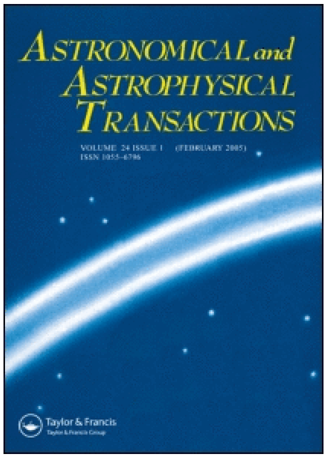

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

The abundance of helium and stellar pulsation

V. V. Kovtyukh [a]
[a] Astronomical Observatory, Odessa State University, Odessa, Ukraine

Online Publication Date: 01 January 1998
To cite this Article: Kovtyukh, V. V. (1998) 'The abundance of helium and stellar pulsation', Astronomical & Astrophysical Transactions, 17:1, 15 - 27
To link to this article: DOI: 10.1080/10556799808235420
URL: http://dx.doi.org/10.1080/10556799808235420

Astronomical and Astrophysical Transactions, 1998,
Vol. 17, pp. 15–27
Reprints available directly from the publisher
Photocopying permitted by license only

THE ABUNDANCE OF HELIUM AND STELLAR PULSATION

V. V. KOVTYUKH

*Astronomical Observatory, Odessa State University, Shevchenko Park, 270014
Odessa, Ukraine*

(Received August 26, 1996)

The radial gradient of the relative amplitude of the light variation (F_V, F_B) for galactic classical Cepheids is found. The gradient has different signs for the two cases: $P > 8^{\mathrm{d}}$ and $P < 8^{\mathrm{d}}$. For $P > 8^{\mathrm{d}}$ the gradient is negative:

$$dF_V/dR_G = -0.009\,\mathrm{kpc}^{-1} \quad dF_B/dR_G = -0.015\,\mathrm{kpc}^{-1}.$$

This gradient is caused by the gradient of the helium content in the primordial clouds, where Cepheids were born. In agreement with theoretical calculations, the amplitude increases with an increasing of the helium content. For this dependence the following calibration was obtained:

$$[\mathrm{He/H}] = (F_V - 0.54)dex \quad Y = 0.55F_V - 0.03.$$

In the case $P < 8^d$ the amplitude (and also the asymmetry parameter $M - m$) is mainly affected by the metallicity. The gradient is:

$$dF_V/dR_G = 0.018\,\mathrm{kpc}^{-1} \quad dF_B/dR_G = 0.025\,\mathrm{kpc}^{-1}.$$

Cepheids with $P < 8^{\mathrm{d}}$ and with known metallicity give the following calibration:

$$[\mathrm{Fe/H}] = 0.739(M - m) + 0.064P - 0.284F_V - 0.425$$

$$[\mathrm{Fe/H}] = 0.931(M - m) + 0.070P - 0.194F_B - 0.593.$$

It is shown that the evolutionary status of Cepheids can be specified using their amplitude (for Cepheids with $P > 8^{\mathrm{d}}$).

KEY WORDS Abundances of stars, classical Cepheids

1 INTRODUCTION

The determination of the helium abundance in the classical Cepheids is an essential problem, because the value of [He/H] becomes a crucial parameter in the theoretical modelling of pulsating stars. Nevertheless, there is no direct method to estimate helium abundance in the F–G supergiants. Here we propose a new method of determining the He content for Cepheids with $P > 8^{\mathrm{d}}$.

From energy considerations, Zhevakin (1954) has shown that the bolometric amplitude of the light variation for Cepheids is mainly defined by the helium content:

$$\delta L_C \approx 0.32\chi_1 \frac{e\Delta M}{m_{\mathrm{He}}} \frac{1}{P} \tag{1}$$

where δL_C is the observed amplitude, P is the period (days), χ_1 is the second ionization potential of He, e is the relative He abundance, ΔM is the mass of the ionized zone, and m_{He} is the mass of the He atom.

The dependence between the amplitude and He abundance follows also from numerical calculations by Stobie (1969). Later, Cogan *et al.* (1980) using updated models have found for a Cepheid with $P = 15^{\mathrm{d}}$ that an increase of the helium mass fraction Y by 0.1 leads to the growth of the bolometric amplitude increasing by $0\overset{\mathrm{m}}{.}25$ and to an increase of the amplitude of the radial velocity by 15 km s^{-1}.

Theoretical estimates show that the He content affects the pulsational amplitude of δ Sct-type stars (Guzik, 1992).

Nevertheless, it was thought that it would not be possible to perform the observational test for this supposition, because the He abundance cannot be directly measured for Cepheids.

It is known that light amplitudes for Cepheids strongly depend on the period P: with increasing period, the light amplitude has a tendency to increase and achieves a maximal value for $P \approx 5^{\mathrm{d}}$. The amplitude decreases in the region of the critical periods 9–10$^{\mathrm{d}}$ and again achieves the maximal value for $P \approx$ 15–20$^{\mathrm{d}}$. A further decrease of amplitude takes place after $P > 20^{\mathrm{d}}$. Therefore we can use a relative amplitude (amplitude parameter, Kraft, 1960; Sandage and Tammann, 1971) instead the amplitude itself (A_V or A_B; $A_V \approx 0.67A_B$):

$$F_V = 10^{0.4(A_V - A_V^{\max})} \tag{2}$$

$$F_B = 10^{0.4(A_B - A_B^{\max})} \tag{3}$$

where A_V, A_B are the observed amplitudes and $A_V^{\max}$, $A_B^{\max}$ are the maximal amplitudes for given P. The upper envelope of the period–amplitude diagram and the observed amplitudes were taken from the compiled catalogues by Berdnikov (1987) and Fernie *et al.* (1995). The so-called s-Cepheids were excluded from the analysis because of the unknown origin of the small amplitudes of these stars.

It appeared that there exists a radial gradient of F_V (F_B) for our Galaxy, and for the two cases $P < 8^{\mathrm{d}}$ and $P > 8^{\mathrm{d}}$ this gradient has different signs (Figures 1 and 2). This means that in these two cases the amplitude is affected by two different factors. Let us consider these two cases separately.

2 CEPHEIDS WITH $P > 8^{\mathrm{d}}$

The dependence of the amplitude upon the galactocentric distance R_G, based on the catalogues by Berdnikov (1987) and Fernie *et al.* (1995), is shown in Figure 1.

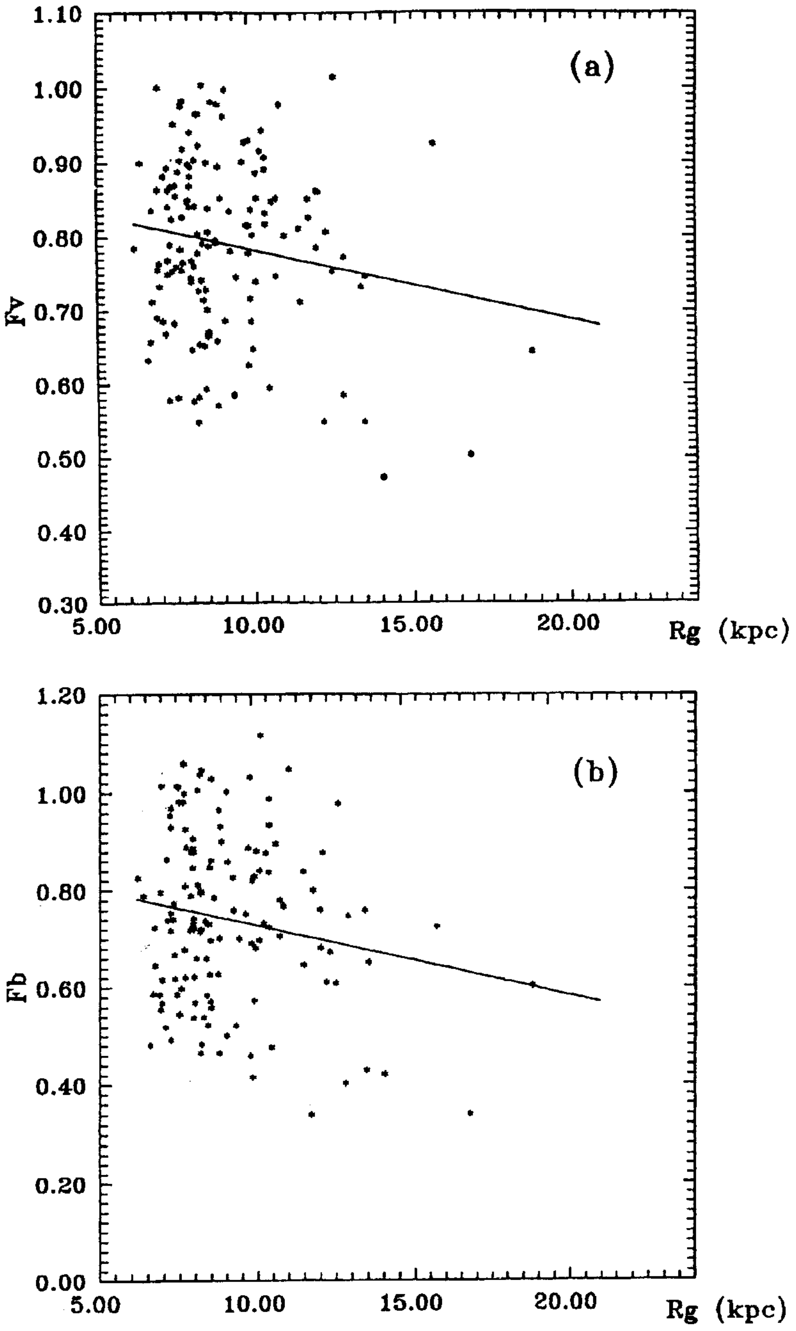

Figure 1 Radial gradient of the relative amplitude for Cepheids with $P > 8^{\mathrm{d}}$. (*a*) for F_V, (*b*) for F_B.

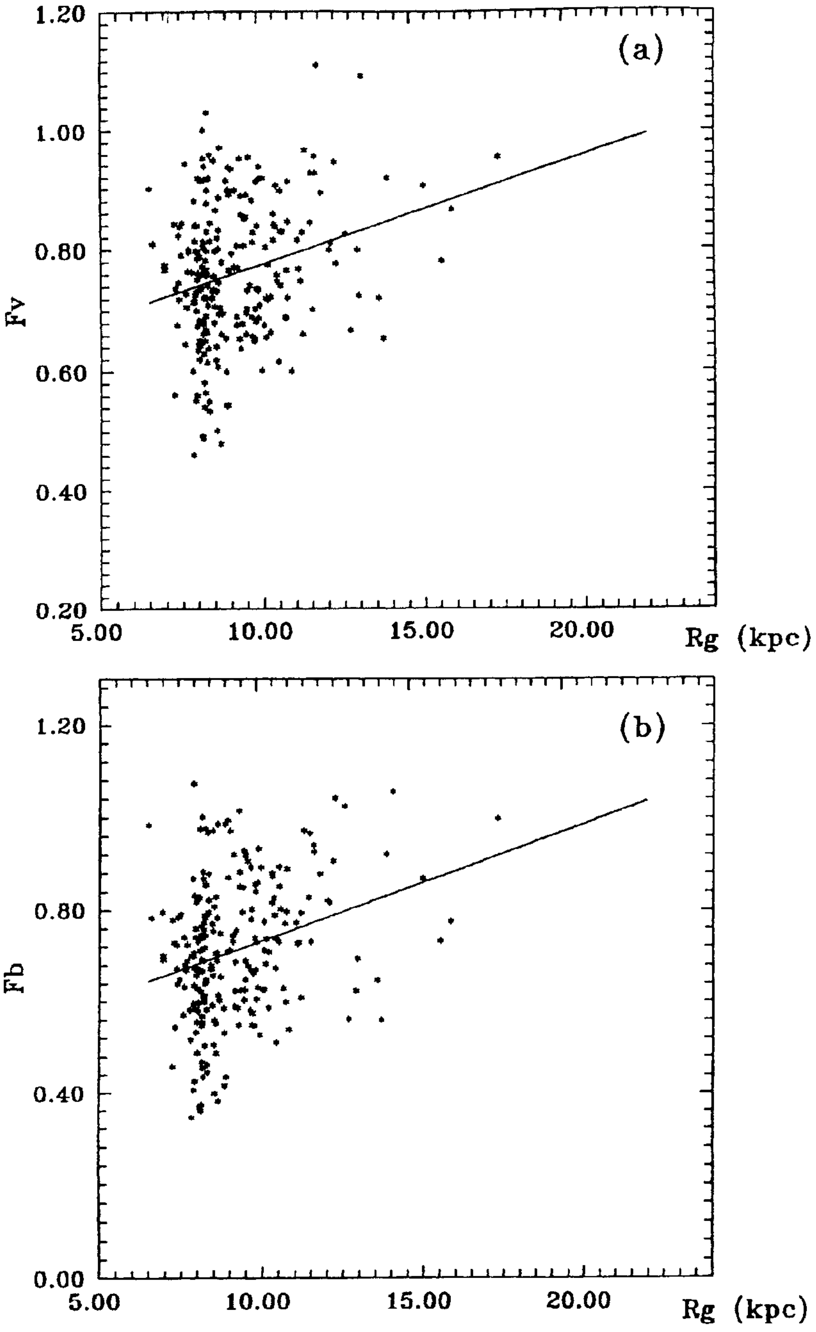

Figure 2 Radial gradient of the relative amplitude for Cepheids with $P < 8^d$. (*a*) for F_V, (*b*) for F_B.

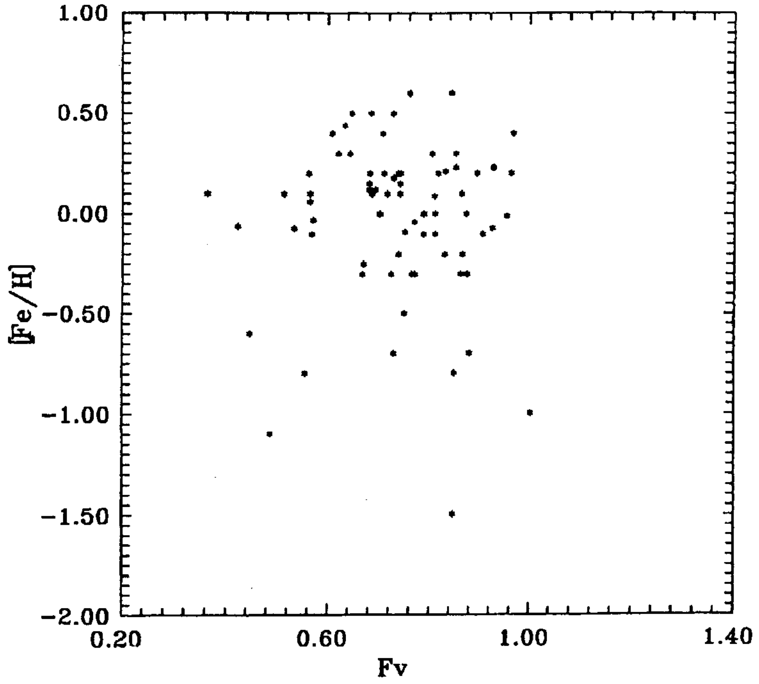

Figure 3 The metallicity vs. F_B for Cepheids with $P > 8^d$.

In this paper we adopt $R_G^{\odot} = 8.5$ kpc for the distance of the Sun to the Galactic centre. The following gradients were obtained:

$$dF_V/dR_G = -0.009\ \mathrm{kpc}^{-1} \pm 0.002 \tag{4}$$

$$dF_B/dR_G = -0.017\ \mathrm{kpc}^{-1} \pm 0.006. \tag{5}$$

Because of the high precision of photoelectric observations, one can consider that the existence of these gradients is reliably established. If we take into account that observational selection impedes the detection of distant small-amplitude Cepheids located at large R_G, the true gradient values can be somewhat higher.

It is known that the amplitude depends upon the helium content or the Cepheids' metallicity (the location of the instability strip is dependent upon the metallicity); in fact both of these parameters may be important. It is easy to show that for Cepheids with $P > 8^d$, the metallicity does not essentially affect the amplitude. In Figure 3 we show the dependence between F_B and the metallicity [Fe/H] determined on the base of fine spectral analysis and the Washington system (Harris, 1981; Giridhar, 1983; Harris and Pilachowski, 1984; Luck and Bond 1989a, b; Giridhar *et al.*, 1991; Luck, 1994).

It is seen that there is no correlation between F_B and [Fe/H]. This is also confirmed by results of Eggen (1994), who has shown the absence of any correlation between ΔM, the metallicity index, and F_B and has explained the misinterpretation of the PLF_B relation as a PL[Fe/H] relation. Nevertheless, he has shown this only for the cases $P > 10^{\mathrm{d}}$ and $P < 10^{\mathrm{d}}$. As we will show below, F_B (F_V) for $P < 8^{\mathrm{d}}$ does depend upon the metallicity.

It is now fairly clear that for the case $P > 8^{\mathrm{d}}$, the amplitude gradient is mainly affected by the gradient of the helium content, which generally exists in the Galaxy (it is found from the H II regions and planetary nebulae studies). As is known, the Cepheids are young objects, which have not had time to evolve far away from their birthplaces (the progenitors of Cepheid variables are main-sequence B-stars and the age of Cepheids is near to 10–100 Myr (Efremov, 1978)). According to modern data, the initial material for stellar formation is the giant molecular clouds, which are linked with O-associations and H II regions (see e.g. Efremov, 1979; Blitz *et al.*, 1982; Elmegreen and Elmegreen, 1983). H II region were used for the determination of the He, O, N, S, Ar abundance gradient. Planetary nebulae were also used for this. In several works (Torres–Peimbert and Peimbert, 1977; Peimbert *et al.*, 1978; Hawley, 1978; Talent and Dufour, 1979; Lichten *et al.*, 1979; Thum *et al.*, 1980; Shaver *et al.*, 1983) the gradient was estimated as -0.02, -0.02, 0.00, -0.01, 0.00, -0.015, 0.00 dex kpc^{-1}, respectively. The average value is -0.009 ± 0.008 dex kpc^{-1} (corrected for the smaller $R_G^{\odot} = 8.5$ kpc). A large deviation, which is comparable with the gradient value itself, requires an additional confirmation of the existence of the gradient and the specification of its value.

Peimbert (1986) in his work demonstrates a strong correlation between [He/H] and [O/H] for the galactic, LMC and SMC objects. At the same time, the gradient [O/H] = -0.06 dex kpc^{-1} has been reliably established (Torres–Peimbert and Peimbert, 1977; Shaver *et al.*, 1983; Maciel and Köppen, 1994). Taking into account this value and the correlation between [O/H] and [He/H], one can independently determine and specify the value of the [He/H] gradient:

$$d[\mathrm{He/H}]/dR_G = -0.009\,\mathrm{kpc}^{-1} \pm 0.002 \tag{6}$$

The existence of the [He/H] gradient also follows from the theory of galactic chemical evolution: $d[\mathrm{He/H}]/dR_G = -0.008$ kpc^{-1} (Matteucci, 1992).

Knowing that the Cepheids (the descendants of B-stars) are formed from molecular clouds with different chemical compositions, one can suppose that the gradient of the He (and other elements) content found using the Cepheids, reflects the galactic gradient. Moreover, the maximal A_B values for the Cepheids from the SMC (having $P \approx 15$–20^{d}) are rather less ($\approx 0^{\mathrm{m}}\!.2$) than for the galactic Cepheids (van Genderen, 1978). This is easily explained by the smaller (~ 0.1 dex) He content in the SMC compared to our Galaxy (Peimbert, 1986, 1993). The Cepheid amplitudes in the LMC for $P > 8^{\mathrm{d}}$ are intermediate between the corresponding Galactic, M31 and SMC values. This agrees with the intermediate value of the He content found for the LMC (Peimbert, 1986, 1993).

Thus, theory and observations confirm a strong relation between the He content and the pulsational amplitude of Cepheids with $P > 8^{\mathrm{d}}$.

Comparing the [He/H] and A_V gradients for Cepheids with $P \approx 15^{\mathrm{d}}$:

$$d[\mathrm{He/H}]/dR_G = -0.009\,\mathrm{kpc}^{-1}, \quad dA_V/dR_G = -0\overset{\mathrm{m}}{.}014\,\mathrm{kpc}^{-1},$$

we find that an increase of the Y content by 0.1 leads to an increase of A_V by up to 0.30 ± 0.09 mag. This estimate is in good agreement with the results of calculations by Cogan *et al.* (1980) for Cepheids with $P = 15^{\mathrm{d}}$ (see text above). We have used the V-amplitude, because it practically coincides with the bolometric amplitude of F–G stars (since Cepheids have $T_{\mathrm{eff}} = 5500$–6500 K; the bolometric correction of V-amplitudes is only 0.0 to -0.05 mag (Kurucz, 1992); we set BC=0.00).

Nevertheless, the observed amplitude is not conditioned only by the primordial He content, but in greater degree by the He abundance which is altered during stellar evolution. Evolutionary changes in helium abundance would not be expected to correlate with the galactocentric radius.

During the evolution from the main-sequence B-star to the red giant phase, a Cepheid rapidly ($\approx 10^3$ yr) crosses the instability strip. In the red giant region Cepheids suffer dredge-up and their envelopes become helium enhanced (Iben, 1965). At this stage, one can expect the appearence of some anomalities in CNO abundances (Luck and Lambert, 1981, 1985; Luck, 1994). These results are also confirmed by theoretical predictions. Therefore, crossing the instability strip for the second time, the Cepheids must pulsate with an increased amplitude. This helium enrichment can grow during the next red giant phase, so the Cepheid's amplitude has a tendency to increase with time.

It is clear that the primordial helium abundance (which is inherent in B-stars and H II regions) can take place only in the Cepheids that cross the instability strip for the first time. These stars are situated close to the lower enveloping line in Figure 1. The number of Cepheids crossing the instability strip for the first time must be small, because of their rapid crossing of the strip. Up to now we have found only two such Cepheids: V1162 Aql ($P = 5\overset{\mathrm{d}}{.}376$, Andrievsky *et al.*, 1996) and V636 Cas ($P = 8\overset{\mathrm{d}}{.}377$, Kovtyukh *et al.*, 1996). Both Cepheids have a small amplitude and solar-like CNO abundances. A search for similar objects is continuing.

The helium abundance in H II-regions is similar to that in B-stars from the solar neighbourhood (the average abundance is $\log \mathrm{He/H} = -1.08$, Adelman, 1986; Lennon and Dufton, 1986; Peimbert, 1993). A similar value (-1.13 dex) was found by Kovtyukh *et al.* (1994) for the B-component of δ Cep (δ Cep-C, B7 IV). One can suppose that Cepheids first crossing the instability strip have the same mean abundance as for the B-stars (-1.08 dex). This value is close to the solar value (-1.01 ± 0.035 dex or $Y = 0.28$, Grevesse and Noels, 1993).

The average F_V amplitude for Cepheids near to the lower enveloping line (ER Aur, YZ Aur, FI Car, YZ Car, SZ Cas, V636 Cas, V641 Cen, Cp Cep, SU Cru, LS Pup, BY Sco, V470 Sco and probably also GT Car, BX Cru, Y Oph; see Figure 1) is equal to 0.54 ± 0.03 (for $R_G = R_G^{\odot}$). V636 Cas has an F_V amplitude of 0.57.

Therefore, comparing (4) and (6) we obtain the following calibration (for Cepheids with $P > 8^{\mathrm{d}}$):

$$\log \mathrm{He/H} = F_V - 1.62\,(\mathrm{dex}) \qquad (7)$$

$$Y = 0.55F_V - 0.03 \tag{8}$$

or relative to the solar abundance:

$$[\mathrm{He/H}] = F_V - 0.54\,(\mathrm{dex}). \tag{9}$$

It is seen that Cepheids with maximal amplitude ($F_V \approx 1.0$) possess a helium overabundance of ≈ 0.45 dex. Thus, after several crossings the instability strip and being in the dredge-up phase, the helium content can be 2.5–3 times higher than the solar value.

Of course, with further gathering of new observational data, these relations can be specified.

The relations (7–9) allow us to obtain the helium content in the upper layers of a Cepheid's atmosphere. This is useful for the calculation of pulsational models of Cepheids. It can also be used as a diagnostic tool in the study of helium abundances in different stellar systems and for the investigation of the He gradient in other galaxies (in particular, M31).

2.1 The Evolutionary Stage of the Cepheids

In Figure 4 we give a histogram of the distribution of the amplitude F_V of Cepheids (the galactic gradient of F_V is excluded). This histogram obviously shows that the amplitude increases during Cepheid evolution. Group I ($0.5 < F_V < 0.6$) contains Cepheids first crossing the instability strip (V636 Cas is included); group II ($0.6 < F_V < 0.9$) contains Cepheids which probably once suffered a dredge-up; group III ($0.9 < F_V$) probably consists of stars that twice (or more) have been in the dredge-up phase. This picture can be more complex due to overlapping of the different stages of the crossing.

According to Iben (1966), the number of Cepheids suffering first dredge-up is aproximately 20 times larger than the number of stars before dredge-up (for masses $\approx 9M_\odot$). In our case, the corresponding ratio is ~ 10, which means that possibly in the first group we have some Cepheids with low amplitude which may be of a different origin (overtone pulsators, close companions, etc). In accordance with evolutionary calculations, the number of Cepheids crossing the instability strip for the second or third time is maximal.

If our suggestions about the Cepheid crossing numbers are correct, we should expect some independent predictions, which may be regarded as consequences of this idea. As one such phenomenon let us consider the changes of the Cepheid periods. The first crossing, from the blue to the red edges of the instability strip, should be characterized by an increasing period. Amoung the first 12 crossing Cepheids, six of them (ER Aur, FI Car, YZ Car, SZ Cas, CP Cep, RY Sco) have a continuous increase of period and six have no visible changes of period, which in the main are in agreement with the results of this paper.

Thus, we can identify the evolutionary stage of Cepheids using the F_V amplitude. This, in particular, enables us to specify the $P - L$ dependence and to determine more correctly the masses and ages of individual Cepheids using their evolutionary tracks.

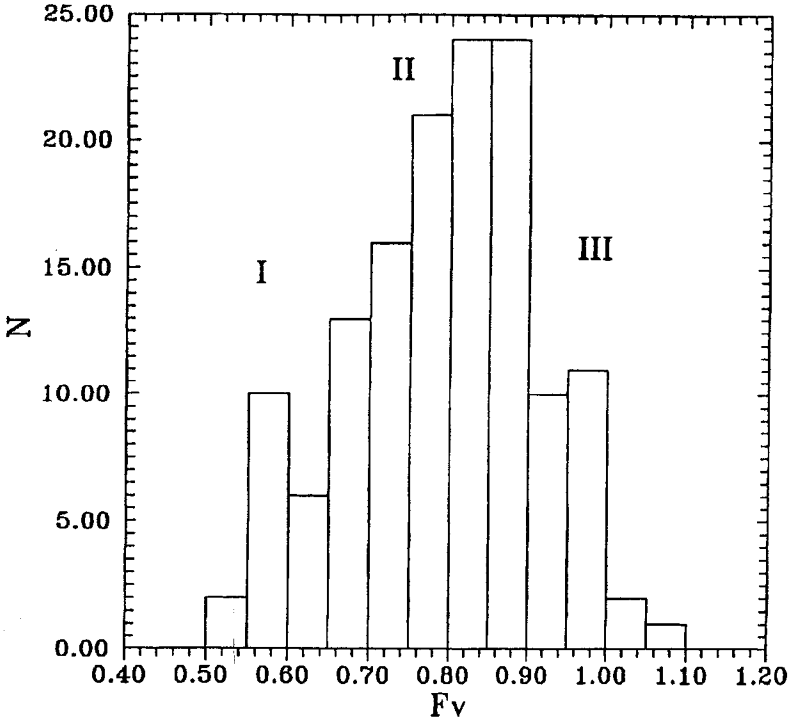

Figure 4 The histogram of the Cepheid amplitude F_V.

3 CEPHEIDS WITH $P < 8^d$

The metallicity, as well as the helium content, strongly affects the pulsational activity of Cepheids with $P < 8^d$. With the metal abundance decreasing, the instability strip is shifted toward the blue region, which probably affects the amplitude. Besides, in the case of small P values, the Cepheids can only partially touch the instability strip. The influence of the metallicity on the Cepheid amplitude is clearly seen for Cepheids from the SMC, which for $P \lesssim 8^d$ have a significantly greater amplitude than Galactic Cepheids (van Genderen, 1978); this is connected with the greater metal deficiency in the SMC ([Fe/H] ≈ -0.5, Luck and Lambert, 1992).

The presence of a positive amplitude gradient for the Galaxy (Figure 2, 232 stars):

$$dF_V/dR_G = 0.018\,\mathrm{kpc}^{-1} \pm 0.005 \tag{10}$$

$$dF_B/dR_G = 0.025\,\mathrm{kpc}^{-1} \pm 0.006 \tag{11}$$

testifies to the dominant role of the metallicity in the influence on the amplitude comparing with the influence of the helium content. The F_B amplitude is more sensitive to the metallicity. Therefore, in this case we have to calibrate the depen-

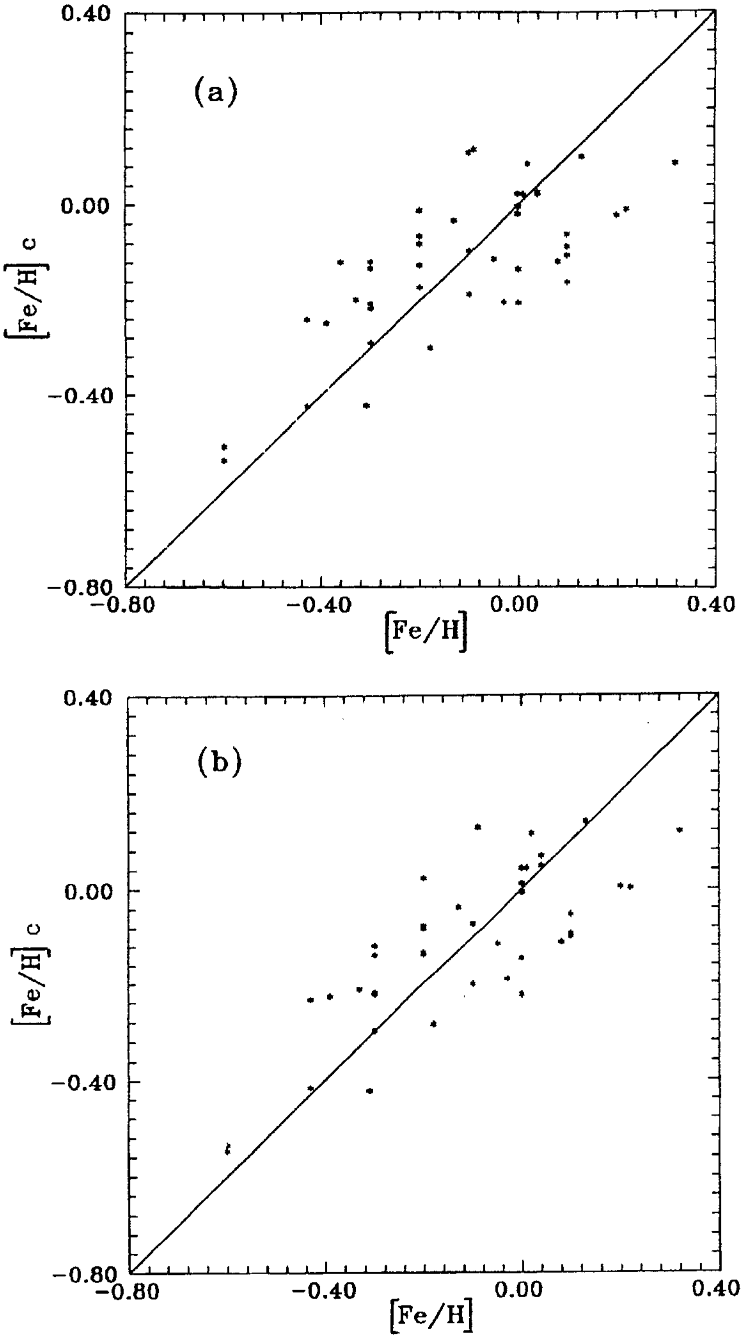

Figure 5 Relation between the spectroscopically determined [Fe/H] values for 42 Cepheids with $P < 8^{\mathrm{d}}$ vs. $[\mathrm{Fe/H}]_C$ calculated from equations (12) and (13) (see text): (*a*) for F_V; (*b*) for F_B.

dence [Fe/H] $\sim A$. A similar calibration has been done by Eggen (1985), but he used Cepheids with $P > 10^{\rm d}$ and $P < 10^{\rm d}$ which is not justified (see text above).

If we plot the asymmetry parameter $M - m$ (that is, the interval between the phases of the maximum (M) and minimum (m) of the light curves) of Cepheids versus the determination of [Fe/H] for Cepheids with $P < 8^{\rm d}$ it is clear that a statistical correlation also exists between these two parameters.

For the [Fe/H] $\sim (A, M - m)$ calibration we used, first of all, well-known determinations of Cepheid metallicity based on high-resolution spectral analysis (Giridhar, 1983; Harris and Pilachowski, 1984; Luck and Bond, 1989a, b; Luck, 1994; Andrievsky *et al.*, 1994) and data of Harris (1981) obtained in the Washington photometrical system.

The following relations between the various parameters of classical Cepheids are taken:

$$\begin{array}{llll} [Fe/H] = 0.739(M-m) & +\,0.064P & -\,0.284F_V & -\,0.425 \\ \qquad\quad \pm\,0.459 & \pm\,0.015 & \pm\,0.182 & \pm\,0.270 \end{array} \qquad (12)$$

$$\begin{array}{llll} [Fe/H] = 0.931(M-m) & +\,0.070P & -\,0.194F_B & -\,0.593 \\ \qquad\quad \pm\,0.441 & \pm\,0.015 & \pm\,0.147 & \pm\,0.230 \end{array} \qquad (13)$$

The asymmetry parameters $M - m$ are also taken from GCVS (Kholopov *et al.*, 1985, 1987). We ignored all s-Cepheids. This solution is valid for Cepheids with $P < 8^{\rm d}$ and has a standard deviation σ (one star) $= 0.15$ dex, which is only slightly poorer than the best spectroscopic determination. This calibration is illustrated in Figure 5 in which the 42 calibrating Cepheids are plotted in the [Fe/H] (observed) versus $[\mathrm{Fe/H}]_C$ (calculated) plane.

4 CONCLUSION

The major conclusions that have been drawn from the present study of the radial gradient of the relative amplitude F_V, F_B for Galactic classical Cepheids are the following:

(1) The gradient F_V, F_B has different signs for two cases: $P < 8^{\rm d}$ and $P > 8^{\rm d}$.

(2) It is shown that for $P > 8^{\rm d}$, the gradient of the amplitude is caused by the general gradient of the He content in our Galaxy. The calibrating relations [He/H] $\sim F_V$ ($Y \sim F_V$) were obtained from the analysis of this gradient and from the study of Cepheids first crossing the instability strip Cepheids. There is good agreement between these relations and theoretical predictions.

(3) It is concluded that the F_V value confidently allows us to determine how many times a Cepheid has been in the red giant phase.

(4) The asymmetry parameter $M - m$ and the amplitude F_V (F_B) of Cepheids with $P < 8^{\rm d}$ is mainly affected by the metallicity. [Fe/H] $\sim (F_V, M - m)$, [Fe/H] $\sim (F_B, M - m)$ calibrations were obtained. This calibrations reproduced [Fe/H] to within their stated errors (±0.15 dex)

The results obtained in the present work allow us to resolve in a first approximation the problem of Cepheid amplitudes and the problem of the helium content in their atmospheres. The relations (7), (8), (9), (12), (13) can be used as a diagnostic tool for the estimation of the metallicity and helium abundance in stellar systems and the nearest galaxies and for a study of the metallicity gradient in the Galactic disk.

For the specification of the main dependences (7–9), an additional search for Cepheids (first crossing the instability strip) is needed. It is also necessary to specify an expression for the gradient of [He/H] in our Galaxy.

Acknowledgements

I would like to express my gratitude to Dr. J. D. Fernie for the Cepheid catalogue, to the European Southern Observatory and personally to Dr. R. West for a C & E E financial grant (A–05–068), which has supported the realization of this programme.

References

Adelman, S. J. (1986) *Astron. Astrophys. Suppl. Ser.* **64**, 173.

Andrievsky, S. M., Kovtyukh, V. V., Usenko, I. A., Klochkova, V. G., and Galazutdinov, G. A. (1994) *Astron. Astrophys. Suppl. Ser.* **108**, 433.

Andrievsky, S. M., Kovtyukh, V. V., and Usenko, I. A. (1996) *Astron. Astrophys.* **305**, 551.

Berdnikov, L. N. (1987) *Perem. Zvezd.* **22**, 505.

Blitz, L., Fich, M., and Stark, A. (1982) *Astrophys. J. Suppl. Ser.* **49**, 183.

Cogan, B. C., Cox, A. N., and King, D. S. (1980) *Space Sc. Rev.* **27**, 419.

Efremov, Yu. N. (1978) *Sov. Astr.* **55**, 272.

Efremov, Yu. N. (1979) *Sov. Astr. Lett.* **5**, 21.

Eggen, O. J. (1985) *Astron. J.* **90**, 1260.

Eggen, O. J. (1994) *Astron. J.* **107**, 2184.

Elmegreen, B. G. and Elmegreen, D. M. (1983) *Month. Not. R.A.S.* **203**, 31.

Fernie, J. D., Evans, N. R., Beatti, B., and Seager, S. (1995) *IBVS*, No. 4148, 1.

van Genderen, A. M. (1978) *Astron. Astrophys.* **65**, 147.

Giridhar, S. (1983) *J. Astron. Astrophys.* **4**, 75.

Giridhar, S., Ferro, A. A., and Goswami, A. (1991) *J. Astron. Astrophys.* 12, 27.

Grevesse, N. and Noels, A. (1993) In: N. Prantzos, E. Vangioni–Flam, and M. Cassé (eds.), *Origin and Evolution of the Elements*, , Cambridge Univ. Press, p. 15.

Guzik, J. A. (1992) *Delta Scuti Star Newsletter*, No. 5, 8.

Harris, H. C. (1981) *Astron. J.* **86**, 707.

Harris, H. C., and Pilachowski, C. (1984) *Astrophys. J.* **282**, 655.

Hawley, S. A. (1978) *Astrophys. J.* **224**, 417.

Iben, I. Jr. (1965) *Astrophys. J.* **142**, 1447.

Iben, I. Jr. (1966) *Astrophys. J.* **143**, 483.

Kholopov, P. N. *et al.* (GCVS) (1985, 1987) *General Catalogue of Variable Stars*, Nauka, Moscow.

Kovtyukh, V. V., Komarov, N. S., and Depenchuk, E. A. (1994) *Sov. Astron. Lett.* **20**, 215.

Kovtyukh, V. V., Andrievsky, S. M., Usenko, I. A., and Klochkova, V. G. (1996) *Astron. Astrophys.*, in press.

Kraft, R. P. (1960) *Stars and Stellar Systems* **3**, 421.
Kurucz, R. L. (1992) In: B. Barbuy, A. Renzini (eds.), *The Stellar Populations of Galaxies*, IAU Symp. 149, 225.
Lennon, D. J. and Dufton, P. L. (1986) *Astron. Astrophys.* **155**, 79.
Lichten, S. M., Rodriguez, L. F., and Chaisson, E. J. (1979) *Astrophys. J.* **229**, 524.
Luck, R. E. (1994) *Astrophys. J. Suppl. Ser.* **91**, 309.
Luck, R. E. and Bond, H. E. (1989a) *Astrophys. J. Suppl. Ser.* **71**, 559.
Luck, R. E. and Bond, H. E. (1989b) *Astrophys. J.* **342**, 476.
Luck, R. E. and Lambert, D. L. (1981) *Astrophys. J.* **245**, 1018.
Luck, R. E. and Lambert, D. L. (1985) *Astrophys. J.* **298**, 782.
Luck, R. E. and Lambert, D. L. (1992) *Astrophys. J. Suppl. Ser.* **79**, 303.
Maciel, W. J. and Köppen, J. (1994) *Astron. Astrophys.* **282**, 436.
Matteucci, F. (1992) G. Longo, M. Capaccioli, and G. Busarello (eds.), *Morphological and Physical Classification of Galaxies*, Kluwer, Dordreht.
Peimbert, M. (1986) *Pub. Astron. Soc. Pacific* **98**, 1057.
Peimbert, M. (1993) *Rev. Mex. Astron. Astrophys.* **27**, 9.
Peimbert, M., Torres-Peimbert, S., and Rayo, J. F. (1978) *Astrophys. J.* **220**, 516.
Sandage, A. and Tammann, G. A. (1971) *Astrophys. J.* **167**, 293.
Shaver, P. A., McGee, R. X., Newton, L. M., Danks, A. C., and Pottasch, S. R. (1983) *Month. Not. R.A.S.* **204**, 53.
Stobie, R. S. (1969) *Month. Not. R.A.S.* **144**, 489.
Talent, D. L. and Dufour, R. J. (1979) *Astrophys. J.* **233**, 888.
Thum, C., Mezger, P. G., and Pankonin, V. (1980) *Astron. Astrophys.* **87**, 269.
Torres-Peimbert, S. and Peimbert, M. (1977) *Rev. Mex. Astr. Astrophys.* **2**, 181.
Zhevakin, S. A. (1954) *Doclady Akadem. Nauk* **99**, No. 2, 217.