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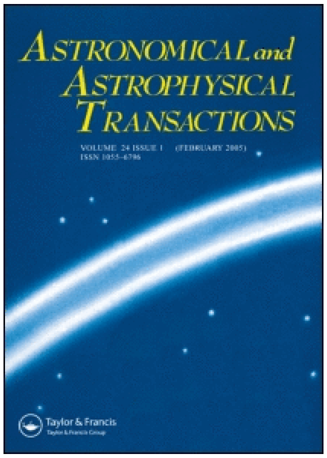

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 January 2003
To cite this Article: Zhugzhda (2003) 'Waves and shear flows', Astronomical & Astrophysical Transactions, 22:4, 593 - 606
To link to this article: DOI: 10.1080/1055679031000124457
URL: http://dx.doi.org/10.1080/1055679031000124457

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Astronomical and Astrophysical Transactions
Vol. 22, Nos. 4–5, August–October 2003, pp. 593–606

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WAVES AND SHEAR FLOWS

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(Received October 16, 2002)

The exact analytical solution of the extended Rayleigh equation for the case of periodic compressible shear flow is found. The dispersion relation of the problem is the Hill determinant. It is found that the sound waves in shear flow have a dispersion and its velocity field contains a solenoidal part. Besides the sound waves, new wave modes, namely phonon, waveguide and vortex wave modes, are revealed. The phonon mode is similar to phonons in the crystal lattice but they are not connected to heat transfer. The vortex mode is a singular solenoidal mode. The vortex modes are negative-energy waves and it is possible that they possess dissipative instability. The absolute phonon-vortex instability appears for a Mach number Ma $\gtrsim 0.4$. The interplay of waves and solar granulation is considered.

Keywords: Shear flows; Waves; Instabilities; Solar atmosphere

1 INTRODUCTION

The Sun and many stars have a convection zone. The convection consists of hot upflows and cold downflows. Also, convection generates turbulence. Both convection and turbulence produce a pattern of alternating temperature fluctuations and alternating shear flows. The exploration of the effect of convection on hydrodynamic waves encounters problems since both temperature and velocity fluctuations affect acoustic-type waves. For example, in the case of the solar photosphere, the local phase velocity of upgoing waves in intergranular lanes is half that in granules. The phase velocity in the intergranular lanes decreases as a result of low temperatures and counter-flow. The scale of velocity and temperature fluctuations is less than a wavelength for 5 min oscillations in the solar atmosphere. So, one needs to know the effect of strong fluctuations of sound velocity on waves whose wavelength is not always much more than the characteristic scale of fluctuations. Therefore a linear theory of waves in shear flows is needed.

The linear theory of shear flows is one of the cornerstones of the theory of turbulence. Despite the efforts of generations of scientists, there are still unsolved problems. First of all the classical linear theory of the shear flow instability (Drazin and Reid, 1981) contradicts some of the laboratory experiments. The classical theory of shear flows was concerned only with exponential instability. It failed to explore waves in shear flows. The difficulties of the shear flow theory arise owing to stratification, which makes the coefficients of the Rayleigh equation dependent on the coordinate. Considerable progress towards the understanding of the physics of shear flows has been made owing to the introduction of the concept of

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negative-energy waves and exploration of the plasma analogy (Craik, 1985; Ostrovskii et al., 1986; Fabricant and Stepanyants, 1998), but some of the basic difficulties have still not been overcome. Also, the classical theory of sound generation by turbulence (Lighthill, 1952) failed to explain some of the laboratory experiments (Laufer and Yen, 1983), tending to look for alternative mechanisms (Mitchell et al., 1999). This initiates the development of the so-called non-modal approach to the problem (Trefethen et al., 1993). It is claimed that modal analysis misses some physics. It seems to me that the current modal analysis is just not good enough to make an exhaustive exploration of the problem. There is no question that the finding of an exact analytical solution for special velocity profiles is the best way to develop the linear theory of shear flow. Only if the dispersion relation is obtained can it be safely suggested that nothing is overlooked.

A new method of the treatment of shear flows is presented in the current paper. It makes it possible to obtain an exact analytical solution for the case of parallel periodic shear flow. The periodic shear flow is a simple model of convection, since it consists of hot and cold flows of opposite directions. The derivation of the dispersion relation for the simplest periodic shear flow allows one to make an exhaustive exploration of wave modes and instabilities. The first steps along these lines were made (Zhugzhda and Stix, 1994; Zhugzhda, 1998) in connection with the problems of helioseimology. The effect of the convection on sound waves was in focus of those papers. Some effects, namely new wave modes, were revealed, but they were not understood properly, partly because we were occupied mostly by the effect of the geometrical dispersion of sound waves.

This paper is organized as follows. The deduction of the exact analytical solution of the Rayleigh equation and the dispersion relation is outlined first. The study of all hydrodynamic wave modes in a periodic shear flow is presented. The waves of negative energy and instabilities are described. Applications to waves in the solar photosphere are discussed.

2 BASIC EQUATIONS

An equilibrium atmosphere with vertical hot upflows and cold downflows is considered. The equilibrium pressure p_0 is constant in the entire atmosphere, while the equilibrium values of temperature T_0, density ρ_0 and vertical velocity $V = (0, 0, V)$ are arbitrary functions of x. All equilibrium variables are independent of the vertical coordinate z. The problem of linear waves in the structured atmosphere is reduced to the extended Rayleigh (ER) equation

$$\frac{d^2p}{dx^2} + \left(\frac{d[\ln(c_0^2)]}{dx} + \frac{2}{V_{ph} - V}\frac{dV}{dx}\right)\frac{dp}{dx} + \left(\frac{k_z^2(V_{ph} - V)^2}{c_0^2} - k_y^2 - k_z^2\right)p = 0, \tag{1}$$

where p is the amplitude of the pressure fluctuations

$$p' = p(x)\, e^{i(\omega t - k_y y - k_z z)},$$

c_0 is the sound speed and $V_{ph} = \omega/k_z$ is the phase velocity. In the limit of a uniform atmosphere, when $V(x) = \text{constant}$ and $c_0(x) = \text{constant}$ the equation is reduced to the standard dispersion relation for the sound waves, $\omega^2 = (k_z^2 + k_\perp^2)c_0^2$, where $k_\perp$ is the 'horizontal' wave number. In the general case of a structured atmosphere it is assumed that the wave amplitude is bounded at infinity: $x \to \infty$. This condition corresponds to the requirement that $k_\perp^2 \geqslant 0$ for the case of the uniform atmosphere.

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The consideration is restricted to the specific case of the periodic shear flow, when the temperature, density and vertical velocity are periodic functions of x, with spatial period 2d:

$$T_0(x + 2d) = T_0(x), \quad \rho_0(x + 2d) = \rho_0(x), \quad V(x + 2d) = V(x). \tag{2}$$

In this special case the ER Eq. (1) belongs to the class of equations with periodic coefficients. The treatment of such equations has been developed especially for the Mathieu–Hill equations, which often appear in physical problems (Ince, 1944). However, Eq. (1) is an extension of the Mathieu–Hill equations, and more complicated algebra is needed to solve it.

The following dimensionless variables are introduced:

$$\xi = k_l x,\ \eta = k_l y,\ \zeta = k_l z,\ k_l = \frac{\pi}{2d},\ \tilde{k}_y = \frac{k_y}{k_l},\ \tilde{k}_z = \frac{k_z}{k_l},\ \tilde{\omega} = \frac{\omega}{k_l \tilde{c}},\ \tilde{V} = \frac{V}{\tilde{c}}, \tag{3}$$

$$\tilde{V}_{ph} = \frac{V_{ph}}{\tilde{c}},\ \tilde{c}_0^2 = \frac{c_0^2}{\tilde{c}^2},\ \tilde{p} = \frac{p}{p_0},\ \tilde{v}_x = \frac{v_x}{\tilde{c}},\ \tilde{v}_z = \frac{v_z}{\tilde{c}},\ \tilde{c}^2 = \langle c_0^2 \rangle, \tag{4}$$

where $\langle c_0^2 \rangle$ is the mean of the sound speed over the space period 2d. The reason for the unusual definition of the lattice wave numher k_l is that in this way the solutions of Eq. (1) is reduced to the form, which is used in the theory of the Mathieu–Hill equations (Ince, 1944). In terms of the dimensionless variables, Eq. (1) is

$$\frac{d^2 p}{d\xi^2} + \left(\frac{d[\ln(\tilde{c}_0^2)]}{d\xi} + \frac{2}{\tilde{V}_{ph} - \tilde{V}} \frac{d\tilde{V}}{d\xi} \right) \frac{dp}{d\xi} + \left(\frac{\tilde{k}_z^2 (\tilde{V}_{ph} - \tilde{V})^2}{\tilde{c}_0^2} - (\tilde{k}_y^2 - \tilde{k}_z^2) \right) p = 0. \tag{5}$$

In the following the tilde is omitted.

3 EXACT ANALYTICAL SOLUTION

To obtain an analytical solution of Eq. (5) the consideration is restricted to the case when the x dependence of the temperature and the flow velocity of the periodic shear flow are given by

$$\tilde{c}_0^2 = 1 + \delta \cos(2\xi), \quad \tilde{V} = V_m + \mathrm{Ma}\cos(2\xi), \tag{6}$$

where Ma is the Mach number. The mean flow velocity

$$V_m = \mathrm{Ma}\,\delta^{-1}[1 - (1 - \delta^2)^{1/2}] \tag{7}$$

is defined by the condition of the zero mean mass flux $\langle \rho_0 V \rangle = 0$. In this case, Eq. (5) is

$$A_1 \frac{d^2 p}{d\xi^2} + A_2 \frac{dp}{d\xi} + A_3 p = 0, \tag{8}$$

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where

$$\begin{aligned}
&A_1 = a_1 + a_2 \cos(2\xi) + a_3 \cos(4\xi),\ A_2 = b_1 \sin(2\xi) + b_2 \sin(4\xi),\\
&A_3 = c_1 = c_2 \cos(2\xi) + c_3 \cos(4\xi) + c_4 \cos(6\xi),\\
&a_1 = V_D - 0.5\delta\ \mathrm{Ma},\ a_2 = \delta V_D - \mathrm{Ma},\ a_3 = -0.5\delta\ \mathrm{Ma},\ b_1 = -2(\delta V_D + 2\ \mathrm{Ma}),\\
&b_2 = -\delta\ \mathrm{Ma},\ c_1 = V_D[V_D^2 + 1.5(\mathrm{Ma})^2]k_z^2 - a_1(k_y^2 + k_z^2),\\
&c_2 = -3\ \mathrm{Ma}\ [V_D^2 + 0.25(\mathrm{Ma})^2]k_z^2 - a_2(k_y^2 + k_z^2),\\
&c_3 = 1.5V_D k_z^2(\mathrm{Ma})^2 - a_3(k_y^2 + k_z^2),\ c_4 = -0.25(\mathrm{Ma})^3 k_z^2,\ V_D = V_{ph} - V_m.
\end{aligned} \tag{9}$$

The mean velocity V_m causes a Doppler shift of frequency $\omega_D = \omega - k_z V_m$ in the laboratory frame.

The general case of oblique wave propagation with respect to the flow direction is considered. It is known from the theory of differential equations with periodic coefficients (Ince, 1944) that there are two kinds of bounded solution of Eq. (8) with a period of either $\pi 0$) or $2\pi(n = 1)$:

$$p = e^{ik_\perp \xi} \sum_{-\infty}^{\infty} C_{2m+n}\, e^{i(2m+n)\xi}, \quad n = 0, 1. \tag{10}$$

The coefficients C_{2m+n} are not arbitrary constants. The solution satisfies Eq. (8) only for a special choice of the constants C_{2m+n}. After substitution of the solution (10) into Eq. (8) and collecting terms with the same powers of the $e^{i\xi}$, the equation is replaced by the expansion in terms of $e^{i\xi}$. Because the solution (10) has to satisfy the equation for all values of ξ, the coefficients of the equation expansion in powers of $e^{i\xi}$ will be equal to zero. This condition provides the infinite set of coupled linear algebraic equations in the coefficients of the solution (10). The set of the algebraic equations can be rewritten as the recursion relations for the coefficients C_{2m+n}:

$$\begin{gathered}
q_m C_m + l^+_{m+2} C_{m+2} + l^-_{m-2} C_{m-2} + r_{m+4} C_{m+4} + r_{m-4} C_{m-4} + s(C_{m+6} + C_{m-6}) = 0,\\
k_{\perp,m} = m + k_\perp, q_m = -k^2_{\perp,m} a_1 + c_1, l^\pm_m = 0.5(-k^2_{\perp,m} a_2 \mp k_{\perp,m} b_1 + c_2),\\
r_m = 0.5(-k^2_{\perp,m} a_3 + k_{\perp,m} b_2 + c_3), s = 0.5 c_4,\\
m = 0,2,4\ldots \ \text{for } n = 0,\\
m = 1,3,5\ldots \ \text{for } n = 1,
\end{gathered} \tag{11}$$

where a_n, b_n and c_n are defined by Eq. (9). The set of linear algebraic equations has a non-trivial solution when its determinant equals to zero. The determinant, which is known in the theory of the differential equations with periodic coefficients as the Hill determinant, is infinite. In the case under consideration the Hill determinant is a dispersion equation for

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hydrodynamic linear waves in periodic shear flow. For the solution (10) with $n = 0$ the dispersion equation is

$$D(\omega) = \begin{vmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & q_{-4} & l_{-2}^{+} & r_0 & s & 0 & \cdots \\ \cdots & l_{-4}^{-} & q_{-2} & l_0^{+} & r_2 & s & \cdots \\ \cdots & r_{-4} & l_{-2}^{-} & q_0 & l_2^{+} & r_4 & \cdots \\ \cdots & s & r_{-2} & l_0^{-} & q_2 & l_4^{+} & \cdots \\ \cdots & 0 & s & r_0 & l_2^{-} & q_4 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{vmatrix} = 0. \quad (12)$$

In fact the dispersion relation for $n = 1$ is reduced to Eq. (12) by the transformations $k_\perp \rightarrow k_\perp \pm 1$, as follows from the general solution (10). Thus, it is sufficient to explore one of them. The dispersion equations defined by Hill determinants are polynomials of infinite degree in the variables ω, k_z and $k_\perp$. The infinite degree appears owing to the infinite number of wave modes in the structured atmosphere. The Hill determinant has to be truncated to consider a finite number of the wave modes.

4 WAVE MODES

The key point of the current paper is the exploration of linear waves in shear flows. In contrast with uniform compressible media, which supports only acoustic waves, the (k, ω) diagrams for periodic shear flow shown in Figure 1 disclose many different wave modes. Most of these have been unknown until now. To understand their properties, analogies with solid-state physics, plasma physics and photonics are used. The different wave modes, namely sound, phonon, waveguide and vortex modes, are found in the periodic shear flow. The only way to explore these wave modes is the solution of the truncated Hill determinant. The truncation is limited in the number of modes and leads to cut-off of high harmonics in the solution (10). The limitation on the number of modes is not essential in most cases, because the high-order modes do not cross the low-order modes, which are of the greatest interest. In contrast, the cut-off of the solution (10) means the replacement of the exact eigenfunction by an approximate eigenfunction resulting in a reduction in the accuracy of eigenfrequencies. To check the convergence of the results the dimension of the truncated Hill

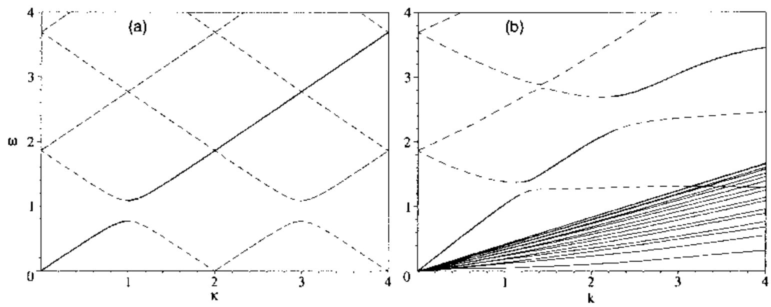

FIGURE 1 The (k, ω) diagrams for (a) horizontal (k_z,$k_y = 0$) and (b) oblique $k_z = k_\perp$, $k_y = 0$ propagation of the sound (——), phonon (– – –), waveguide (········) and vortex (——) modes in the periodic shear flow for $\delta = 0.6$ and $Ma = 0.6$ and the dimension of the determinant (12) is $T = 40$.

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determinant is varied in the range $T = 10$–200. The results of the numerical solution of the truncated Hill determinant (12) are presented on Figure 1 as (k, ω) diagrams, where the total wave number is $k = (k_\perp^2 + k_y^2 + k_z^2)^{1/2}$. Two representative (k, ω) diagrams for transverse and oblique propagation of waves in periodic shear flow are shown in Figure 1. The diagrams are plotted for the Doppler-shifted frequency ω_D. The Doppler shift is small for small values of δ and Ma owing to the low mean velocity V_m (Eq. (9)).

4.1 Sound, Phonon and Waveguide Modes

There is no question that there is a sound mode in the periodic shear flow because, in the limit of a uniform atmosphere $\delta, Ma \to 0$, the ER Eq. (1) reduces to the dispersion equation for the sound waves $\omega = k$ (the dimensionless sound speed tends to unity in this limit).

The acoustic mode is shown by solid curves in the (k, ω) diagrams in Figure 1. The main distinctions of the sound mode in the periodic shear flow from the conventional uniform media are the occurrence of wave dispersion and frequency gaps, where the propagation of waves is inhibited. The dispersion manifests itself as the difference in the sound curve on the (k, ω) diagrams in Figure 1 from a straight line. By this it is meant that the phase speed of sound waves differs from the mean sound speed (4) as the result of dispersion. The deviation of the phase speed from the sound speed increases with increasing $k_\perp$. The dispersion appears owing to the effect of both velocity and temperature variations in the shear flow.

In the case of the transverse propagation of sound waves ($k_z = 0$), the (k, ω) diagram shown in Figure 1(a) looks like the conventional Brillouin zone diagram well known in solid-state physics. To be more specific, the curve for sound waves is not continuous owing to the avoided crossing with another wave mode, which in the case of a crystal lattice is known as a phonon mode. It is not surprising that the Brillouin diagram appears in the problem, because the shear flow is periodic. The similarity of the (k, ω)-diagram in Figure 1(a) to the conventional Brillouin zone diagrams appears because of the occurrence of an alternating acoustic-type modes of negative group velocity $d\omega/dk < 0$ and of finite frequency for $k = 0$. To follow the crystal lattice analogy, this mode is called the ‘phonon’ wave mode. The avoided crossing of sound and ‘phonon’ modes creates a frequency gap, where waves are evanescent. The frequency gap is analogous to ‘forbidden energy gaps’ in the crystal lattice. In the special case of transverse propagation, the frequency gap appears as the result of only temperature variations. The waves are not affected by transverse flow because the effect of flow on waves is connected with the Doppler effect, which is absent in the case of transverse propagation. Far from the intersection with the phonon mode, the curve for the sound mode is almost straight and the phase speed differs only slightly from the sound speed.

The term ‘phonon mode’ is chosen to emphasize the similarity of some of its properties to phonon modes in a crystal lattice, but hydrodynamic ‘phonons’ are not connected with the heat transfer. They can transfer only acoustic energy. There is an infinite number of phonon modes, which differ in their frequencies for $k = 0$. The directions of group and phase velocities of phonon modes are opposite as it usually is. The frequency gap increases with increasing δ and Ma, but it does not depend on the Mach number Ma of the shear flow in the special case of transverse propagation $k_z = 0$.

Even in the case of the transverse propagation the (k, ω) diagram in Figure 1(a) differs from the conventional Brillouin zone diagram, since there is one more acoustic-type mode, which is absent in a one-dimensional lattice. The first of these modes starts from the same frequency $\omega = 2$, as the phonon mode and is almost parallel to the sound mode in the case of transverse propagation $k_z = 0$. The nature of this acoustic mode becomes clear in the case of quasilongitudinal propagation, when a one-dimensional structured

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atmosphere can be considered as a multilayered waveguide for acoustic waves. The multilayered waveguide has been studied in great detail in photonics and electronics (Yeh et al., 1977). The third mode of acoustic type in the periodic shear flow is an ordinary waveguide mode with low-frequency cut-off.

4.1.1 Solenoidal Components of Acoustic-type Modes

Sound, phonon and waveguide modes appear only in a compressible atmosphere, because the gas pressure works as a restoring force for all these modes. There is a crucial peculiarity of all acoustic-type modes in periodical shear flow. They are accompanied by vertical oscillations, while conventional sound waves in a uniform atmosphere have a pure potential. To show this effect, the velocity field of the wave mode has been decomposed into its solenoidal and potential parts. The ratio of the amplitude of the solenoidal part to that of the potential part of the velocity field for the acoustic and first phonon modes is plotted in Figure 2. The calculations for two sets of flow parameters are presented, namely $\delta = \mathrm{Ma} = 0.1$ and $\delta = \mathrm{Ma} = 0.3$. In the case of oblique propagation $k_\perp = k_z$, the solenoidal part increases from about 0.1 to 0.3, when the Mach number and the amplitude of temperature variations increase from 0.1 to 0.3. The solenoidal component of the velocity field appears to be due not only to shear flow. Even in the case of motionless media the solenoidal component appears as a result of temperature variations. In the case of oblique propagation shown in Figure 2(a), the ratio of the solenoidal part to the potential part of the velocity field is reduced by half if the amplitude of temperature variations is kept constant, while the Mach number decreases to zero. However, in the case of transverse propagation, temperature variations alone are not sufficient to create the solenoidal component of velocity field. The solenoidal component appears only when both temperature and velocity variations affect the waves propagating across the flow. As shown in Figure 2(b), the ratio of the components appears even more than in the case of oblique propagation. This is an unexpected property of acoustic-type waves because their eigenfrequencies are not affected by flow in the case of transverse propagation. It is instructive to point out that the solenoidal velocity field is of a small scale in comparison with the potential field. The difference in the scales increases for acoustic waves in the limit of long wavelengths. As can be seen in Figure 2, the long-wavelength acoustic waves in periodic shear flow appear to be almost solenoidal. In the case of transverse propagation shown in Figure 2(b), the ratio increases also for $k = 2$. This happens because the

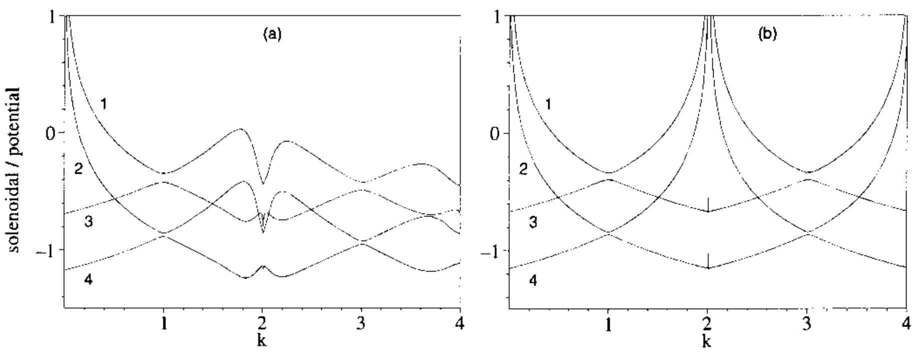

FIGURE 2 The ratio of the amplitudes of the solenoidal part to the potential part of the velocity field for (a) oblique ($k_\perp = k_z$) and (b) transverse ($k_z = 0$) propagations for two sets of flow parameters, namely $\delta = \mathrm{Ma} = 0.3$ and $\delta = \mathrm{Ma} = 0.1$: curves 1 and 2, acoustic modes; curves 3 and 4, first phonon modes.

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(k, ω) diagram is a Brillouin diagram in this case and, consequently, it is invariant with respect to the transformation $k = k \pm 2$. The occurrence of the solenoidal component of acoustic-type waves in shear flow is crucial for nonlinear effects because a direct interaction with flow appears to be possible. Until now, it was assumed that the coupling of turbulent solenoidal disturbances and acoustic waves occurs only via pressure fluctuations.

4.2 Vortex Wave Mode

The most intriguing mode in the periodic shear flow is the vortex wave mode, which is located at the bottom of the (k, ω) diagram in Figure 1(b). We present a detailed exploration of the vortex mode because of its crucial role for the flow stability.

A vortex mode appears owing to shear flow. It is absent for the case of transverse propagation of waves, which is shown in Figure 1(a). In this case the waves are not affected by flows because the projection of the component of flow velocity along the wave propagation equals zero: $v_z = 0$. The key point of the treatment of thc vortex mode is the exploration of the Hill determinant convergence. As the dimension of truncated Hill determinant, increases, the number of vortex modes increases, the separation between them in the (k, ω) diagram decreases, and they are clustered in the vicinity of the line $\omega = \mathrm{Ma}\, k_z$.

Thus, the convergences of the Hill determinant for modes of acoustic type and vortex modes differ from one another. The reason for the distinction is that in the case of vortex modes the ER Eq. (1) has an infinite number of singular points because of the factor $V_{ph} - V$ in front of the second derivative. The location of the singular points in the flow is defined by the condition $V_{ph} - V = 0$. If the phase velocity is less than the maximum flow velocity Ma, as happens for vortex eigen solutions of a truncated determinant, the singular points have to be located in pairs close to the maximum or minimum of the flow velocity depending on the sense of wave propagation along the z axis. In the limit of the infinite Hill determinant, the pairs of singular points joint in one point as the phase velocity approaches Ma. The eigenfunctions of vortex modes have to contain singularities, where the amplitudes of variables became infinite. As a consequence, the expansions of the eigenfunctions for the vortex modes in the space harmonics have to contain all high harmonics, and, in turn, all terms of the general solution (10) of the ER Eq. (1) have to be taken into account in the case of a singular solution of the equation. The truncation of the general solution (10) and the Hill determinant smooths out the singularities of the eigenfunctions. To obtain further insight into the problem, we turn our attention to the eigenfunctions of the vortex modes.

5 INSTABILITY OF SHEAR FLOW

Now, bearing in mind all wave modes of the structured atmosphere, we arrive at the key problem, namely the problem of periodic shear flow stability. As shown later the instability of the periodic shear flow is defined completely by the negative-energy waves. The concept of negative-energy waves was first proposed for waves in electron beams and has since been broadly used in plasma physics and electronics. Later, negative-energy waves were introduced into hydrodynamics in connection with waves in shear flows (a detailed discussion of the subject has been given by Craik (1985), Ostrovskii et al. (1986) and Fabricant and Stepanyants (1998). The negative-energy waves play a significant role in hydrodynamic instabilities of shear flows. They may exist only in non-equilibrium media, and in flow media in particular. Negative-energy waves are involved in all kinds of instability in flows, because

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only they can withdraw an energy from the flow. This is the reason why, because we have a dispersion relation for periodic shear flows, the search for negative-energy waves was chosen as a first step in the exploration of flow instability. Negative-energy waves can appear only when the phase velocity of waves is less than the velocity of flow. So, the wave modes, which are located in the (k_z, ω) diagram below the line $\omega = \mathrm{Ma}\, k_z$, will be the focus of attention. The stability of shear flow depends on whether there is crossing of phonon and vortex modes. Figure 1(b) is plotted for $\mathrm{Ma} = 0.6$ and $\delta = 0.6$, when there is crossing. The crossing is replaced by avoided crossing for fewer values of Ma and δ. Figure 3 shows the dependence of the occurrence of crossing on the the choice of flow parameters.

5.1 The Concept of Similarity of the Effects of the Determinant Truncation and Dissipation

The effect of truncation of the Hill determinant has received much consideration. The necessity for truncation of the Hill determinant seems to be a shortcoming of the current method. The effect of truncation causes significant complications for the treatment of vortex modes. However, precisely in the case of vortex modes, truncation helps us to escape from the search for singular solutions of the ER equation. Smearing of the singularities of eigenfunctions by truncation of the determinant is similar to smoothing out singularities by viscosity or other dissipative effects. The reason for this is that the high space harmonics in the expansion of the eigenfunction are rapidly damped owing to viscosity. The amplitudes of high harmonics rapidly decrease with increasing number of harmonics, starting from a certain harmonic that is defined by the value of viscosity. Thus, the effect of dissipation calls for the 'truncation' of the wave functions, but the 'truncation' of the expansion of the eigenfunction in terms of space harmonics (10) as the result of the dissipation is smoothed, contrary to abrupt truncation. Consequently, the approximate solutions defined by the truncated Hill determinant have to be similar to solutions for viscous (dissipative) periodic shear flow. Therefore the $(k, \mathrm{Re}(\omega))$ diagrams of dissipative shear flow are similar to the (k, ω) diagrams obtained for inviscid adiabatic shear flow. The concept of similarity of the viscous solution to the solution defined by the truncated Hill determinant for inviscid shear flow offers probabilities of qualitative exploration of stability of viscous (dissipative) periodic shear flow.

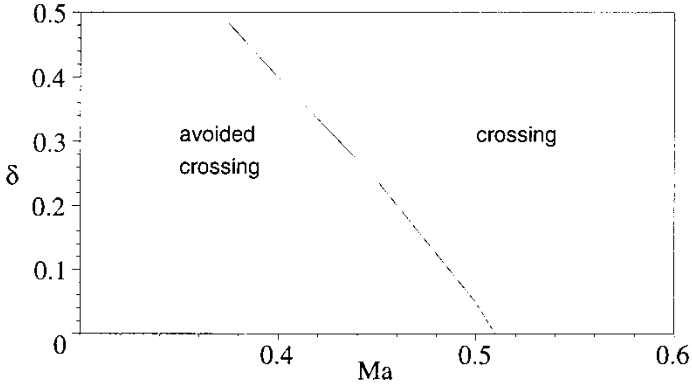

FIGURE 3 The occurrence of the crossing or avoided crossing of the vortex and phonon modes as a function of the parameters Ma and δ for the case $k_\perp = k_z$.

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5.2 Avoided Crossing: Dissipative Instability of Vortex Waves

In the case of uniform flow, $V(x) = \text{constant}$, negative-energy waves appear when their phase speed is less than flow velocity: $V_{ph} < V$. In the case of vortex modes, the dimensionless phase speed is less than the maximum flow velocity Ma, but the wave function covers also parts of flow where $V_{ph} > V$. So the vortex waves are not necessarily negative-energy waves, when the condition $V_{ph} < Ma$ is met. The wave energy in the linear approximation (see, for details, the paper by Ostrovskii et al. (1986)) is given by

$$E = \omega \frac{\partial D(k,\omega)}{\partial \omega} A^2, \tag{13}$$

where $D(\omega)$ is the dispersion relation and A is the wave amplitude. The essential point is that Eq. (13) is obtained as an expansion of the Langrangian in terms of the small wave amplitude A. The question arises whether the dispersion relation (12) can be substituted into Eq. (13). Except for some factor, dispersion relations have to be the same, irrespective of how they are obtained. So the relations for dispersion $D(\omega)$ given in Eqs. (12) and (13) differ by a factor that can be negative or positive. Consequently, only the change in the sign of the wave energy can be found from Eq. (13) if the dispersion Eq. (12) is substituted. In actual fact, the true sign of the energy of acoustic and waveguide modes is known; namely, their energy is positive since their phase velocities are greater than the maximum flow velocity: $V_{ph} > Ma$. Consequently, after substitution of Eq. (12) into Eq. (13) it is necessary to look whether there is a change in sign of Eq. (13) on going from acoustic waves to vortex waves. The vortex waves appear negative since there is a change in sign in Eq. (13) when the phase velocity V_{ph} crosses the value Ma on the way from acoustic to vortex modes.

The amplitude of negative-energy waves increases as their energy is depleted. This is a well-known dissipative instability (Craik, 1985; Ostrovskii et al., 1986; Fabricant and Stepanyants, 1998). It appears when damping due to viscosity or radiation transfer occurs. The dissipative instability is convective. This means that the waves are amplified because of the instability. Consequently, the system does not necessarily become unstable owing to dissipative instability, because the amplified waves can leave a system of finite size before their amplitude increases sufficiently to make the system unstable. If there is a wave reflection from the system boundaries or, generally speaking, a positive feedback exists, the dissipative instability becomes absolute. All vortex modes are negative-energy modes and dissipative instability of vortex modes is possible for the region of avoided crossing shown in Figure 3. The dissipative instability of vortex modes is peculiar because the effect of resonance absorption of the waves in these modes takes place. It was established that the eigenfunctions of vortex modes have singularities, which are smoothed out owing to the Hill determinant truncation. Therefore, vortex modes are subjected to resonance absorption. The theory of resonance absorption in hydrodynamics and its analogy with electrodynamics has been given by Fabricant and Stepanyants (1988). Resonance absorption can be positive or negative. The sign of the absorption is defined by the second derivative of the flow velocity:

$$\frac{d^2V(\xi)}{d\xi^2} > 0 \longrightarrow \text{positive absorption}, \tag{14}$$

$$\frac{d^2V(\xi)}{d\xi^2} < 0 \longrightarrow \text{negative absorption}. \tag{15}$$

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Thus, the negative-energy vortex modes of positive phase velocity $0 < V_{ph} \leqslant Ma$ are affected by negative absorption because the second derivative of flow velocity (6) is given by

$$\frac{d^2V(\xi)}{d\xi^2} = -4Ma\cos(2\xi). \tag{16}$$

Negative-energy waves under the effect of negative absorption become damped waves; therefore vortex waves of positive phase velocity are damped owing to the effect of negative resonance 'absorption'. All vortex modes remain damped in the limit of the infinite Hill determinant, when they tend to two limiting modes $\omega = \pm k_z$ Ma. Thus we arrive at the conclusion that the degenerative vortex mode of inviscid periodic shear flow is damped owing to the effect of resonance absorption.

Now, with the concept of similarity of the effects of truncation and dissipation of the determinant in mind, let us explore the effect of non-resonant dissipation on vortex modes. As the viscous or other dissipation is sufficiently small, the resonance dissipation of vortex modes dominates. With increasing truncation, the phase velocities of all vortex modes decrease. As a result the singular points are shifted from the middle of the flow stream to the interface with counter-flows. The second derivative of flow velocity (Eq. (16)) defines not only the sign of resonance dissipation but also the amount of dissipation, which is proportional to it. When the singular points of the eigenfunctions of vortex modes are shifted from the middle of the flow stream, the value of the second derivative (Eq. (16)) decreases. Also the eigenfunctions of vortex modes widen and cover partly the regions of counter-flow, while the phase velocity decreases owing to increasing truncation. As a consequence, regular dissipation comes into play. There are reasons to believe that regular dissipation due to viscosity or other dissipative processes can overcome resonance dissipation for high-number vortex waves. In this case the sign of dissipation is changed and dissipative instability of vortex modes appears. So, we reach the conclusion that the dissipative instability of vortex modes is possible when regular dissipation exceeds resonance dissipation for sufficiently strong dissipative processes.

5.3 Mode Crossing: Phonon–Vortex Instability and Dissipative Instability of Phonon Mode

In the case of crossing of phonon and vortex modes, an alternative method of discrimination between positive- and negative-energy waves (Ostrovskii et al., 1986) can be applied. The flux of wave energy is given by

$$S = V_{gr}E, \quad V_{gr} = -\frac{\partial D(k,\omega)}{\partial k} \Big/ \frac{\partial D(k,\omega)}{\partial \omega}, \tag{17}$$

where V_{gr} is the group velocity. When the group velocity approaches infinity, it changes sign and, simultaneously, the energy also changes sign. This method helps when negative and positive energy waves are coupled.

Figure 4 shows part of the (k,ω) diagram in Figure 1(b), where the coupling of phonon and vortex modes occurs. This is a typical (k,ω) diagram for the cases when there is coupling between positive- and negative-energy waves. Interconversion of waves with opposite signs of energy occurs at the point of the curve $\omega = \omega(k)$ where the group velocity V_{gr} approaches infinity. In Figure 4 at a point of this sort, the phonon mode of positive energy and vortex mode of negative energy merge. The frequency becomes complex after this branch point. The instability appears in the range of wave numbers between two branch

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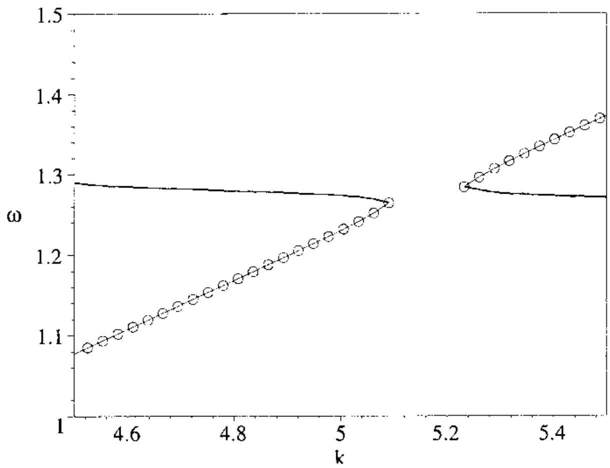

FIGURE 4 The coupling of the positive-energy phonon mode and negative-energy vortex mode is shown as a small part of the (k, ω) diagram in Figure 1(b): ○, parts of the mode curves where the energy is negative.

points in Figure 4, where the frequencies are complex. The instability of coupled waves of this kind is well known in electrodynamics (Lifschitz and Pitaevsky, 1981). The Kelvin–Helmholz instability is also an example of this type of instability. Instabilities of this kind are called absolute instabilities to distinguish them from convective instabilities, which can only amplify waves. In the limit of the infinite determinant, the coupling of phonon and vortex modes dies out, and the range of unstable wave numbers shrinks to zero. Thus, the phonon–vortex instability appears only in the case of a truncated determinant which is similar to the viscous case. This instability produces pressure fluctuations together with vortical disturbances. The phonon–vortex instability appears for subsonic flows, as can be seen in Figure 3, where the region of mode crossing is shown as a function of the flow parameters Ma and δ. It is well known that acoustic-type instabilities appear in the case of supersonic flows (Fabricant and Stepanyants, 1998). To my knowledge, subsonic acoustic-type instabilities have not been found in shear flows to date. Up to the present acoustic noise in turbulent shear flows has been considered in the framework of Lighthill mechanism, which has nothing to do with instabilities of flows. So, the revealed phonon–vortex instability can be considered as one more mechanism responsible for acoustic noise generation. When phonon modes enter the region below the line $\omega = \mathrm{Ma}\, k_z$ in the (k, ω) diagram, their phase velocity turns out to be less than Ma, and singular points of eigenfunctions appear. The resonance absorption is negative in this case, as follows from Eq. (15). Consequently, the phonon modes undergo a resonant instability, since they are positive-energy waves. The resonant instability is of a convective type, which manifests itself only in wave amplification. The resonant instability of phonon modes occurs for both a truncated and an infinite determinant, that is the resonant instability works in both inviscid and viscous shear flows. The resonant phonon instability can be involved with acoustic–vortex instability in the generation of acoustic noise by compressible turbulence.

6 WAVES AND GRANULATION

The observations clear show the interrelation between granulation and p-modes (Rimmele et al., 1995; Espagner et al., 1996; Hoekzema et al., 1998a,b; Hoekzema and Rutten,

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1998). Without question, phonon mode oscillations in granules and lanes are different. The limited space resolution of current observations provides no way for detailed exploration of the effect. For example, the width of lanes is about 100–200 km, that is beyond the resolution of current observation. The granulation has its greatest impact on waves, when both temperature fluctuations and flows are taken into account. The temperature in the granules and lanes are about 6000 K and 4000 K respectively. Consequently, the local sound velocity in lanes is less than in granules by approximately 20%. However, this is not a whole story. The upflows and downflows affect hydrodynamic wave propagation owing to the acoustic Doppler effect. For example, upgoing waves slow down in the lanes owing to the downflows. If the velocity of downflows is about $3\,\mathrm{km\,s^{-1}}$, while the velocities of upflows in the granules are about $1\,\mathrm{km\,s^{-1}}$, the local phase velocities of upgoing waves in the laboratory frame in lanes are half those in granules. This is true for high-frequency phonon modes in the photosphere, but a similar problem exists for 5 min oscillations in the subphotospheric layers. It is interesting that downgoing waves do not meet such a strong inhomogeneity, because in this case the effects of the temperature fluctuations and flows on local phase velocity of sound waves are opposite and compensate each other. The model of granulation based on the simplest periodic shear flow (Eq. (16)) is in conflict with observations, because it assumes that granules and lanes are of the same size. Figure 5 shows the periodic shear flow with wide hot upflows and thin cold downflows, which is used as the one-dimensional model of granulation. Figure 6 shows the dependence of the velocity amplitudes of upgoing and downgoing acoustic waves of periods $P = 200$ s and horizontal wavelengths $\lambda = 4400$ and 8800 km. The effect of the capture of upgoing waves by intergranular lanes and downgoing waves by granules is very pronounced. The amplitude of upgoing waves in the intergranular lanes is about twice that in the granules. If high-frequency phonon modes do not undergo reflection in the photosphere, because their frequency is above the cut-off frequency in the photosphere, the energy flux in the intergranular lanes has to be much larger than in the granules. The essential enhancement of energy flux of high-frequency phonon modes is bound to occur under intergranular lanes in the chromosphere, because the distance between lanes is about 1000 km, which compares with the thickness of chromosphere. The current model does not take into account that downflows in some lanes are rather fast. The effect of the capture of upgoing waves has to be reinforced in the lanes with fast downflows. This effect deserves more exploration in connection with bright points in the chromosphere. The phase velocities of upgoing and downgoing waves in terms of the mean sound speed in the photosphere are $V_{ph}/\overline{c_0} = 1.04$ and -1.02 (for $P = 200$ s) and 1.02 and -0.99 (for $P = 180$ s). Thus, phase velocities of high-frequency phonon modes differ only slightly from the mean sound

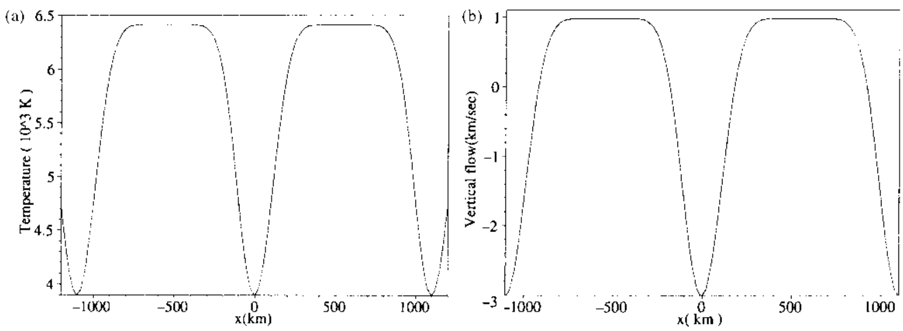

FIGURE 5 One-dimensional model of solar granulation: the dependences of (a) the temperature and (b) the vertical velocity of flows on the horizontal coordinate.

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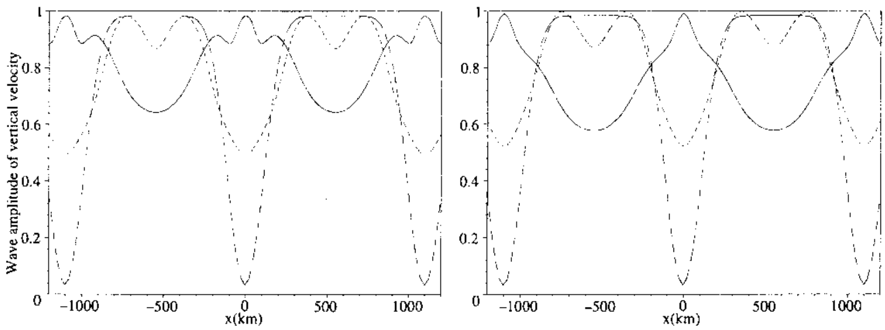

FIGURE 6 The amplitude of the vertical velocity of acoustic waves in arbitrary units against the horizontal coordinate for waves with (a) a period $P = 200$ s and horizontal wavelength $\lambda_\perp = 4400$ km and (b) a period $P = 180$ s and horizontal wavelength $\lambda_\perp = 8800$ km: ——, upgoing waves; – – –, downgoing waves; ·······, temperature profile of the granulation model in arbitrary units.

speed and corrections to the eigenfrequencies due to granulation are rather small. Another important spin-off of the exploration of periodic shear flow is the revelation of the phonon–vortex instability, which appears in the case of mode crossing. The diagram of the dependence of the appearance of the crossing on flow parameters shown in Figure 3 is calculated for simple periodic flow defined by Eq. (6). In the case of asymmetrical shear flow shown in Figure 5, the diagram in Figure 3 works if the amplitudes of temperature and velocity variations are replaced by $\delta = (\delta_1 + \delta_2)/2$, $\mathrm{Ma} = (\mathrm{Ma}_1 + \mathrm{Ma}_2)/2$, where δ_1 and Ma_1 represent upflow and δ_2 and Ma_2 represent downflow. The instability appears for downgoing waves. So, for the mean model of granulation shown in Figure 3 the instability does not appear. It can appear in the lanes where the velocity of downflow exceeds $2.5\,\mathrm{km\,s^{-1}}$. Also it has to be taken into account that the temperature 'granules' and 'intergranular lanes' of the model correspond to temperatures at the optical depth $\tau = 0.7$, while the temperature difference between them at the same geometrical depth can be much greater. So, there are reasons to believe that instability occurs in the solar photosphere. At least, it occurs in the numerical simulations of compressible shear flows (Passot and Pouquet, 1987).

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