Lecture 2. Heat transport in stars. Radiation diffusion and convection. Equations of stellar structure. Sources of stellar energy. Hydrogen burning. Solar neutrino problem.

Heat transport in stars: Radiation diffusion

$$L(r) = -4\pi r^2 D_r \nabla \varepsilon_r,$$ Radiation diffusion eq.

$$\varepsilon_r = a_r T^4,$$ Radiation energy density

$$D_r = cl/3 = c/(3\kappa\rho),$$ Diffusion coefficient

$$\kappa = (\kappa_T + \kappa_{ff} + \kappa_{bf} + \kappa_{bb} ...),$$ Rosseland mean opacity

$$\kappa_T \simeq 0.2[cm^2/g](1 + X_H),$$ Thomson scattering

$$\kappa_{ff} \approx 7 \cdot 10^{22}[cm^2/g]\rho/T^{7/2}$$ Bremsstrahlung (Kramers)

In convectively stable regions (hot main sequence stars, outer layers of Sun)

From radiation diffusion to temperature gradient

$$\varepsilon_\gamma = a_r T^4,$$

$$\nabla \varepsilon_\gamma = 4 a_r T^3 \nabla T,$$

$$L(r) = -4\pi r^2 D_\gamma 4 a_r T^3 \nabla T,$$

$$\frac{dT}{dr} = -\frac{3\kappa\rho}{4 c a_r T^3} \frac{L(r)}{4\pi r^2}$$

Lesson: For radiation to be efficient in the heat transport (i.e. to maintain constant heat flux $L/4\pi r^2$) , temperature gradient must satisfy this eq. In many real stars, especially at advanced evolutionary stages, density decreases slower than T^3, so dT/dr becomes too big and other heat transport mechanisms start working – convection (non-radial motions of gas)

Heat transport in stars: Convection

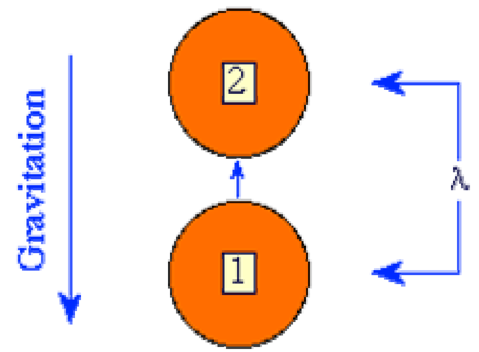
Conditions for Convection

CONVECTION

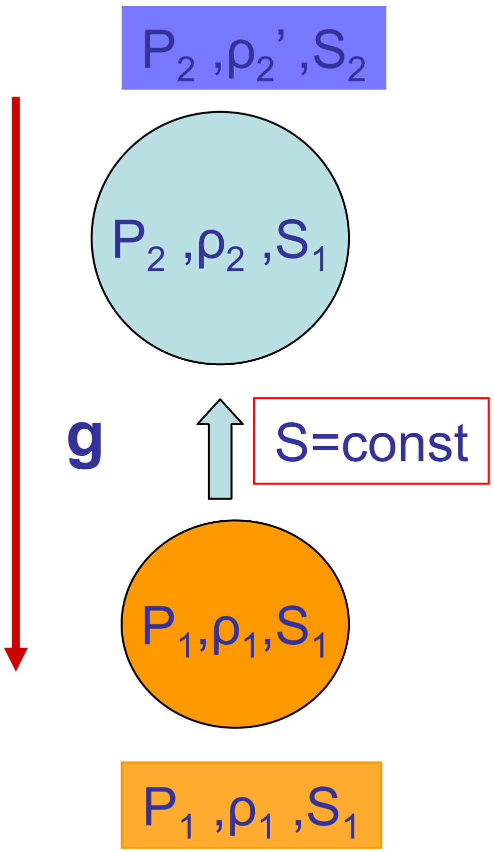
Microconvection
Convecting blobs small relative to unstable region; spherical "mixing length" theory may be adequate.

Macroconvection
Convecting blobs large relative to unstable region; multi-D hydrodynamics may be required.

Thought Experiment: a blob of fluid moves upward due to some small stimulus. If it is less dense than the surrounding material at position 2, it will be driven further upward by bouyancy forces, indicating that the region is *convectively unstable*.



Convection: Schwarzschield



Assumes constant chemical composition.

Blob's entropy is conserved.

Blob will move upwards if $\rho_2<\rho_2'$ **(convective instability)**

or downwards if $\rho_2>\rho_2'$ **(convective stability)**

Ideal gas: $S/k_B=5/2+\ln(T^{3/2}/\rho)+\text{const} = 5/2+\ln(P^{3/2}/\rho^{5/2})+\text{const} \Rightarrow \rho\sim\exp(-S/k_B)P^{3/5}$

$\rho_2-\rho_2'=P_2^{3/5}(\exp(-S_1/k_B)-\exp(-S_2/k_B)) \Rightarrow$

$\rho_2<\rho_2'$ if $S_2<S_1$

$\rho_2>\rho_2'$ if $S_2>S_1$

Convection makes dS/dr=0!

Convection: Ledoux

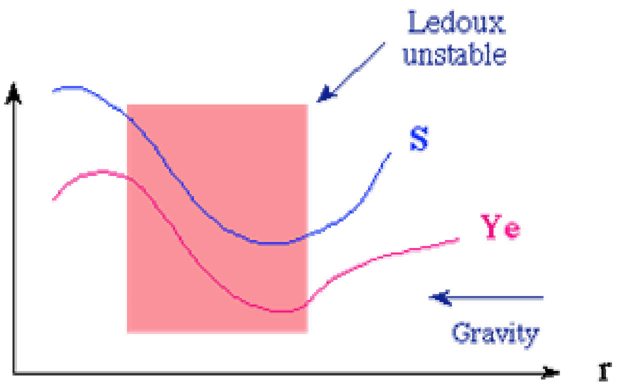
Generally one finds that

$$\left.\frac{\partial \rho}{\partial T}\right|_{P, Y_e} \leq 0 \qquad \textit{and} \qquad \left.\frac{\partial \rho}{\partial Y_e}\right|_{P, S} \leq 0$$

Chemical composition Y_e of external medium is allowed to changes

The Ledoux condition for instability:

$$\frac{dS}{dr} + c\frac{dY_e}{dr} \leq 0$$



Notice that a region could, e.g., be Schwarzchild stable but Ledoux unstable.

semiconvection

Convection: Adiabatic temperature gradient

$$dS/dr \geq 0 \Rightarrow \left(\frac{\partial S}{\partial T}\right)_P \frac{dT}{dr} + \left(\frac{\partial S}{\partial P}\right)_T \frac{dP}{dr} \geq 0$$

$$\left|\frac{dT}{dr}\right| \leq \left|\frac{\left(\frac{\partial S}{\partial P}\right)_T}{\left(\frac{\partial S}{\partial T}\right)_P} \frac{dP}{dr}\right| \equiv \left|\frac{dT}{dr}\right|_{ad} = \left(1 - \frac{1}{\Gamma}\right) \frac{T}{P} \frac{dP}{dr},$$

$$\Gamma = \frac{d \ln P}{d \ln \rho}$$

NB: Due to convection, adiabatic temperature gradient is an upper limit on temperature gradients in stationary stars!

Convection: Resume

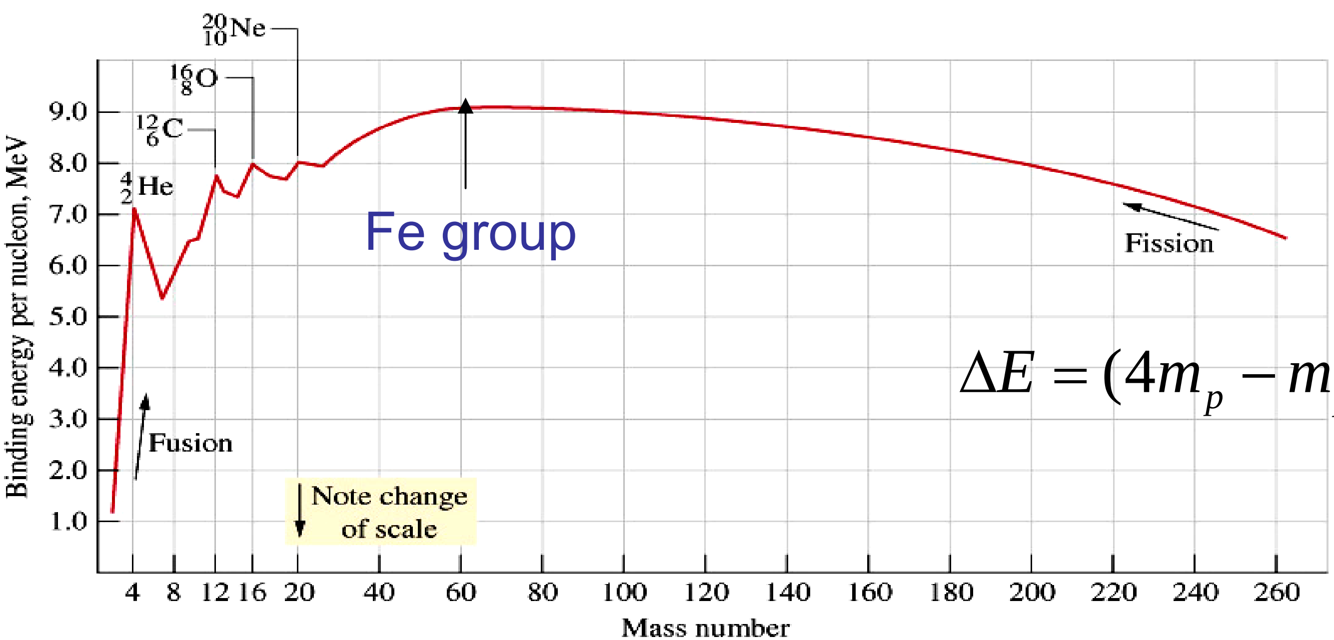
- Appears when $dS/dr < 0$ (Schwarzshield, if chemically homogeneous) or $dS/dr + c dY_e/dr < 0$ (Ledoux, if gradient of chemical composition). "Semiconvection" if Schw. stable but Ledoux unstable
- Tends to establish isentropic distribution ($dS/dr=0$) or adiabatic temperature gradient, and chemical homogeneity
- Heat convective diffusion $D_c=1/3\ V_c l$ is much more efficient than radiation diffusion
- Provides fresh material to the nuclear reaction zone
- Difficult to treat as no microscopic theory exists and the entire star should be considered

Equations of stellar structure

$$\frac{dP}{dr} = -\frac{Gm(r)}{r^2}\rho,$$	Hydrostatic equilibrium
$$\frac{dm}{dr} = 4\pi r^2 \rho,$$	Mass conservation
$$\frac{dT}{dr} = -\frac{3}{4a_r c}\frac{\kappa\rho}{T^3}\frac{L(r)}{4\pi r^2},$$	Radiation diffusion
$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \varepsilon^+,$$	Energy generation
$$P = P(\rho, T, X, Y, Z),$$	Equation of state
$$\kappa = \kappa(\rho, T, X, Y, Z),$$	Rosseland mean opacity
$$\varepsilon = \varepsilon(\rho, T, X, Y, Z),$$	Energy generation per gram
$$m(R) = M, P(R) = 0$$	Boundary conditions

Sources of stellar energy

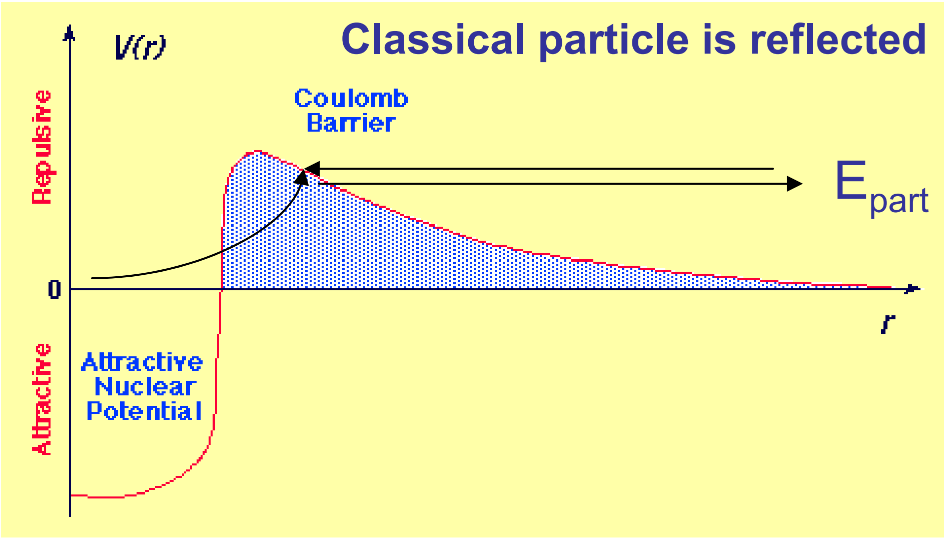
- Thermal (KH) : $Q \sim -U \sim GM^2/R$, $t_{KH} = Q/L \sim 3\ 10^7$yrs is too short
- Nuclear (Eddington, ca 1921): binding energy ~ a few MeV per baryon (~1 GeV) ➔ efficiency $\eta = \Delta E/mc^2 \sim 0.007$ (cf. chemical reactions – fire! ~ 1 eV/GeV ~ 10^{-9}!). Expected lifetime is bln. years!



$$\Delta E = (4m_p - m_{He^4_2})c^2 \simeq 27.3 MeV$$

Hydrogen burning

- **Problem**: central temperature T_c~ 1keV (Sun: 14 10^6K) is apparently too small for nuclear reactions: Coulomb barrier for p+p reaction is about $e^2/(10^{-13}cm)$~1 MeV
- **Solution**: Atkinson & Houtermans (1929) after Gamow's theroy of α-decay – subbarrier quantum transition



Real particle pentrates the barrier

Barrier penetration in more detail

$p = \hbar k, \quad k = 2\pi/\lambda, \qquad \psi \sim e^{ikx} = e^{ip/\hbar} = e^{\frac{i}{\hbar}\int p dx}, \quad p^2/2m = E_k = E_0 - U \;\Rightarrow p = \sqrt{2m(E_0 - U)},$

Coulomb : $U = Z_1 Z_2 e^2/r > 0$ (repulsion). Find $r_1 = Z_1 Z_2 e^2/E_0$, where $p = 0$.

For $r < r_1$ (classically forbidden!) $\quad p = i\sqrt{2m(U - E_0)}$, and for probability :

$$w(r < r_1) = |\psi^2| \sim \exp\left(-\frac{1}{\hbar}\int_r^{r_1}\sqrt{U - E_0}\,dx\right) \neq 0!!!!$$

The result is : $w(0) \sim e^{-\sqrt{\frac{E_G}{E_0}}}$, $\quad E_G \sim Z_1 Z_2 e^4/\hbar^2 m_p \sim \alpha^2 m_p c^2$ - Gamow's energy

($\alpha = \dfrac{e^2}{\hbar c} \approx 1/137$ - fine structure const.)

In stars $\quad E_0 \sim kT\,(\sim 1\,\text{keV}) \ll U \sim \alpha m_p c^2\,(\sim \text{MeV})$, *but* $E_G/E_0 \sim 1/137$ and probability is quite high!

p-p cycle (H.Bethe, 1939, Nobel prize 1967)

(1906-2005)

$p + p \rightarrow {}^2H + e^+ + \nu_e$ 99.77%

$p + p + e^- \rightarrow {}^2H + \nu_e + 0.42MeV$ 0.23%

$^2H + p \rightarrow {}^3He + \gamma + 5.49MeV$

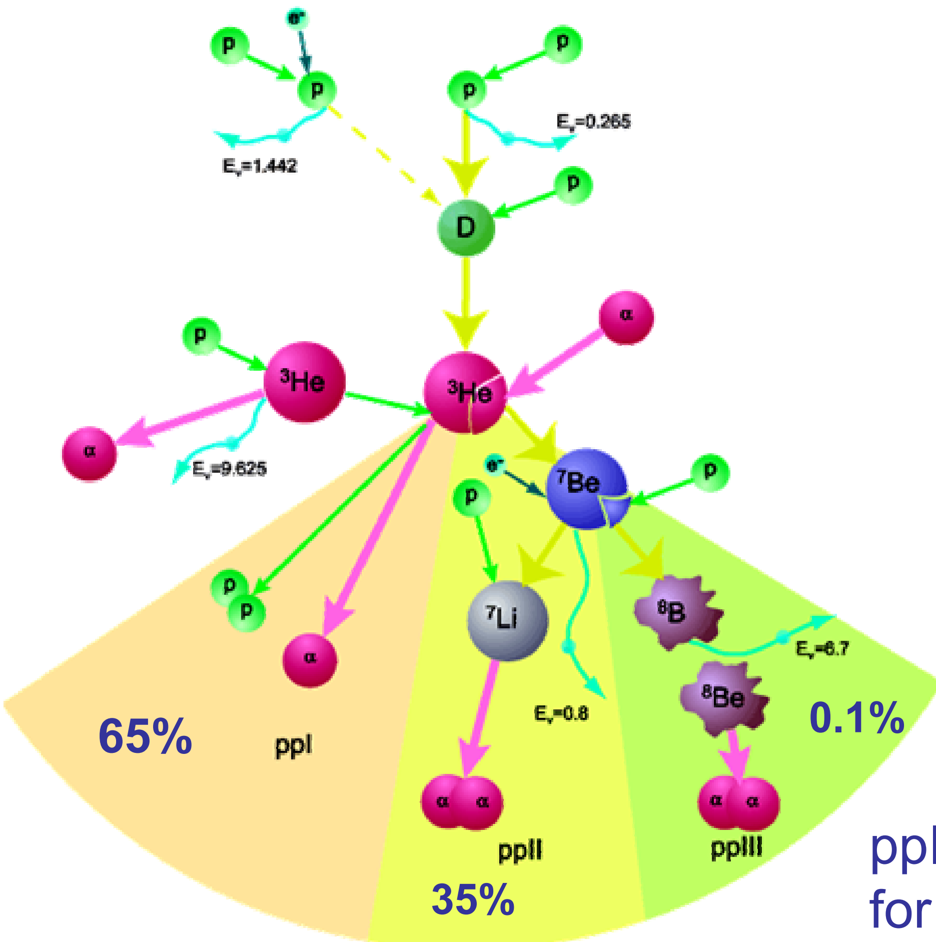
$^3He + {}^3He \rightarrow \alpha + 2p + 12.86MeV$ 84.92%

$^3He + p \rightarrow \alpha + e^+ + \nu_e$ $\approx 10^{-5}\%$

$^3He + \alpha \rightarrow {}^7Be + \gamma + 1.59MeV$ 15.08%

1. $4p \rightarrow He^4 + 2e^- + 2\nu_e + 26.7MeV$
2. 1^{st} reaction is the slowest ($\tau = 1/(n\sigma v) \sim 10^{10}$yrs) due to weak interactions
3. Deuterium (2d reaction) is rapidly (<1 s) converted into Helium-3
3. $\varepsilon \sim \rho T^{4...8}$ [erg/g/s]
4. 2 neutrinos carry away 0.6 MeV

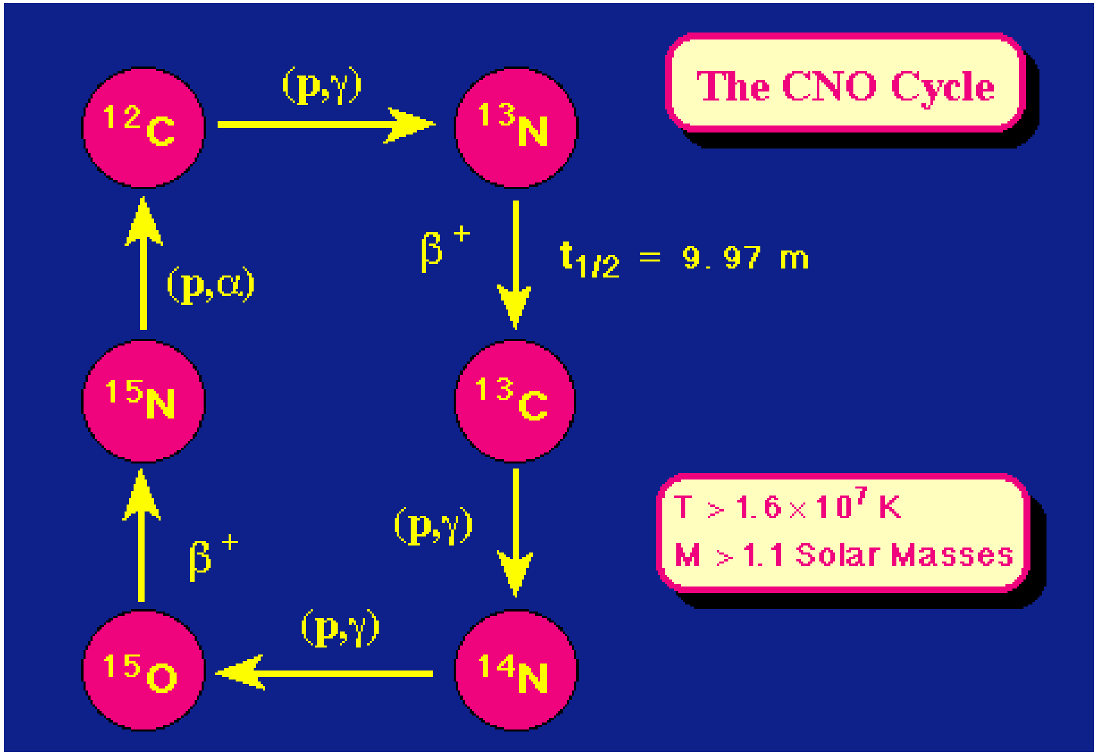
P-p 'chains'



Percents are given for solar center :
X=0.5, Y=0.5,
ρ=100 g/cm^3
T=15 mln K

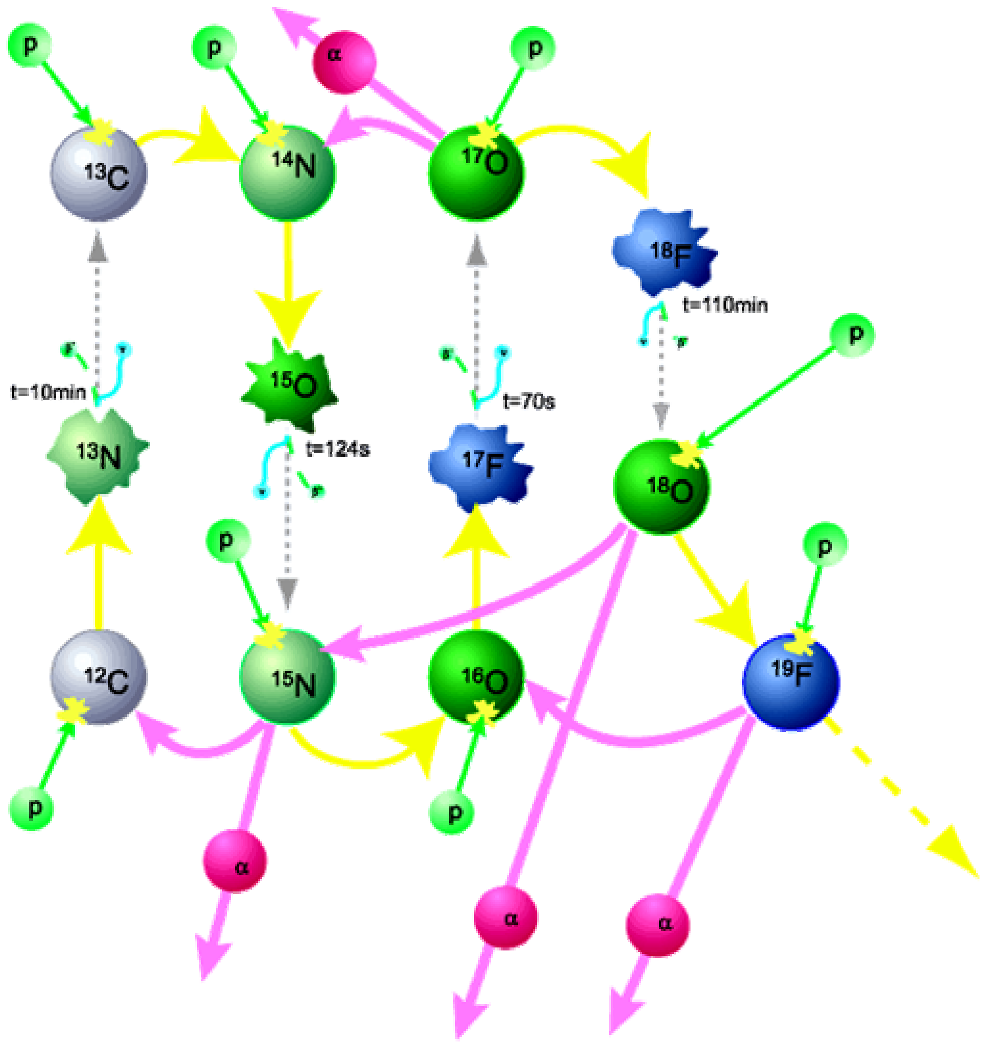
ppIII important for solar neutrinos

CNO-cycle (prevalent in more massive stars)



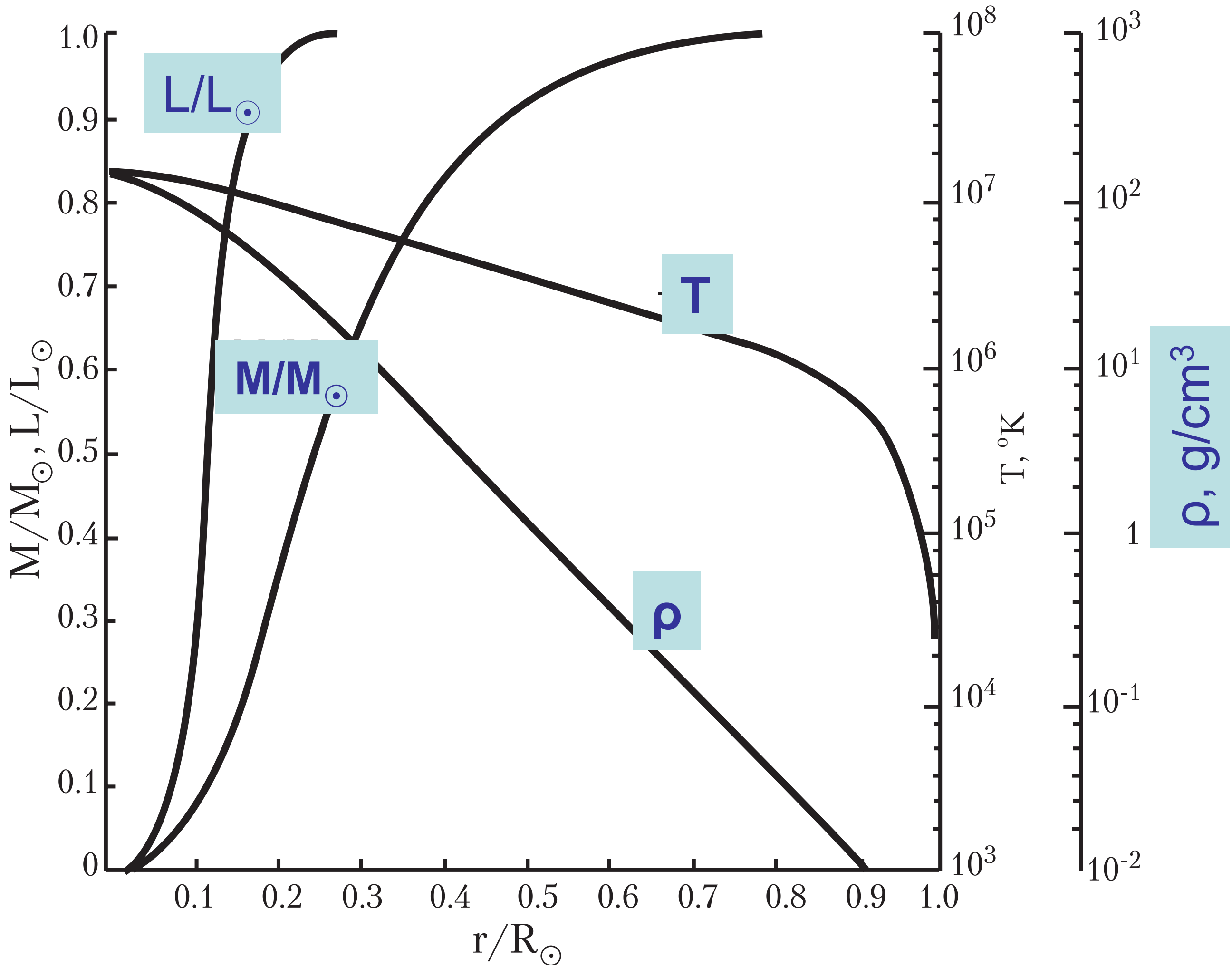
1. Dominant at T>20 10^6K
   (M>1.5 $M_\odot$)
2. C^{12} is catalizer
3. 4p→He^4+2e+2v_e+25MeV
4. ε~ρT^{10-20}

CNO(F) 'chains'

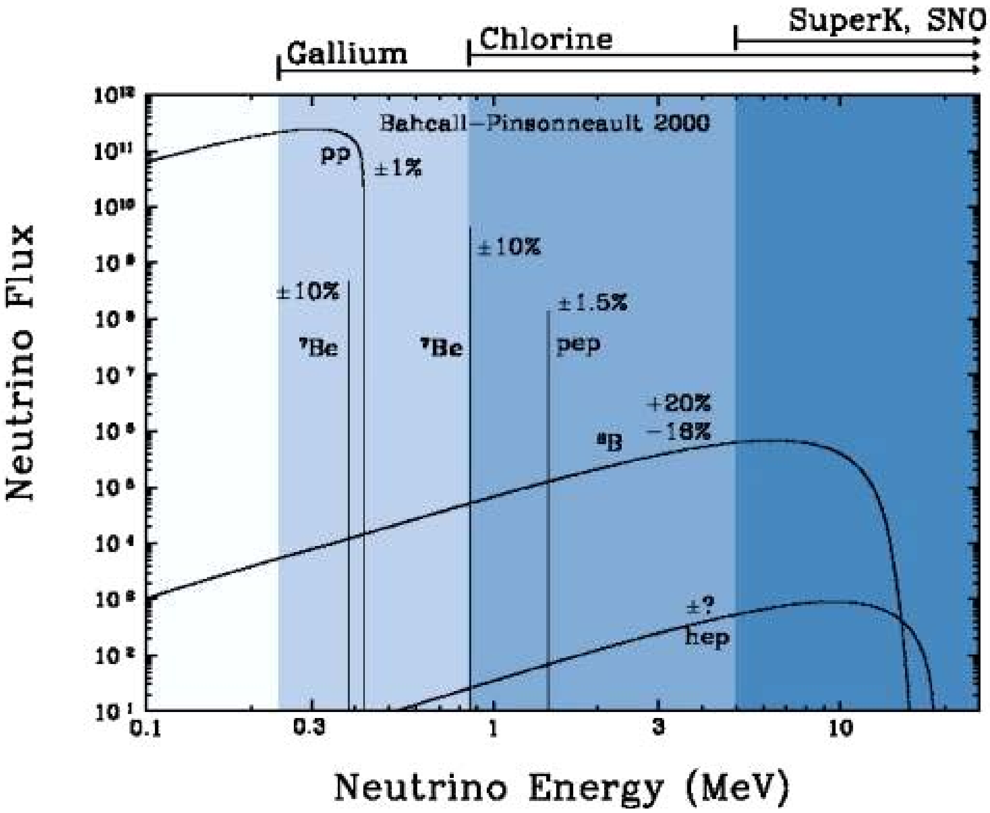


p
p
α
p
^{13}C
^{14}N
^{17}O
^{18}F
t=110min
p
t=10min
^{15}O
t=70s
^{13}N
t=124s
^{17}F
^{18}O
p
p
^{12}C
^{15}N
^{16}O
^{19}F
p
p
α
α
α

Solar Model: Structure



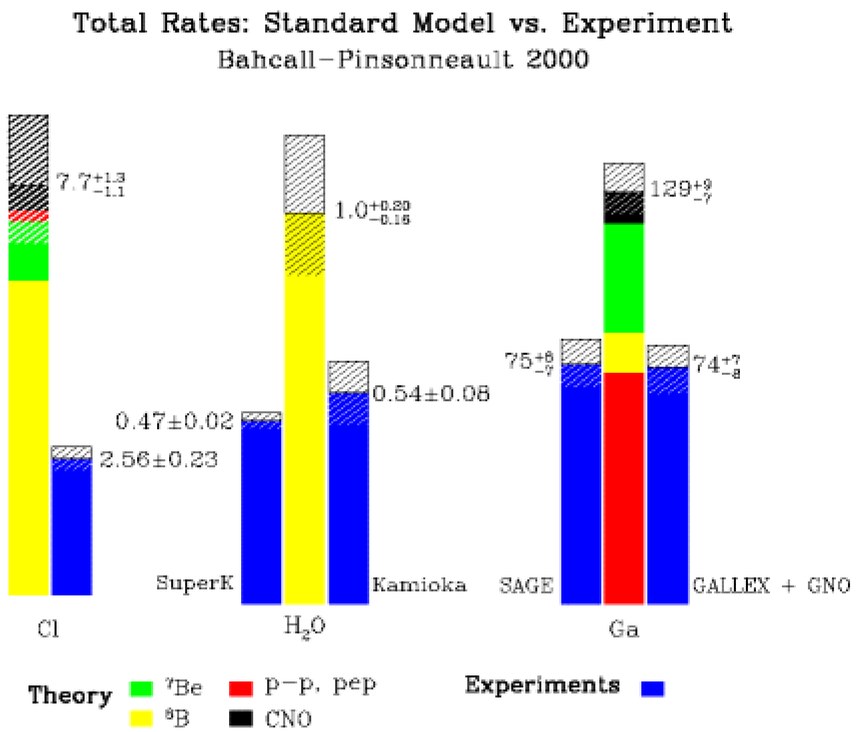
Solar neutrinos



J. Bahcall
1935-2005

Pp-neutrino luminosity of Sun:
$dN_\nu/dt=2L_\odot/(26.7MeV)$~$2x10^{38}$ s^{-1}

Solar neutrino: Problem



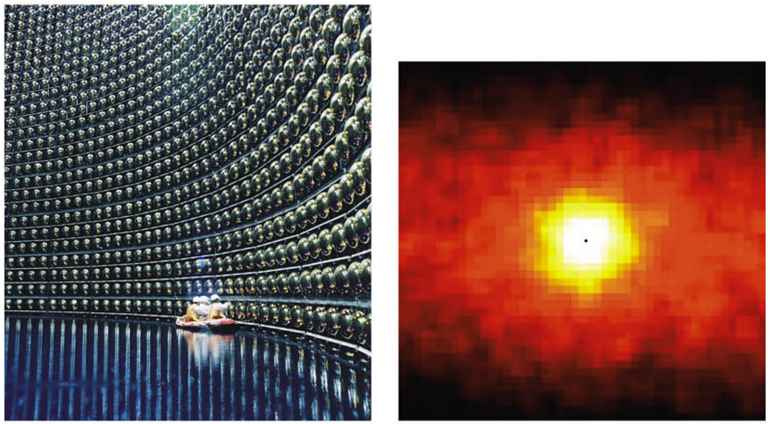
Measured flux of ν_e-neutrinos in all experiments is

~2 times smaller than predicted.

Idea: Neutrino oscillations if m_ν nonzero.
(Pontecorvo 1968;
In matter:
Mikheev, Smirnov 1986
Wolfenstein 1978)

Solar neutrino: experiments (1/2 Nobel prize 2002)

R.Davies M.Koshiba

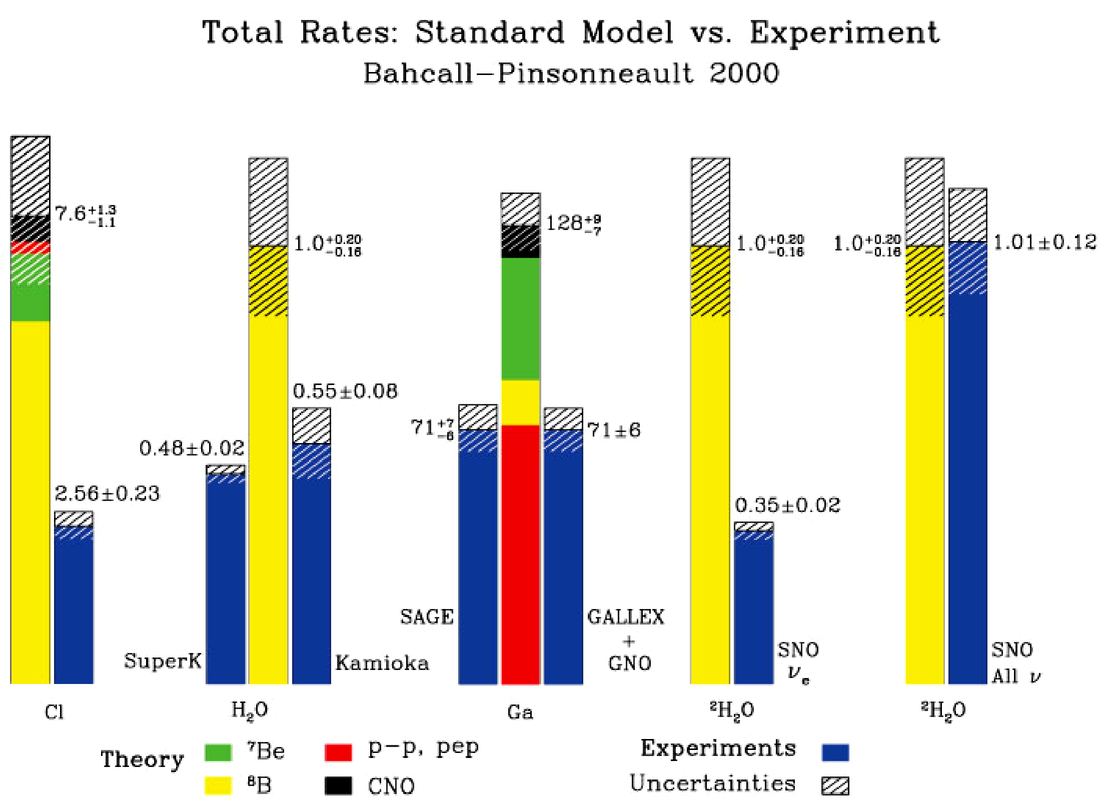


(a) (b)

SuperKamiokande water detector (Japan). Detect mostly electronic neutrinos.

Sudbury heavy water neutrino detector (Canada) (1000 tons). Can detect neutrinos of all 3 species (ν_e, ν_μ, ν_τ)

Solar neutrinos: Solution



Conclusions:

1. Standard Solar Model is secure
2. Neutrinos must oscillate➔ they must have nonzero mass ➔
3. Standard Model of nuclear physics must be corrected