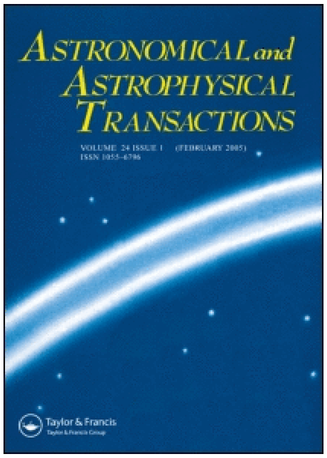

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COUPLED OSCILLATIONS IN THE EARTH–MOON SYSTEM AND GEOPHYSICAL PROCESSES

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In the Earth–Moon system, oscillations of close frequencies can be transformed as a result of the degree of coupling. The observed extended period of the Chandler oscillations can be explained by coupling of the oscillations. The problem of amplitude and period variations of the main oscillations of the Earth poles is treated on the basis of parametric excitement. For the first instability region an equation is obtained to determine the amplitudes of possible steady-state solutions. The variation limits of Chandler period oscillations and the relation between amplitudes of a possible steady-state solution for two periodic polar motion components are computed. The possible influence of energy exchange in coupled Earth–Moon oscillations on excitement and self-excitement in geophysical processes is discussed. As an illustration the interaction between the Earth's angular velocity variations and variations in the Siberian anticyclone parameters are considered.

Keywords: Coupled oscillations; Earth–Moon system; Geophysical processes

1 INTRODUCTION

The problem of oscillation interaction in the Sun–Earth–Moon system is closely connected with the influence of gravitational forces on the present geophysical processes. The mechanism of this influence is not quite clear for each individual case: is gravitation only a trigger in extreme geophysical events, or is it actively participating in the formations of irregularities and tensions in the atmosphere or in other mobile envelopes of the Earth?

The present authors proposed in 1995 a new approach to this problem (see Kurbasova and Rykhlova (2001) and Kurbasova et al. (2002) for details), the essence of which can be formulated in the following way.

(i) The Earth–Moon system is coupled so that oscillations of close frequencies can be transformed as a result of the degree of coupling and of the variations in physical situation in the system.
(ii) The amplitude and period variations of the main oscillations of the Earth's poles can be explained by the parametric influence.

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(iii) The periodic exchange processes between Earth and Moon include all the envelopes of the Earth, its core and its atmosphere.

2 PROPER OSCILLATIONS OF THE COUPLED EARTH–MOON SYSTEM

The motion of the Earth and the Moon along heliocentric orbits is restricted by mutual attraction in the Earth–Moon system. The action of this is similar to coupling the Earth and Moon motion around the Sun. In this case the coupling is an idealized model of internal interaction forces. Such a model allows the main interaction effect to be considered without probing into the physical nature of these forces.

With these assumptions the mathematical description of the motions of the Earth and the Moon under the influence of the Sun represent Lagrange equations, their right-hand sides identified with the coupling, and the Lagrange indefinite multiplier with the coupling coefficient. For this coefficient the present authors obtained (Kurbasova and Rykhlova, 2001).

$$\lambda = \frac{0.5(1+\mu)}{1-\mu}, \tag{1}$$

where $\mu = \mathrm{m_M}/\mathrm{m_E}$ is the ratio of the mass of the Moon to the mass of the Earth.

If the masses of the Moon and the Earth are not represented as material points, then the system of two coupled bodies can be regarded as two separate (partial) systems determined by the parameter σ, which is called the degree of coupling of the system (Mandelstam, 1955, 1977). For the Earth–Moon system it is

$$\sigma = \frac{2\lambda}{|\nu_1^2 - \nu_2^2|}, \tag{2}$$

where ν_1 and ν_2 are the partial frequencies of the Earth and the Moon respectively. Therefore, the coupling of the system is determined not only by the coupling coefficient but also by the closeness of the partial frequency values. The connection between the partial and proper frequencies of the Earth–Moon system is determined as a result of solution of the system of equations

$$\begin{aligned} \ddot{\xi} + \nu_1^2\xi - 2\lambda(\nu_2^2 - \nu_1^2)\eta &= 0,\\ \ddot{\eta} + \nu_2^2\eta - 2\lambda(\nu_2^2 - \nu_1^2)\xi &= 0. \end{aligned} \tag{3}$$

Here ξ and η are coordinates characterizing small deviations from equilibrium motion.

Solving the above equations, we obtain the possible proper oscillations ω_1 and ω_2 of the system for various ratios of the partial frequencies ν_1 and ν_2.

$$\nu_2 > \nu_1, \quad \omega_1^2 = \mathrm{k}_1\nu_1^2 - \mathrm{k}_2\nu_2^2, \quad \omega_2^2 = \mathrm{k}_1\nu_2^2 - \mathrm{k}_1\nu_1^2, \tag{4}$$

$$\nu_2 = \nu_1 = \omega_1 = \omega_2, \tag{5}$$

$$\nu_2 < \nu_1, \quad \omega_1^2 = \mathrm{k}_1\nu_2^2 - \mathrm{k}_2\nu_1^2, \quad \omega_2^2 = \mathrm{k}_1\nu_1^2 - \mathrm{k}_2\nu_2^2. \tag{6}$$

The coefficients k_1 and k_2 ($\mathrm{k}_1 > \mathrm{k}_2$) are roots of the equation

$$\mathrm{k}^2 - \mathrm{k} - 1.050\,43 = 0 \tag{7}$$

and depend on the value of the coupling coefficient ($\lambda = 0.512\,453$).

In the case of the Sun–barycentre of the Earth–Moon system this coefficient is similar to equation (1) and is given by

$$\lambda_S = \frac{0.5(1 + \mu_S)}{1 - \mu_S}, \tag{8}$$

where $\mu_S = (m_E + m_E)/m_S$ and m_E, m_M and m_S are the masses of the Earth, the Moon and the Sun, respectively.

The value for $\lambda_S = 0.500\,00$ using equation (7) is

$$k^2 - k - 1 = 0. \tag{9}$$

The roots of equation (9) will be the Fibonacci numbers ($k_1 = \Phi$; $k_2 = \Phi^{-1}$). These numbers are often seen in descriptions of parameters and proportions of the Solar System. For example, the rotation periods and throbbing periods of planets from a geometric progression with the denominator Φ, and others.

If we assume that $\nu_1 = 0.905\,88$ cycles years^{-1} ($T_{\nu_1} = 403.2$ days) and $\nu_2 = 1$ cycles year^{-1} ($T_{\nu_2} = 365.25$ days) and introduce them into equation (4), we obtain two proper frequencies of the Earth–Moon system: $\omega_1 = 0.840\,04$ ($T_{\omega_1} = 434.8$ days) and $\omega_2 = 1.055\,94$ ($T_{\omega_2} = 345.9$ days).

In the real system, variations in the physical conditions (energy parameters for example) lead to deviations in the proper motion frequencies from the average values determined by the system of equations (3).

3 PARAMETRIC INFLUENCE ON MAIN POLAR OSCILLATIONS

The proper oscillation frequency ω_0 and the damping coefficient δ are defined by the physical properties of the system only. By considering the external influence on the Earth–Moon system the power and parametric influences should be distinguished. The influence of power does not change the values of the parameters ω_0 and δ. The parametric influence on the contrary changes ω_0 and δ. A pure power influence on the Earth–Moon system is possible only in the ideal case of a linear system.

Considering the parametric influence, various ways to accumulate energy by the system should be classified in order to study resonance effects.

Kurbasova et al. (2002) solved the problem concerning the source and mechanism of excitement for Chandler oscillations for the first domain of parametric excitement assuming the system to be conservative. As the interaction of the Moon with the Earth is determined only by the distance between their mass centres, the description of parametric influence is made using one differential equation:

$$\ddot{x} + \omega_0^2[1 + e\cos(2\omega t)]\, f(x) = 0, \tag{10}$$

where ω_0 is the proper frequency of oscillations by sufficiently small amplitudes, e is the eccentricity of lunar orbits, ω is the frequency of oscillations excited by the change in ρ, $f(x) = x + \gamma x^3$ is the description of the nonlinear characteristic of the system and γ is determined from

$$\gamma = \pm\frac{[1 + (4\lambda)^2]^{1/2}}{2}. \tag{11}$$

By solving equation (10), the amplitude is obtained for a stationary solution:

$$A^2 = \frac{4}{3\gamma}\left(\frac{\omega^2}{\omega_0^2(1+e/2)} - 1\right). \tag{12}$$

If the mean frequency decreases on increase in the amplitude ($\gamma < 0$), the oscillations due to the parametric influence will be

$$A^2 = \frac{4}{3\gamma}\left(1 - \frac{\omega^2}{\omega_0^2(1 \pm e/2)}\right). \tag{13}$$

In this case the ω_1 and ω boundaries for the first domain of parametric excitement of Chandler oscillations are

$$\omega_1 = \omega_0\left(1+\frac{e}{2}\right)^{1/2}, \qquad \omega_2 = \omega_0\left(1-\frac{e}{2}\right)^{1/2}, \tag{14}$$

Table I shows the interval boundaries for the first domain of parametric excitement (the periods P_1 and P_2) for the average period of Chandler oscillations ($P_0 = 435.3$ days) and three values of the parameter e for the possible change in the eccentricity value of the lunar orbit.

Consideration of dissipation and deformation of amplitude caused by forced processes may extend the interval limits of the average period.

The character of the processes with parametric influence and a given modulation depth e can be described in the following way. The existence in the system of oscillations with frequency ω in the frequency interval $\omega_1 - \omega_2$ started an oscillating process with amplitude A that depends on the degree γ of nonlinearity of the systems. For a certain γ the oscillations may increase or decrease to zero if variations in ω cross the boundary of the first domain of parametric excitement.

As the system is not isochronic, the parametric excited oscillations lead to variations in the frequency of the proper oscillations and the system itself reaches the boundary of the domain of parametric excitement. This leads to a decrease in the energy that the system uses to vary parameter ρ, restricting the increase in the amplitude of the proper oscillation with frequency ω.

In the idealized system Sun–barycentre of the Earth–Moon system the average proper oscillation is 1 cycle year^{-1} and the only energy parameter is the distance between the Sun and the

barycentre. So the description of parametric resonance for this model is equation (10).

The relation between the stationary amplitudes of Chandler oscillations and oscillations with a frequency of 1 cycle year^{-1} is according to equation (12)

$$\frac{A_y}{A_z} = \left(\frac{\gamma_S e}{\gamma e_E}\,\frac{(1+e/2)}{(1+e_E/2)}\right)^{1/2}, \tag{15}$$

TABLE I Interval Boundaries for the First Domain of Parametric Excitement of Chandler Oscillations ($P_0 = 435.3$ days).

e	P_1 (days)	P_2 (days)
0.0448	430.5	440.3
0.0549	429.4	441.4
0.0650	428.4	442.6

where e_E is the eccentricity of the orbit along which the barycentre of the Earth–Moon system moves and γ_S is determined according to equation (11) if $\lambda = \lambda_S$.

Assuming that $e_S = 0.016\,751$ we obtain an amplitude of Chandler oscillations according to equation (15) that is 1.8 times larger than the oscillation amplitude with a 1 year period. This means that with $A_y = 0.17$ the amplitude with a 1 year period is $A_z = 0.094$.

4 GEODYNAMIC ASPECTS OF GEOPHYSICAL PROCESSES

In coupled oscillations it is possible to transfer energy from one wave to another during a restricted time interval without change in the total energy for the whole oscillation. For the coupled Earth–Moon system with two degrees of freedom it is possible to estimate their interaction according to the degree of energy transfer (Kurbasova et al., 2002). Proper oscillations of the Earth in the Earth–Moon system can be described in the following equations:

$$\varphi_1(t) = A(t)\cos[\omega_1 t - \psi(t)], \tag{16}$$

$$A^2(t) = -\frac{\varphi_0^2}{(|\kappa_2| + \kappa_1)^2}\{\kappa_2^2 + \kappa_1^2 + 2\kappa_1|\kappa_2|\cos[(\omega_2 - \omega_1)t]\}, \tag{17}$$

$$\tan[\psi(t)] = -\frac{|\kappa_2|\sin[(\omega_2 - \omega_1)t]}{\kappa_1 + |\kappa_2|\cos[(\omega_2 - \omega_1)t]}. \tag{18}$$

Here A(t) is the amplitude of proper oscillations of the Earth, φ_0 is the starting value of the amplitude, $\psi(t)$ is the oscillation phase, κ_1 and κ_2 are coefficients showing the distribution of the amplitude along frequencies ω_1 and ω_2; ω_1 is the frequency of proper oscillation of the Earth ($P_1 = 435.5$ days) and ω_2 is the frequency of oscillation of the lunar orbit line of nodes ($P_2 = 346.6$ days).

The frequencies ω_1 and ω_2 are not very different. The amplitude A(t) can therefore be regarded as changing with time with a period $2\pi/(\omega_1 - \omega_2)$.

The time for transferring energy from the Moon to the Earth is

$$\tau = \frac{\pi}{\omega_2 - \omega_1}. \tag{19}$$

The full cycle of exchange of energy is $P = 4.65$ years.

Real systems show deviations in the values of the proper frequencies from their average values. This causes various durations P of the energy transfer cycle. As a result, the predominant wave frequency does not remain constant at various time intervals of the energy exchange process; the modulating wave period varies from 5.40 to 4.19 years, with variations in the Chandler oscillations period in the limits from 420 to 448 days, so that coupled oscillations with close frequencies may appear. An analysis of experimental data confirms the theoretical conclusions.

5 ANALYSIS OF EXPERIMENTAL DATA

Yearly average data concerning the polar coordinates Z ($z_i = (x_i^2 + y_i^2)^{1/2}$), together with variations in the angular velocity $\Delta\omega$ of Earth rotation, the integral seismic energy E, the southern oscillation index I, were studied. Details of these data and analytic method have been given by Kurbasova et al. (1997). Data concerning the minimal distance ρ_{min} and maximal distance ρ_{max} between the mass centres of the Moon and the Earth are taken from astronomical year books. Parameters of the Siberian anticyclone (longitude shift S_λ and latitude shift S_φ of the centre and intensity anomalies S_a) from 1891 to 1999 are from the work of Sorkina (1972).

Table II shows comparison results of oscillations with periods of about 4 years detected in the spectrum of yearly average data. The comparison was performed by the two-channel autoregressive analytic method.

In the first channel are given the yearly average data of real distances: the minimal distance ρ_{min} and the maximal distance ρ_{max} between the mass centres of the Earth and the Moon. Variations in these parameters are decisive in the gravitational interaction of the Earth and the Moon.

The two-channel spectral analysis detects and compares oscillations of similar frequency in series of data in both channels. The degree of similarity of oscillations with similar periods (given in the third column) is determined by the square of the coherent modulus (given in the fourth column). Relative shifts (coherence phases) of compared oscillations are presented in the fifth column. In the sixth column are given the time intervals over which the comparison was performed.

An analysis of Table II shows that the available data contain a variation with a period about 4 years. As these oscillations are revealed in geodynamic and in geophysical data, they may be of common origin, for example the gravitational interaction in the Sun–Earth–Moon system.

The rotational velocity of the Earth around its axis is substantial larger than the velocity of exchange processes in the Earth–Moon system. This is why the removal of energy by gravitational periodic processes is negligible.

However, if the velocity of accumulation energy in a powerful anticyclone is substantially smaller than the velocity of energy redistribution in waves with similar frequencies, the reaction of the atmosphere will be an extreme displacement of inhomogeneity in the atmosphere. In this case a rotating atmosphere can be regarded as one form of parametric enhancement, and the moving inhomogeneity may be regarded as a parameter changing with time.

It is obvious that the excess of outgoing power/incoming power is provided by the Earth, that is the Earth maintains rotation of the atmosphere with an angular velocity ω.

TABLE II Two-channel Spectral Analysis of Comparison Results of Oscillations with Periods of About 4 Years.

First channel	Second channel	P (years)	KMK (%)	Relative displacement in channel (years)	Comparison results of data (years)
ρ_{min}	Z	4.43	71	−0.10	1922–1993
	$\Delta\omega$	4.47	69	−1.52	1922–1991
	I	4.47	97	−0.55	1922–1960
	E	4.45	98	−0.68	1922–1990
	S_λ	4.45	84	−0.02	1922–1999
	S_φ	4.43	82	−0.42	1922–1999
	S_a	4.43	89	0.19	1922–1999
ρ_{max}	Z	4.41	84	1.79	1922–1993
	$\Delta\omega$	4.41	73	0.66	1922–1991
	I	4.39	98	1.7	1922–1960
	E	4.43	98	1.58	1922–1990
	S_λ	4.41	78	−2.13	1922–1999
	S_φ	4.39	91	1.85	1922–1999
	S_a	4.39	78	−1.95	1922–1999

The analysis of individual (mostly by size) displacements of the Siberian anticyclone centre indicate their non-accidental origin. The maximal displacement originating simultaneously at latitude and longitude happened in February 1947 (Fig. 1). This time coincides with the longest synodic and the shortest sidereal months in the 35 year cycle starting in 1912, if the coincidence of a new Moon with apogee (winter months) is taken as the starting point.

The oscillation amplitude of the duration of the synodic month is the largest at the start and end of the 35-years period. In 1912 the difference between the longest and the shortest synodic months reached 12 h 45 min (Dolgorukov, 1912).

3–4 years after the start of the period the difference between the longest and shortest synodic months was only 4 min.

Variations in the lunar periods lead to a change in the velocity of energy exchange processes between the Earth and the Moon, which are decisive in the origin of parametric excitation in an atmosphere with a 'ready' anticyclone hearth. Peculiarities of energy accumulation in the anticyclone hearth determine the size of displacement. In 1912, at the start of the 35 year cycle, the maximal displacement of the Siberian anticyclone was only with respect to longitude.

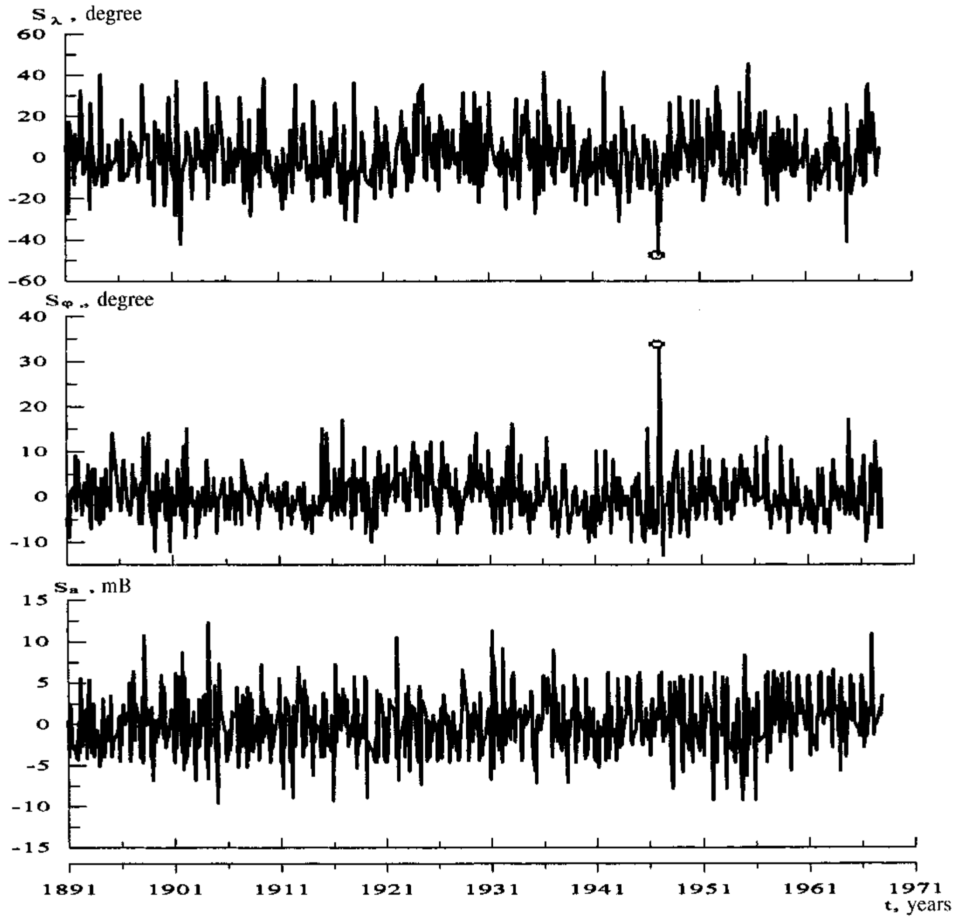

FIGURE 1 Variations in the mean monthly data S_λ, S_φ and S_a.

6 CONCLUSIONS

(i) Oscillations in the Earth–Moon system obey the main principle of dynamics coupled systems. As the result of mutual influence of coupled frequencies proper oscillations of the system originate, based on a complicated mechanism of energy redistribution including all the moving parts of the system.

(ii) In the complicated processes involving geodynamic and geophysical parameters, variations are present determined by the oscillations generated by the energy exchange processes between the Earth and the Moon.

(iii) Coupling of geodynamic oscillations may under certain conditions lead to a transfer of energy between oscillations having similar frequencies. Oscillations may be enhanced by rotation of the atmosphere and by a moving inhomogeneity (parametric enhancement). Redistribution of energy in oscillations with a similar frequency creates conditions for the origin of a trigger effect that may lead to geodynamical processes.

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