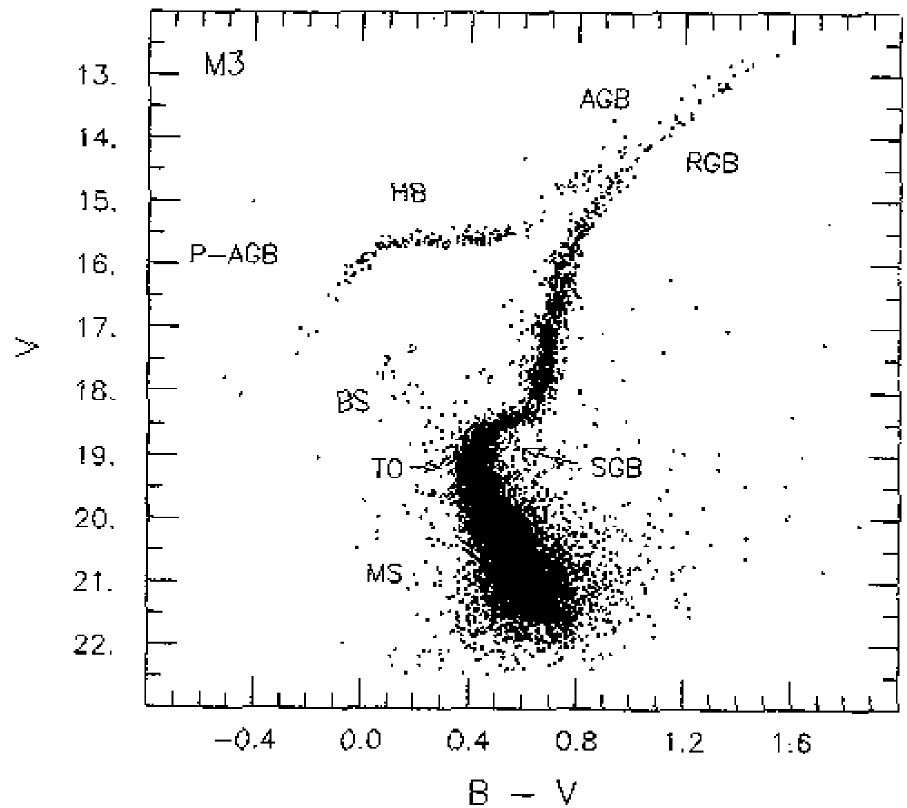
# Red giants



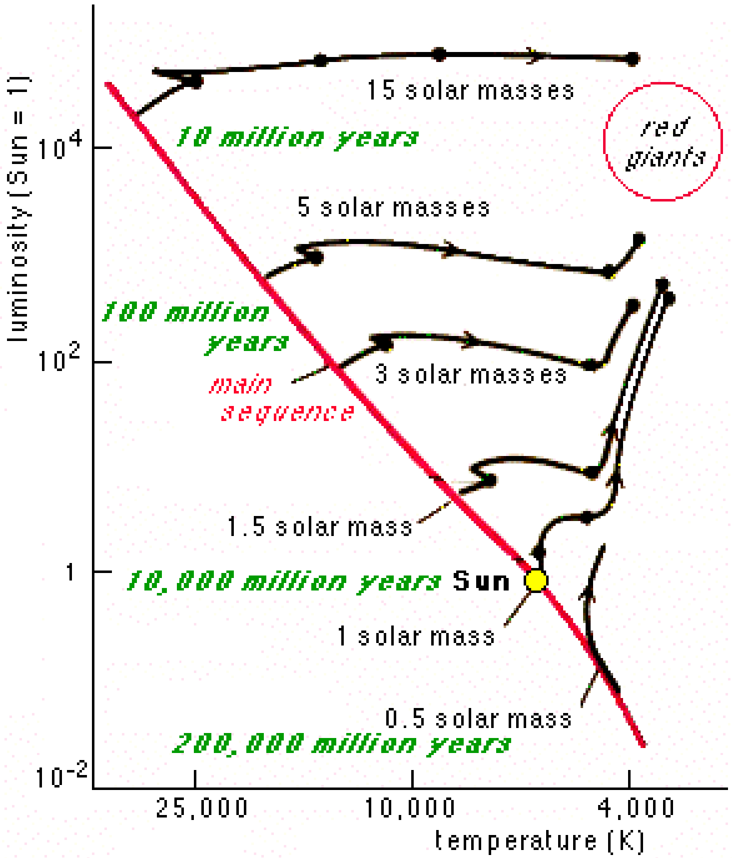



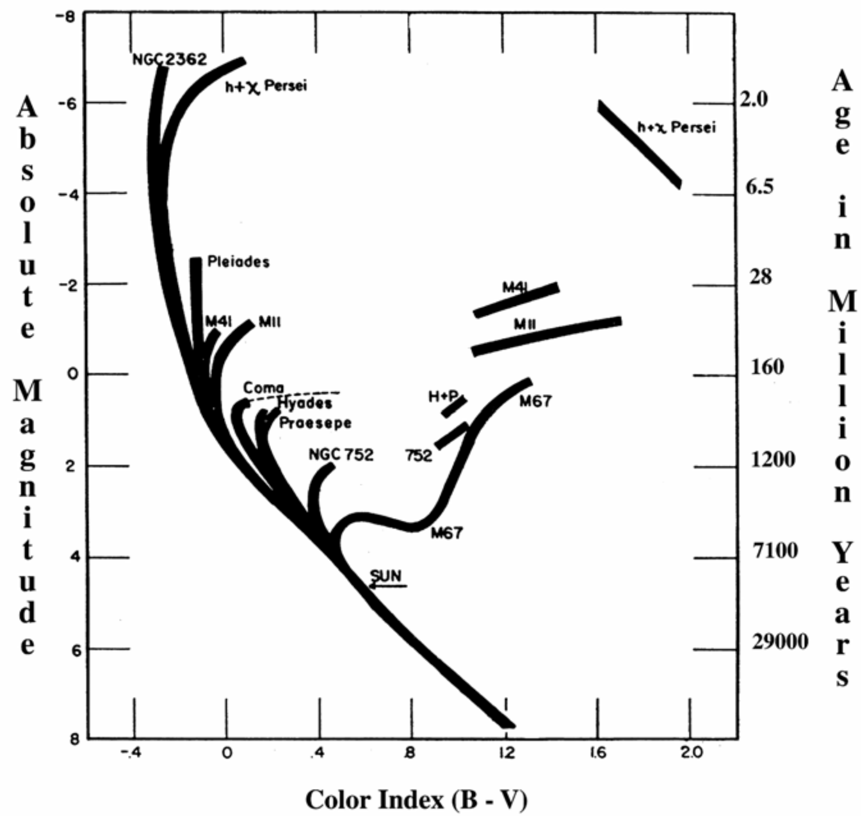

**Bethelgeise**

# HR diagram for stellar clusters

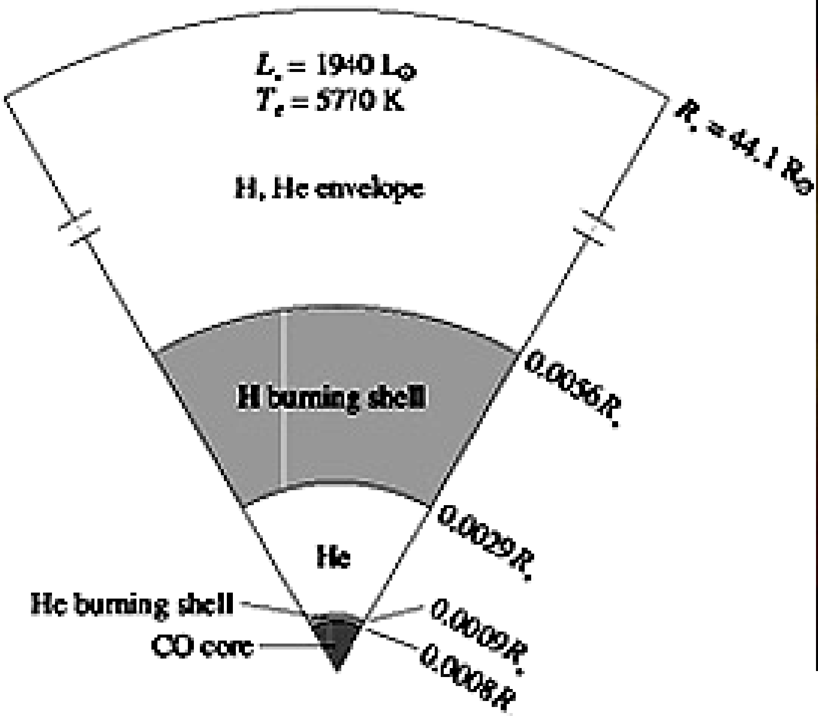


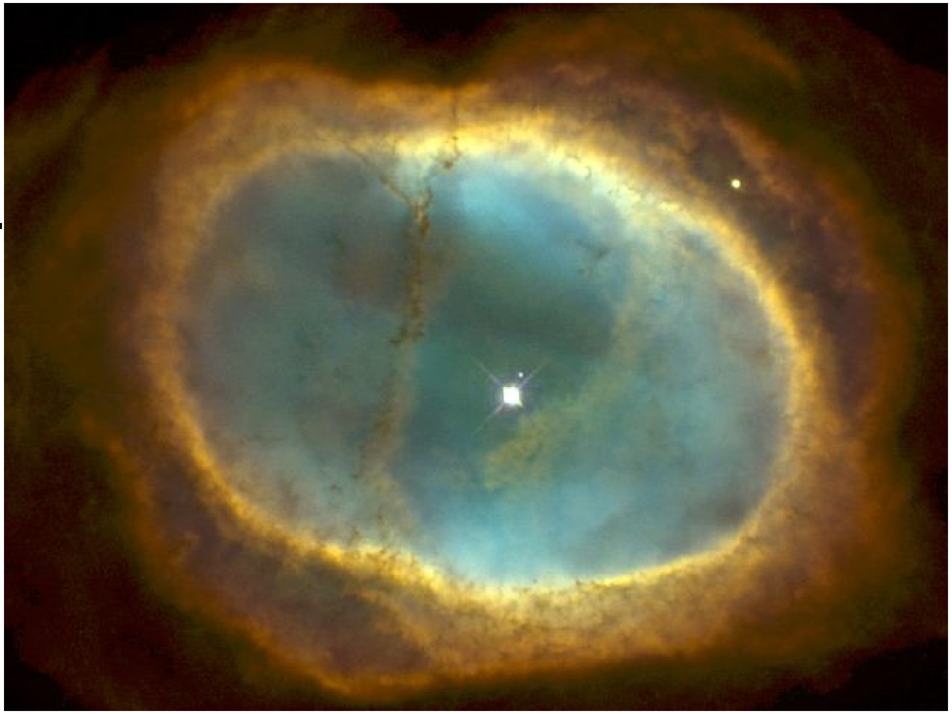
# Hydrogen burning time ~1/M$^2$ ➔ Turn-off point as a measure of the cluster age (Sandage)



# AGB stars and planetary nebula formation



**Thermal instability of H burning shell leads to pulsations (Mira!) and subsequent planetary nebula formation**

# Zoo of planetary nebulae