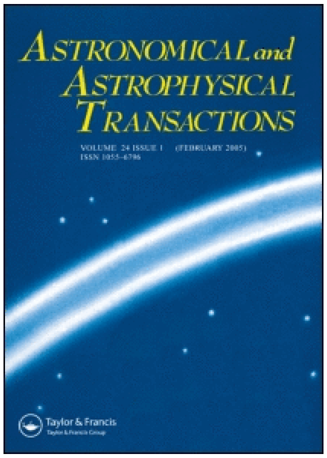

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METHODS OF DETERMINATION OF THE ORIENTATION OF A SPACE SYSTEM OF COORDINATES ON THE BASIS OF LUNAR OCCULTATIONS

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The algorithm for determining the orientation of the system of coordinates in the Hipparcos catalogue relative to the dynamic system for reduction of 150 000 lunar occultations is given.

Keywords: Space catalogue; Occultations; System of coordinates

The algorithm for calculating the orientation of a space system of coordinates on the basis of lunar occultation, is as follows.

Step 1. Transform the star coordinates from the system of initial catalogue into the system of the Hipparcos catalogue.

Step 2. Reduce the coordinates of the occulted stars into apparent positions on the moment of occultation with the known formulae (Zagrebin, 1996) to give $\alpha_{*\mathrm{v}}$, $\delta_{*\mathrm{v}}$.

Step 3. Interpolate from the ephemeris of the geocentric coordinates and radius of the Moon on the moment of occultation with the appropriate theory to give α, δ and R.

Step 4. Calculate the apparent topocentric coordinates and radius of the Moon by the known formulae (Zagrebin, 1966) to give α', δ' and R′.

Step 5. Calculate the apparent topocentric differences between α and δ of the occulted star and the coordinates of the centre of the Moon mass:

$$\begin{aligned} (\Delta\alpha\cos\delta)' &= (\alpha_{*\mathrm{v}} - \alpha'), \\ (\Delta\delta)' &= (\delta_{*\mathrm{v}} - \delta'). \end{aligned} \tag{1}$$

Step 6. Calculate the constituents of topocentric optical libration of the Moon considering the influence of physical libration of the Moon (PhLL) by the appropriate formulae to give l″, b″ and C″ (The Astronomical Almanac, 1984). In this case, corrections for physical libration are calculated with the Eckhardt (1981) tables.

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Step 7. Calculate the coordinates of the Moon's limb points, where occultation of a star occurs, in the Hayn (1917) system to give P and D.

Step 8. Choose Δh heights from the charts of lunar marginal zone (Watts, 1963; Nefedjev and Rizvanov, 2002) by the P and D values for appropriate points of the Moon's edge. Let Δh be the observed distance from the Earth to the Moon: $\Delta h' = \Delta h\ R/R$. Solution of the main problem, namely studying the accuracy of the orientation of the Hipparcos system of coordinates, is achieved in the following way.

Step 9. Calculate the values O−C from α and δ, considering the corrections from the charts of Moon marginal zone. The values $(\Delta\alpha\cos\delta)'$ and $(\Delta\delta)'$ in Eq. (1) are differences in appropriate star coordinates and the centre of the Moon's mass without considering the heights of the Moon's marginal zone. From α and δ the constituents of the heights of the points of the Moon's marginal zone above an average sphere of the lunar figure and topocentric lunar radius R'_{Moon} are

$$\begin{aligned}(\Delta\alpha\cos\delta)'_{\mathrm{h}} &= (\Delta h' + R'_{\mathrm{Moon}})\sin\theta'_{*},\\ (\Delta\delta)'_{\mathrm{h}} &= (\Delta h' + R'_{\mathrm{Moon}})\cos\theta'_{*}.\end{aligned} \tag{2}$$

according to charts of the Moon's marginal zone, where θ'_{*} is the positional angle of the star for projections of the Moon's centre of masses on the celestial sphere. Then the apparent values O−C at the moment of observation will be

$$\begin{aligned}(\Delta\alpha_{\mathrm{O-C}}\cos\delta)'_{\mathrm{v}} &= (\Delta\alpha\cos\delta)' - (\Delta\alpha\cos\delta)'_{\mathrm{h}},\\ (\Delta\delta_{\mathrm{O-C}})'_{\mathrm{v}} &= (\Delta\delta)' - (\Delta\delta)'_{\mathrm{h}}.\end{aligned} \tag{3}$$

We reduce these to the average distance from the Earth to the Moon:

$$\begin{aligned}(\Delta\alpha_{\mathrm{O-C}}\cos\delta)'_{\mathrm{m}} &= \frac{(\Delta\alpha_{\mathrm{O-C}}\cos\delta)'_{\mathrm{v}} \times 932.58''}{R'},\\ (\Delta\delta_{\mathrm{O-C}})'_{\mathrm{m}} &= \frac{(\Delta\delta_{\mathrm{O-C}})_{\mathrm{v}} \times 932.58''}{R'}.\end{aligned} \tag{4}$$

Step 10. The first way to determine the orientation of the system of coordinates is as follows. Let the orientation of the catalogue system of coordinates relative to the dynamic system be specified by the rotation angles ϵ_{x}, ϵ_{y} and ϵ_{z} around the x, y and z axes of the dynamic system of coordinates; then for the corrections $\Delta\alpha_{\mathrm{O-C}} = \alpha_{\mathrm{cat}} - \alpha_{\mathrm{dyn}}$ and $\Delta\delta_{\mathrm{O-C}} = \delta_{\mathrm{cat}} - \delta_{\mathrm{dyn}}$ we have the following equations which hold for small rotations (Nefedjev et al., 2002):

$$\begin{aligned}\Delta\alpha_{\mathrm{O-C}}\cos\delta &= \sin\delta\cos\alpha\epsilon_{\mathrm{x}} + \sin\delta\sin\alpha\epsilon_{\mathrm{y}} - \cos\delta\epsilon_{\mathrm{z}},\\ \Delta\delta_{\mathrm{O-C}} &= -\sin\alpha\epsilon_{\mathrm{x}} + \cos\alpha\epsilon_{\mathrm{y}}.\end{aligned} \tag{5}$$

They can be used to calculate the rotation angles ε_{x}, ε_{y} and ε_{z} of the catalogue system of coordinates relative to the dynamic system. Further, since

$$\begin{aligned}\epsilon_{\mathrm{x}} &= -\Delta\varepsilon,\\ \epsilon_{\mathrm{y}} &= \Delta L\sin\varepsilon,\\ \epsilon_{\mathrm{z}} &= \Delta A - \Delta L\cos\varepsilon,\end{aligned} \tag{6}$$

we obtain n conditional equations to determine the adjustments enumerated below, where n is the number of observations:

$$\begin{aligned} \Delta\alpha_{O-C} &= -\Delta A + \Delta L \cos\varepsilon(1 + \tan\varepsilon \tan\delta \sin\alpha) - \Delta\varepsilon \tan\delta \cos\alpha, \\ \Delta\delta_{O-C} &= -\Delta D + \Delta L \sin\varepsilon \cos\alpha + \Delta\varepsilon \sin\alpha, \end{aligned} \tag{7}$$

where ΔA and ΔD are corrections to the equinox and the equator of the catalogue, and ΔL and $\Delta\varepsilon$ are corrections to the Moon's longitude and inclination of the ecliptic to the equator. The rates of change in the angles ϵ_x, ϵ_y and ϵ_z are determined by the following equations:

$$\begin{aligned} \omega_x &= -\Delta\dot{\varepsilon}, \\ \omega_y &= \Delta\dot{L} \sin\varepsilon, \\ \omega_z &= \Delta\dot{A} - \Delta\dot{L} \cos\varepsilon, \end{aligned} \tag{8}$$

where the points indicate rates of change of appropriate values. The rotation angles ϵ_{xt}, ϵ_{yt} and ϵ_{zt} for time t are (Batrakov et al., 1999)

$$\begin{aligned} \epsilon_{xt} &= \epsilon_x + \omega_x(t - t_0), \\ \epsilon_{yt} &= \epsilon_y + \omega_y(t - t_0), \\ \epsilon_{zt} &= \epsilon_z + \omega_z(t - t_0). \end{aligned} \tag{9}$$

Thus, putting Eqs. (6), (8) and (9) into Eqs. (7), Eqs. (7) reduce to the forms

$$\begin{aligned} \Delta\alpha_{O-C} = {} & -[\Delta A + \Delta\dot{A}(t - t_0)] \\ & + [\Delta L + \Delta\dot{L}(t - t_0)] \cos\varepsilon(1 + \tan\varepsilon \tan\delta \sin\alpha) \\ & - [\Delta\varepsilon + \Delta\dot{\varepsilon}(t - t_0)] \tan\delta \cos\alpha, \\ \Delta\delta_{O-C} = {} & -\Delta D + [\Delta L + \Delta\dot{L}(t - t_0)] \sin\varepsilon \cos\alpha \\ & + [\Delta\varepsilon + \Delta\dot{\varepsilon}(t - t_0)] \sin\alpha. \end{aligned} \tag{10}$$

The terms $(\Delta\alpha_{O-C} \cos\delta)'_m$ and $(\Delta\delta_{O-C})'_m$ represent $\alpha_{cat}-\alpha_{dyn}$ and $\delta_{cat}-\delta_{dyn}$ respectively. Thus, substituting these values into the left-hand sides of Eqs. (10), we obtain a system with 2n conditional equations of the form (10). Let us solve these by the least-squares method and determine the required angles of orientation of the system of coordinates of the Hipparcos catalogue relative to the dynamic system of coordinates, ϵ_x, ϵ_y, ϵ_z, ω_x, ω_y, ω_z, taking into consideration Eqs. (6) and (8).

Step 11. The second way to determine the orientation of the system of coordinates is as follows. Let (Morrison, 1979)

$$\Delta\sigma = \sum_{k=1}^{N} \frac{\partial\sigma}{\partial Q_k} \Delta Q_k, \tag{11}$$

where σ is the angular distance from a star to the ephemeris centre of the Moon's mass at the moment of occultation, $\Delta\sigma = (O - C)_\sigma$, $\partial\sigma/\partial Q_k$ is the known factor for the corrections ΔQ_k to parameters under study and N is the number of parameters. At the present time, lunar and planet ephemerides have higher accuracy, and the positions of stars were determined with the accuracy of a millisecond in the Hipparcos catalogue. Thus, the number of corrections in question can be minimized. To a first approximation, it was decided to decrease N to 3 in conditional Eq. (11) and to find the corrections only to some parameters of the theory of the Moon's movement, to the origin of the right ascension of the Hipparcos catalogue, to the inclination of the equator to the ecliptic, to the system of coordinates of charts of the Moon marginal zone and, probably, to the equator of the Hipparcos catalogue. In these conditions, the conditional Eq. (11) will take the form (Soma, 1985)

$$\Delta\sigma = \frac{\partial\sigma}{\partial\lambda}\Delta\lambda + \frac{\partial\sigma}{\partial\alpha_0}\Delta\alpha_0 + \frac{\partial\sigma}{\partial\varepsilon}\Delta\varepsilon. \tag{12}$$

Here $\Delta\lambda = \Delta w_1^0$, $\Delta\alpha_0 = (\Delta A + \Delta\alpha_s \sin\alpha_{cat} + \Delta\alpha_c \cos\alpha_{cat}) - [\Delta R_0 + \Delta R_{1c}\cos\theta'_* + \Delta R_{2s}\sin(2\theta'_*) + \Delta R_{2c}\cos(2\theta'_*)]$, $\Delta\varepsilon = \Delta\varepsilon_0$, w_1^0 is a constant member of the Moon's mean longitude, $(\Delta A + \Delta\alpha_s \sin\alpha_{cat} + \Delta\alpha_c \cos\alpha_{cat})$ is a constant correction to the equinox of the catalogue and variables dependent on the $\sin\alpha_{cat}$ and $\cos\alpha_{cat}$ corrections to the right ascension of a catalogue, $[\Delta R_0 + \Delta R_{1c}\cos\theta'_* + R_{2s}\sin(2\theta'_*) + \Delta R_{2c}\cos(2\theta'_*)]$ is a constant and variable correction to the system of coordinates of the charts of the Moon's marginal zone, ε_0 is a constant member of inclination of the equator to the ecliptic, $\Delta\alpha_s$ is a systematic correction in the right ascension of Hipparcos varying as $\sin\alpha$, $\Delta\alpha_c$ is a systematic correction in right ascension of Hipparcos varying as $\cos\alpha$, ΔR_{1c} is the latitude component of the shift of centre of the map's lunar marginal zone datum, ΔR_{2s} is the longitude component of the correction to the ellipticity of the map's lunar marginal zone datum and ΔR_{2c} is the latitude component of the correction to the ellipticity of the map's lunar marginal zone datum. The solution of conditional Eq. (12) will be found by the least-squares method through the iteration; that is, firstly the worst corrections will be determined, and then after this, other required values will be found. In other words, the solution will be found through successive approximations.

Thus, the algorithm of reduction in the lunar occultation with the aim of estimating the accuracy of orientation of the space system of coordinates was constructed by several methods.

The computer-readable version of the relief charts of the Moon's marginal zone built up by the 'absolute' method through binding stars to the fundamental system of coordinates and referred to the centre of the Moon's mass and principal axes of its inertia was constructed for the first time in world astronomy. They have been described in detail by Nefedjev and Rizvanov (2002). The charts of the Moon's marginal zone in computer-readable form constructed at the Engelhardt Astronomical Observatory (Nefedjev and Rizvanov, 2002) are comparable with Watts's (1963) charts but, in contrast, they do not require special corrections, which are connected with carrying out additional calculations.

Taking into account the irregularity of the Moon's marginal zone, points outside the picture plane will be used. Hence, it is as if a three-dimensional model of the charts of the Moon's marginal zone will be involved to reduce the observations.

Reduction in the lunar occultations will be executed from two charts (that by Watts and that by Nefedjev and Rizvanov). That will allow us to obtain more certain information as well as to estimate the advantages and defects of these charts.

The use of 150 000 lunar occultations will allow us to obtain more authentic information about the orientation of the system of coordinates of the Hipparcos catalogue.

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