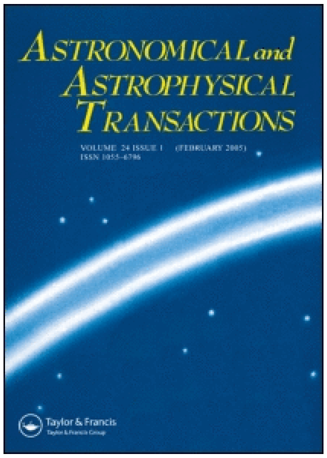

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INFLUENCE OF COMPUTER REDUCTION ON THE RESULTS OF COMPUTER CALCULATIONS FOR LONG-TERM INTEGRATION

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Computer reduction is one reason why many computer models of celestial mechanics exhibit chaotic behaviour. This makes the build-up of long-term prognoses of behaviour of small bodies in the Solar System more complicated. Utilizing stochastic variation in the initial conditions of the model within the accuracy of the computing system's bit grid we can smooth the influence of computer reduction and construct more valuable prognoses.

Keywords: Computer reduction; Celestial mechanics; Computer calculations; Long-term prognoses

The mathematical models of classical celestial mechanics describing the evolution of the substantial systems in space are nonlinear, and it is impossible to derive an analytical solution. In such conditions the solution of the task and research on its complex dynamics may be approached with computer simulation.

Any computing approximation of a nonlinear model as a numerical algorithm is sensitive to the accuracy of the initial data. The results of numerical procedures on such conditions reflect a state that does not behave in the same way as the initial determined system. This is a limitation for obtaining a long-term prognosis, even for elementary nonlinear dynamic systems. Adequate deterministic description and solution is precluded because infinite computing and observational accuracy are not possible (Ford, 1983).

When computer models and appropriate technologies of a computing experiment imitating the dynamics of a continuous mathematical model are used, the information obtained about its evolution states has not a continuous but a discrete character. This discrete character appears for both estimated and computed constants, for parameters and variables and for methods, operations and model equations.

The build-up of a numerical solution by integrating dynamic equations with a numerical method is carried out on a grid; that is, the determined numerical algorithm gives a solution at isolated points. This, in our judgement, is the limitation of the determined numerical algorithms. In such an approach the solution has a selective dot character. The increase in error of

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such a solution is especially strong in close encounters, when there is an intensive energy exchange between bodies.

The determined numerical solution contains errors; according to the classical error theories they are as follows:

(i) unremoveable (errors in the model and initial data);
(ii) errors in the numerical method errors;
(iii) computing errors or round-off errors (Bachvalov et al., 2002).

The integration errors increase and at some $T > T_{crit}$ the global error (over all interval of integration) exceeds the local error (of the order of a step) for long-term integration. The integration errors have a random character. Logically, for their estimation, control and recording, stochastic approaches and methods ought to be developed.

Stochastic variation in the initial conditions of the model allows errors caused by digitization of the model to be minimized to some extent. On the other hand, stochastic variation in the initial conditions of the model allows various features of the nonlinear celestial mechanics system to be revealed as well as the influence of the degree of these uncertainties on the system stability with respect to initial data to be defined. Stochastic modelling on the basis of Monte Carlo methods allows a statistical probability solution of dynamics equations to be found (Neiman, 1971).

Information of the investigated system state may be obtained only by appropriate system preparation and observation. The specificity of these processes is the integral part of the objective system description of the system that was repeatedly emphasized (Winer, 1961; Mishev, 1998).

The reduction in or obtaining of information in computer dynamic systems is the ‘mastering’ part of the information, obtained or worked out during the recursive process. Computer reduction appears in computer modelling, for example, when decimal places are discarded or lost in arithmetic operations (round-off error), owing to an inadequate model for the derivation operator by the difference operator (method or algorithm error). Other examples can be discussed. Such a process can be called genetic evolution, when part of the information on the research object is eliminated owing to discarding some of the tags from generation to generation (from iteration to iteration) during evolution. In this case the object is considered to be the machine solution of the system of equations.

Any physical measurements are connected to obtaining and processing information. They also require entropy and power expenditures: ‘one has to pay for obtaining information on [the] rise [in] entropy’ (Volkshtain, 1986). The processes of measurement and information processing of the system by technical tools are in essence irreversible. Apparently, the process of computer reduction is irreversible; in this sense the process of computer reduction occurs because of the increase in the computer’s dynamic system (Nicolis and Prigogine, 1990).

The computer’s dynamic system is dissipative; the dissipation arises because some of the information, for example round-off and method errors, disappears. To remove or bypass completely the process of computer reduction is obviously not possible. The confirmation is given by the second law of thermodynamics.

As an algorithm of integrating dynamics equations the Everhart (1985) algorithm (RADA27) is chosen. The equations are solved on the basis of Monte Carlo methods using the Bernouilli scheme.

Taking into account features of the behaviour of celestial mechanics systems, it is convenient to carry out research on their evolution using the space of Keplerian motion elements a, e, i, Ω, ω and ν.

TABLE I Values of the Keplerian Elements of the Brooks 2 Comet on January 25, 1800 and Limiting Values for Figures 1 and 2 (a_{min}, a_{max}, etc.).

Keplerian element	Value	Minimum	Maximum
a_0 (a.e.)	9.089 03	9.089 029	9.089 031 5
e_0	0.375 5	0.375 49	0.375 51
i_0 (deg)	6.25	6.249	6.251
q_0 (a.e.)	5.676 1	5.676 09	5.676 11
ω_0 (deg)	351.391	351.39	351.392
Ω_0 (deg)	197.761	197.76	197.762

Research into the reduction process of the build-up of the numerical solution of the two-body problem (the Brooks 2 comet and the Sun) was carried out. The equations of motion of the N-body problem are integrated numerically at $N = 2$. The initial conditions are represented in Table I (Belyaev et al., 1996).

The integration interval of the equations of motion or the considered evolution period is 275 years. In the first stage the research of the given task solution behaviour was carried on without varying the initial conditions, that is at $a_0 =$ constant, $e_0 =$ constant, $i_0 =$ constant, $\Omega_0 =$ constant, $\omega_0 =$ constant and $v_0 = 75.0^\circ$ (constant) (determined initial conditions).

Let us consider the evolution image of Keplerian elements for the Brooks 2 comet within the framework of the given two-body problem (Fig. 1). The precise analytical solution of the given two-body problem is shown in the figure by dashed lines (Duboshin, 1968).

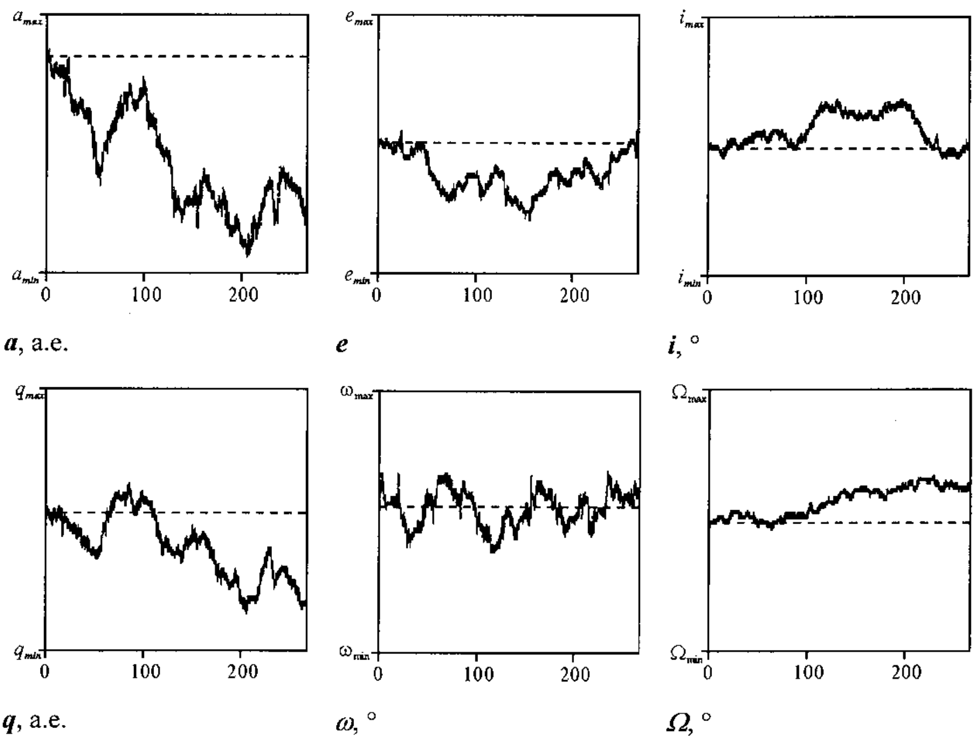

FIGURE 1 Evolution image of Keplerian elements for the Brooks 2 comet (two-body problem with the determined initial conditions).

The given image shows the reduction process of computer solution of the considered two-body problem within the framework of the computer model. In the given figure it is possible to estimate the degree of influence of computer reduction on evolution parameters of the system.

In the subsequent stage, research on the computer reduction process for the considered two-body problem was carried out with variation in the initial conditions $a_0 = \text{constant}$, $e_0 = \text{constant}$, $i_0 = \text{constant}$, $\Omega_0 = \text{constant}$, $\omega_0 = \text{constant}$ and v_0 is a random variable (stochastic initial conditions). As a stochastic variable the true anomaly v was selected. The area of variation was $v_0 \in 75.0^\circ \pm 10^{-14\circ}$. The probability distribution in the area is accepted as uniform. It should be noted that the computer bit grid of a computer, on which the computer model was constructed, allows lossless accuracy to be encoded by a binary code of number less than 1 (mantissa) from 15 to 16 by decimal signs after a comma. The statistical images of the evolution of Keplerian elements for the Brooks 2 comet for a given task are represented in Figure 2.

These figures show that, by varying the initial data within the area of accuracy of the computer bit grid, we may smooth the influence of computer reduction. Finally, we construct a more valuable prognosis of the behaviour of a system. It is obvious that there are conditions approximating model and analytical solutions; on the average the model solution is closer to the analytical solution (Fig. 2), in comparison with the determined case (Fig. 1). In the presented case the solution will be given by the probability characteristics, which can be estimated on statistical images. In the given approach (Fig. 2) the influence of computer

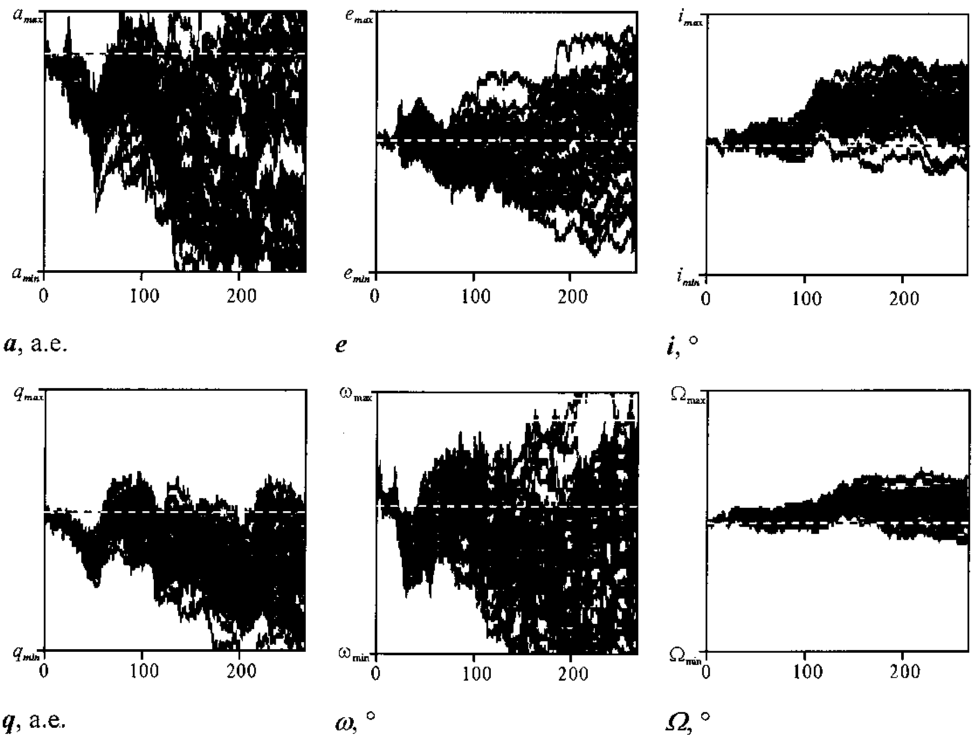

FIGURE 2 Statistical images of the evolution of Keplerian elements for the Brooks 2 comet (two-body problem with stochastic initial conditions).

reduction no longer has a random character, owing to selectivity of one concrete implementation of machine solution, as in the former case (Fig. 1).

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