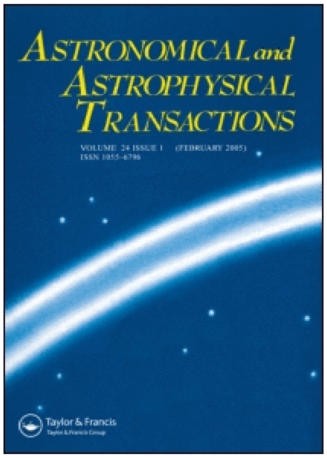

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ON THE NEW THEORY OF GEOSTATIONARY SATELLITE MOTION

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Lagrange's equations for geostationary satellite motion in spherical coordinates connected with the Laplacian plane are considered. The first integral of system of equations has been found. The intermediate orbit of satellite has been constructed.

Keywords: Geostationary satellite; Orbit; First integral

1 INTRODUCTION

The need to create a quick algorithm for investigation of the orbital evolution motion of geostationary satellites (GSs) is dictated by constantly increasing population of the geostationary orbit (GEO) and its environment. The motion theory of a GS is rather complicated. The existence of longitude-dependent terms in the Earth's potential function induces long-term large-scale oscillations in GS longitudes λ. Because of the commensurability of the GS's mean motion n and the Earth's rotation $\dot{S}$, small divisors ($\dot{\lambda} \approx n - \dot{S}$) occur. Therefore, to define the longitude at any time moment, it is necessary to solve the differential resonant equation of second order, the coefficients of which depend on time.

The lunar–solar attractions cause essential changes in the space orientation of the GS's orbital plane. The solar radiation pressure causes variations in the orbital eccentricity e. The task is simplified if an appropriate coordinate system and suitable intermediate orbit are selected. In our theory the Laplacian plane is adopted as fundamental. This plane is inclined to the Earth's equator by nearly 7° and passes through the node line of the equator and ecliptic. The inclination i of the GS's orbit, referred to this plane, will be changed by an amplitude less than $0\overset{\circ}{.}4$; the longitude Ω of the ascending node and the argument ω of the perigee become nearly linear functions of time. Fortunately none of these elements depends on λ, because the amplitudes of short-term perturbations are negligibly small. Therefore for the elements i, Ω, ω and e the intermediate orbit can be constructed taking into account the maximum residuals that arise from all perturbations.

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2 BASIC EQUATIONS AND THEIR SOLUTION

The perturbing function for the intermediate orbit with maximum terms can be represented as follows:

$$R = R_0(c_0, i, e) + R_1(c_1, i, e)\cos(\omega + \Omega + \omega_L + \Omega_L) + R_2(c_2, i, e)\cos(\omega + \Omega - \lambda_S), \quad (1)$$

where c_0, c_1 and c_2 are the constant parameters of each term, the orbital elements of the Moon are indicated by the subscript L, and λ_S is the longitude of the Sun. The complete expression for R has been given by Kiladze et al. (1999). Let us introduce the following definitions:

$$z = \Omega - \Omega_L, \quad R = \Omega + \omega - \Omega_L - \omega_L, \quad x = \Omega + \omega - \lambda_S, \quad \dot{\Omega} = -\kappa \cos i. \quad (2)$$

Lagrange's equations for the new variables z, i, R and e can be written as follows:

$$\frac{dz}{dt} = \dot{z} + b\frac{\cos(2i)}{\sin i}\cos z, \quad (3)$$

$$\frac{di}{dt} = b\cos i \sin z - A\dot{R}e\tan\left(\frac{i}{2}\right)\sin R, \quad (4)$$

$$\frac{dR}{dt} = \dot{R} + b\left[3\sin i\cos i + \cos(2i)\tan\left(\frac{i}{2}\right)\right] + \frac{A\dot{R}\cos R - D\dot{x}\cos x}{e}, \quad (5)$$

$$\frac{de}{dt} = A\dot{R}\sin R - D\dot{x}\sin x, \quad (6)$$

where D is the light pressure; κ, A, b, $\dot{x}$, $\dot{\Omega}_L$ and $\dot{R}$ are constants, which for resonant GSs have the following numerical values: $D \approx 0.0001$ rad, $\kappa \approx 3.26 \times 10^{-4}$ rad, $A \approx 2.7 \times 10^{-4}$ rad, $b \approx 0.62 \times 10^{-5}$ rad, $\dot{x} \approx -0.0169$ rad, $\dot{\Omega}_L \approx -9.24 \times 10^{-4}$ rad, $\dot{R} \approx -0.7 \times 10^{-5}$ rad.

As a rule GSs move in circular orbits; then the second term in Eq. (4) will be equal to zero. In this case Eqs. (3)–(6) are divided into two systems, each of which contains two equations only.

Multiplying Eqs. (3) and (4) by $b\sin i\cos i\sin z$ and $-\dot{z}\sin i - b\cos(2i)\cos z$ respectively and summing, as a result we have an expression in the form of a full differential that gives us the first integral of the system:

$$\frac{\kappa}{2}\cos^2 i + (\dot{\Omega}_L + b\sin i\cos z)\cos i = C. \quad (7)$$

If we save the second term in Eq. (4), then Eq. (7) will be written

$$\frac{\kappa}{2}\cos^2 i + (\dot{\Omega}_L + b\sin i\cos z)\cos i = C + A\int e\dot{R}\tan\left(\frac{i}{2}\right)\sin R(\dot{z}\sin i + b\cos(2i)\cos z)\,dt. \quad (8)$$

The additional term in Eq. (8) is the periodic function of the time with an amplitude not more than $Ce \times 10^{-8}$. Therefore Eq. (7) is quite correct and has a satisfactory accuracy.

Using Eq. (7), the solutions of Eqs. (3) and (4) have been expressed by the elliptic integral:

$$t - t_0 = \int_{V_0}^{V} \frac{\sin i\,di}{[((\kappa^2/4) + b^2)\cos^4 i - \kappa\dot{\Omega}_L\cos^3 i + (\kappa C - \dot{\Omega}_L^2 + b^2)\cos^2 i + 2C\dot{\Omega}_L\cos i - C^2]^{1/2}}. \quad (9)$$

After taking the integral (9), the solution for Ω also can be obtained (Kiladze et al., 1999).

To solve the system of Eqs. (5) and (6) the new variables p and q are introduced:

$$p = e \cos R + A, \quad q = e \sin R, \tag{10}$$

and also

$$d\tau = (\dot{R} + \delta)\, dt, \tag{11}$$

where

$$\delta = b \sin i \cos z \left(3 \cos i + \frac{2 \cos^2 i - 1}{\cos i + 1} \right). \tag{12}$$

Then Eqs. (7) and (8) may be rewritten as follows:

$$\begin{aligned} \frac{dp}{d\tau} &= -q + D\dot{x} \sin (R - x), \\ \frac{dq}{d\tau} &= p - D\dot{x} \cos (R - x) - A\delta. \end{aligned} \tag{13}$$

The solution of the linear system of Eqs. (13) may be obtained without any mathematical difficulties because on right-hand sides of these equations the variables determined from Eqs. (5) and (6) are entered as free terms.

Then, by integrating Eq. (11) the time can be defined as a function of the variable τ:

$$t = \int \frac{d\tau}{\dot{R} + \delta}. \tag{14}$$

Finally, the solution of these elements can be constructed by adding all necessary perturbations and the evolutions of i, Ω, e and ω can be calculated for a period starting several tens of years ago.

The software constructed on the basis of the described solution has permitted us to study the long-term orbital evolution of all uncontrolled known GSs and to compare it with NASA's orbital two line elements. Integral (7) has been used for identification of GS observations.

3 RESONANT EQUATION FOR LONGITUDE AND ITS SOLUTION

To investigate the motion of a GS the resonant equation of longitude ($\lambda = M + \omega + \Omega - S$, where M is the mean anomaly and S the sidereal Greenwich Time) was used by Gedeon (1969). Gedeon's equations have been derived in adopted coordinate systems and with adding lunar–solar pertubations:

$$\frac{d^2\lambda}{dt^2} = -n^2 - \sum_{lmkpq} A_{lmkpq} \sin [m\lambda - m\lambda_{lmkpq} + (k - m)\Omega] + LSP, \tag{15}$$

where n is mean diurnal motion of the GS, the coefficients A_{lmkpq} depend on Hansen's coefficients, the inclination functions (Gaposchkin, 1973), the geopotential parameters and the semimajor axis expressed as the mean radius of the equator of the Earth, and l, m, k, p and q are indices of summation of the geopotential harmonics. LSP designates the influence of lunar–solar perturbations, the maximum term of which is equal to $\dot{\Omega}^2 f(\Omega)$, where $f(\Omega)$ is a periodic function. It should be noted that the presence of Ω in Eq. (15) is a consequence of the Laplacian plane introduction. Taking into account its weak change is simpler than the change in inclinations referred to equators.

If Ω were constant, then the equation of motion (15) would have a first integral analogous to Jacobi's integral:

$$\left(\frac{d\lambda}{dt}\right)^2 = C_1 - \Pi(\lambda), \tag{16}$$

where C_1 is a constant of integration and

$$\Pi(\lambda) = 2\sum_{lmkpq} \frac{A_m(m-k)}{m} \cos\left[m\lambda - \varphi_m - (m-k)\Omega\right] \tag{17}$$

with

$$A_m = -n^2 A_{lmkpq}, \quad \varphi_m = m\lambda_{lmkpq}.$$

However, Ω is a function of time. Therefore, it is necessary to take into account the variation in C_1

$$\frac{dC_1}{dt} = 2\dot{\Omega}\sum \frac{A_m(m-k)}{m} \sin\left[m\lambda - \varphi_m - (m-k)\Omega\right] + 2\dot{\lambda}\dot{\Omega}^2 f(\Omega). \tag{18}$$

The derivative dC_1dt is also used by us as a small parameter for constructing the motion theory for a GS. A detailed description of this process has been given by Kiladze and Sochilina (1996). The constructed intermediate orbit has become the basis of appropriate software. This software was successfully used to investigate the motion of about 700 uncontrolled GSs.

4 CONCLUSIONS

The scientific importance of the integral of motion (7) must be especially mentioned. For a time after Newton had resolved the two-body problem in celestial mechanics, only one integral of motion appeared (Jacobi, in the nineteenth century). During the development of science, every new integral of motion has its own (sometimes unusual) application. The integral (7) has an additional application as the criterion for the identification of GSs that had been lost and after a long time were rediscovered.

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