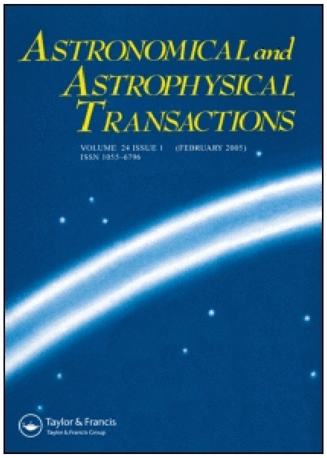

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NEW HARMONIC COORDINATES FOR THE SCHWARZSHILD GEOMETRY AND THE FIELD APPROACH

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New harmonic coordinates for the Schwarzshild geometry are obtained. Unlike the case of Fock's harmonic coordinates, test particles falling into a black hole approach the event horizon and cross it at a finite value of the time parameter. The metric in the new coordinates is investigated. The Schwarzshild solution in both Fock's and the new coordinates is presented. In terms of the field description of GR (that is, such a formulation of GR where all the dynamic fields, including the gravitational one, are considered in a given background space-time). These field configurations are compared, and the particle trajectories in the auxiliary background world are discussed. Due to the fact that the Fock solution satisfies the harmonic conditions, an erroneous opinion that a falling particle cannot approach the event horizon in the field description, could have arizen. With the help of the new solution satisfying the same conditions, it is clearly shown that a particle passes through the horizon without obstacles.

KEY WORDS Black holes, co-ordinate systems, Schwarzshild geometry

1. INTRODUCTION

In order to simplify the general relativity (GR) equations, a certain choice of coordinates is frequently made. Both in the past and nowadays, harmonic coordinates are very popular. Great attention was spared to harmonic coordinates by Fock (Fock, 1959). Different theoretical questions are now being investigated with their help (see, for example Nakanishi (1986); Ruiz (1986)). Using the De Donder (harmonic) coordinate conditions, an approximation method was developed allowing us to investigate the detailed structure of the gravitational field outside an isolated system (Blanshet and Damaour, 1986). Harmonic coordinates are effectively used in construction of a relativistic theory of reference frames for the Solar system bodies (Kopejkin, 1988; Brumberg and Kopejkin, 1989).

In the present paper we propose new harmonic coordinates for the Schwarzshild geometry. What is the goal of their search? What is their advantage compared with the well-known Fock harmonic coordinates (Fock, 1959)?

Let us discuss the latter. The Fock harmonic coordinates, like the Schwarzchild ones (Landau and Lifschitz, 1975), are singular at the event horizon. This is indicated by the following. In the coordinate time a falling particle approaches

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the Schwarzshild sphere is infinitely long. Discontinuity of geodesic lines in the coordinate diagram reflects this fact. Many coordinates without such a singularity were discovered. Among them there are the well-known Lemaitre, Edington-Finkelstein, Kruskal-Szekeres, Novikov coordinates (Landau and Lifshitz, 1975; Misner *et al.*, 1973; Novikov and Frolov, 1986). However, so far we did not know a harmonic coordinate system which were as good at the event horizon as the enumerated ones.

Here such coordinates are found.† In Sec. 2 the metric in the new coordinates is suggested and its properties are investigated. Constructing the new coordinate system we wanted, above all, to obtain good description for the motion of particles falling into a black hole. Unlike the Fock coordinates, in the new coordinates (one can call them contracting coordinates) a falling particle arrives at the event horizon at a finite value of the time parameter. On the coordinate diagram a geodesic line can be followed continuously through the Schwarzchild sphere.

As it is shown in Zel'dovich and Grishchuk (1986) and (1988), GR admits the so-called field description. This is a description where dynamical fields are considered in a given background (auxiliary) space-time. This problem was treated by many authors, see, for example Rosen (1940); Kraichnan (1955); Gupta (1957); Burlankov (1963); Ogievetsky and Polubarinov (1965); Deser (1970). Now it has been elaborated with an exhaustive completeness (Grishchuk *et al.*, 1984; Grishchuk and Petrov, 1987; Popova and Petrov, 1988). Remaining equivalent to ordinary geometric formulation of GR, the field description is used for investigation of different theoretical questions (Grishchuk and Petrov, 1986; Grishchuk and Popova, 1986). One of the significant results of these investigations is the following. If two solutions in the geometric description of GR are connected by a coordinate transformation, then in the field description they are connected by a gauge transformation (by definition, such transformations do not touch the coordinates and the background fields).

On the basis of the Fock coordinates (Fock, 1959) (in which test particles infinitely long approach the event horizon) an opinion that black holes are impossible, could have emerged. Due to the fact that the authors of RTG (see, for example in Logunov and Mestvirishvili (1986)) give an especial sense to a flat background space-time and the harmonic conditions, they obtain such a conclusion. Here, in Sec. 3, using the field formulation of GR, we compare the suggested solution with Fock's one. The transition from one to the other (when the motion of test particles is also taken into account) is interpreted in terms of gauge transformations. In our solution, satisfying the same harmonic conditions, particles penetrate through the event horizon without obstacles. Thus, the Einstein equations with the supplementary harmonic conditions, and the structure of a flat background space-time, do not exclude black holes.‡

In the Appendix it is shown how the new coordinates were found. Their relation to the Fock coordinates is presented. Besides that, brief and clear

† See also brief communications in Petrov (1990), Vlasov (1990) and besides that in Chugreev (1989).

‡ Critique of suggestions of RTG is also contained in Zel'dovich and Grishchuk (1986) and (1988); Burlankov (1989); Grishchuk (1990).

instructions for obtaining expanding harmonic coordinates are given. These coordinates will be convenient for the description of test particles ejected from under the event horizon.

2. THE SCHWARZSHILD GEOMETRY AND THE HARMONIC CONDITIONS

Let us write the Schwarzshild metric in the Fock coordinates (Fock, 1959):

$$ds^2 = \frac{r-\alpha}{r+\alpha} c^2\, dt^2 - \frac{r+\alpha}{r-\alpha} dr^2 - (r+\alpha)^2 (d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{2.1}$$

where $\alpha \equiv GM/c^2$. After the coordinate transformation

$$x = r\sin\theta\cos\varphi, \qquad y = r\sin\theta\sin\varphi, \qquad z = r\cos\theta \tag{2.2}$$

(where the coordinate t is unaffected) the metric (2.1) satisfies the harmonic conditions:

$$(\sqrt{-g}\, g^{\mu\nu})_{,\nu} = 0, \tag{2.3}$$

where $g = \det g_{\mu\nu}$.

To simplify the presentation we consider a test particle, falling radially into a black hole. Besides that, to eliminate cumbersome expressions we confine ourselves to the "parabolic orbit" case when a particle begins its motion from at rest at the infinity $r = \infty$. Then the equation of motion of a test particle has the form:

$$\begin{aligned} ct = -2\alpha\Bigg[&\frac{2}{3}\left(\frac{r+\alpha}{2\alpha}\right)^{3/2} + 2\left(\frac{r+\alpha}{2\alpha}\right)^{1/2} + \ln\left|\frac{r}{\alpha} - 1\right| \\ &- 2\ln\left|\left(\frac{r+\alpha}{2\alpha}\right)^{1/2} + 1\right|\Bigg] + \text{const.} \end{aligned} \tag{2.4}$$

(On "parabolic orbit" in the Schwarzshild coordinates see in Misner *et al.* (1973).) The existence of the term $-2\alpha \ln|r/\alpha - 1|$ leads just to the situation that a particle falls to the event horizon $r = \alpha$ infinitely long in the coordinate time.†

Now let us write the new solution:

$$\begin{aligned} ds^2 = {} & \frac{r-\alpha}{r+\alpha} c^2\tau^2 - \frac{8\alpha^2}{(r+\alpha)^2} d\tau\, dr - \left(1 + \frac{2\alpha}{r+\alpha} + \left(\frac{2\alpha}{r+\alpha}\right)^2 + \left(\frac{2\alpha}{r+\alpha}\right)^3\right) dr^2 \\ & - (r+\alpha)^2 (d\theta^2 + \sin^2\theta\, d\varphi^2). \end{aligned} \tag{2.5}$$

It describes the same physical space-time corresponding to the metric (2.1). (On the relation to the solutions (2.1) and (2.5) see the Appendix.) We point out some properties of (2.5).

First of all we note that after the transformation (2.2) the metric (2.5) also satisfies the conditions (2.3). The metric determinant has the same value as that in (2.1), namely $\det g_{\mu\nu} = -(r+\alpha)^4 \sin^2\theta$. Unlike (2.1), the metric coefficients

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† All test particles and photons radially falling into a black hole have the same asymptotical behavior with respect to t in the neighbourhood of the event horizon.

(2.5) are infinite only at the true singularity $r = -\alpha$, but remain finite at the event horizon $r = \alpha$.

The equation of the "parabolic orbit" has the form:

$$c\tau = -2\alpha\left[\frac{2}{3}\left(\frac{r+\alpha}{2\alpha}\right)^{3/2} + 2\left(\frac{r+\alpha}{2\alpha}\right)^{1/2} + \ln\left|\frac{r}{\alpha}+1\right| - 2\ln\left|\left(\frac{r+\alpha}{2\alpha}\right)^{1/2}+1\right|\right] + \text{const}, \tag{2.6}$$

where, unlike (2.4), the logarithmic term is absent. Hence, in the coordinate diagram (τ, r) a falling particle trajectory may be followed continiously through the Schwarzshild sphere.

Both the form of the metric (2.5) and the structure of the light cones:

$$c\frac{d\tau}{dr}\bigg|_1 = \frac{(r+\alpha)^2 + (2\alpha)^2}{r^2 - \alpha^2}, \qquad c\frac{d\tau}{dr}\bigg|_2 = -\frac{r+3\alpha}{r+\alpha}$$

clearly show the following. In the domain $r < \alpha$ both r and τ are spacelike (as in Finkelstein's solution (Finkelstein, 1958)). It is permissible, because the metric signature in the domain $r < \alpha$ remains correct, as we have seen above. However, when $r < \alpha$ the description of particle motion is somewhat unusual: evolution of the spacelike coordinate r is considered in terms of another spacelike coordinate τ. Still, one should mention that a particle approaches the true singularity $r = -\alpha$ at an infinite value of the parameter τ (see (2.6)).

It follows from above that the sections $\tau = \text{const}$ are spacelike both outside and inside the event horizon. (This is also true for the solution (Finkelstein, 1958).) If some events belong to the surface $\tau = \text{const}$, then in this sense one can speak about their simultaneity outside the event horizon, on it and inside it. It may be useful for investigations using the $(3+1)$-decomposition procedure where a Space-time is interpreted as a foliation of spacelike surfaces, which are defined often by $t = \text{const}$ (see in Misner *et al.* (1973)).

For the Schwarzshild geometry, relations between the Schwarzshild coordinates and many others was investigated very carefully (Landau and Lifshitz, 1975; Misner *et al.*, 1973; Novikov and Frolov, 1986). After the simple substitution $r \to r - \alpha$ the coordinates of (2.1) transform into the Schwarzshild ones. Therefore, using the relation between (2.1) and (2.5) (see the Appendix), it is easy to define the relation of the new coordinates to all the well-known ones. Note only that the coordinates in (2.5) cover the half of the whole Schwarzshild geometry. It is similar to, for example the case with the contracting Lemaitre or Edington-Finkelstein coordinates (Landau and Lifshitz, 1975; Misner *et al.*, 1973).

Both solutions (2.1) and (2.5) with $r > \alpha$ describe the same physical space. As it follows from the fact that $\omega^\alpha = 0$, this space is static. (Here, ω^α is differential invariant related to the timelike Killing vector field (Novikov and Frolov, 1986).) However, one has to note that due to the mixed τr-term the metric (2.5), unlike (2.1), can be called only stationary (not static) for $r > \alpha$.

Thus we have two harmonic coordinate systems for the Schwarzshild geometry. A question arises: is there a contradiction with Fock's theorem about uniqueness of a coordinate system for an isolated physical system (Fock, 1959)? By studying the theorem conditions, which were discussed in detail in Belinfante and Garrison

(1962), we discover the following: None of the metric (2.1) and (2.5) satisfies all the conditions in the whole physical space-time. That is, the question disappears by itself and there is no contradiction.

At the end of the section let us make the following remark. Both for (2.1) and for (2.5) with $-\alpha \leq r \leq \alpha$ arbitrary two points 1 and 2 such that $r_2 = -r_1$, $\theta_2 = \pi - \theta_1$, $\varphi_2 = \pi + \varphi_1$ correspond to the same harmonic coordinate point x, y, z (2.2) that is, not all the points of the physical space-time are in the one to one correspondence with the harmonic coordinates. The left side of (2.3) is proportional to the delta-function $\delta(r)$ for both the metrics (2.1) and (2.5). As the result, the conditions (2.3) are not true at $r = 0$. (This property was noted in Loskutov (1990).) It is possible that there exist harmonic coordinates without these defects, but we have no need to search them.† Indeed, for both solutions, excluding the domain $-\alpha \leq r \leq 0$ from consideration, one can use the harmonic coordinates for $r > 0$. This is perfectly sufficient for an investigation of test particle trajectories in the neighbourhood of the event horizon.

3. A PARTICLE IN THE GRAVITATIONAL FIELD AND GAUGE TRANSFORMATIONS

Linearizated GR is usually used in problems with weak gravitational fields. For construction of such a theory the metric tensor $g_{\mu\nu}$ involved in GR is decomposed into a sum of the Minkowsky tensor and the gravitational field tensor $h^{\mu\nu}$. The coordinate system is chosen in such a way that $|h^{\mu\nu}| \ll 1$. The equations are then expanded in powers of $h^{\mu\nu}$ and, with fairly good precesion, only linear terms are preserved. The linearized theory admits gauge transformations which act only upon dynamic variables and do not affect the coordinates (Landau and Lifshitz, 1975; Misner *et al.*, 1973)

$$h'_{\mu\nu} = h_{\mu\nu} + \xi_{\mu,\nu} + \xi_{\nu,\mu}. \tag{3.1}$$

Here ξ^{μ} means four arbitrary functions, which are so small that $|h'^{\mu\nu}| \ll 1$. Still, in the literature in connection with (3.1), as a rule, there does not emerge a question of gauge transformations for nongravitational dynamic fields or for particle motion. Nevertheless, exact analysis shows that one has to take into account such transformations (for weak fields and the transformation (3.1) see in Mashhoon and Grishchuk (1980)). The exact (not approximate) field formulation of GR (Grishchuk *et al.*, 1984) also admits gauge transformations. Unlike (3.1), they are finite and also affect dynamic nongravitational fields.

Here in terms of the field approach both (2.1) and (2.5) are presented, and test particle motion is considered. After that, transition from one of these field configurations to the other is discussed in terms of finite gauge transformations. Before that to make this discussion clearer, we will show the effect of these transformations for an arbitrary field configuration and a test particle trajectory in an auxiliary background world.

Appealing to Grishchuk *et al.* (1984), let us define in a background world with the metric $\gamma_{\mu\nu}$ the action for a test particle interacting with the gravitational field

† Let us say only that one has to search these coordinates in a wider class of functions. As it follows from the Appendix, for the metric one has to admit dependence on τ.

$h^{\mu\nu}$:

$$S_{m+g} = -cm \int ds. \tag{3.2}$$

Here m is the particle mass and s is a parameter along its trajectory. The field $h^{\mu\nu}$, the metric $\gamma_{\mu\nu}$ and the background world coordinates x^α are involved in (3.2) in the form $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$. The functions $g_{\alpha\beta}$ depend on $h^{\mu\nu}$ and $\gamma_{\mu\nu}$, more exactly, they are defined by the relations:

$$\begin{aligned} &\sqrt{-g}\, g^{\mu\nu} = \sqrt{-\gamma}\,(\gamma^{\mu\nu} + h^{\mu\nu}) \equiv \tilde{\gamma}^{\mu\nu} + \tilde{h}^{\mu\nu}, \qquad g_{\alpha\beta} g^{\beta\gamma} \equiv \delta^\gamma_\alpha, \\ &g \equiv \det g_{\mu\nu}, \qquad \gamma \equiv \det \gamma_{\mu\nu}, \qquad \tilde{h}^{\mu\nu} \equiv \sqrt{-\gamma}\, h^{\mu\nu}. \end{aligned} \tag{3.3}$$

The field formulation of GR is connected with the geometric one (with the metric $g_{\mu\nu}$) just by the relations (3.3). The action (3.2) in terms of the geometric formulation is given in Landau and Lifshitz (1975). Variation of S_{m+g} with respect to the coordinates gives the equations of motion for a test particle. Their solutions are the vector components of the particle "4-velocity" $u^\alpha \equiv dx^\alpha/ds$.

Let us present the action (3.2) in the more suitable form $\int L\, d^4x$. One easily arrives at:

$$\begin{aligned} &S_{m+g} = -c \int \rho g_{\mu\nu} u^\mu u^\nu \sqrt{-g}\, d^4x, \\ &\rho \equiv \frac{m\delta(\vec{r} - \vec{r}_0)}{\sqrt{-g^{(3)} g_{00}}} \frac{ds}{cdt}, \qquad g^{(3)} \equiv \det g_{ab}, \end{aligned} \tag{3.4}$$

where $\delta(\vec{r} - \vec{r}_0)$ is the Dirac delta-function, and g_{ab} is spatial part of the tensor $g_{\alpha\beta}$ defined in (3.3). Then as it follows from Grishchuk *et al.* (1984), the theory based on the action (3.4) and, consequently, (3.2) is invariant under gauge transformations for the gravitational field $h^{\mu\nu}$ and for the material fields $\varphi^A \in \rho, u^\alpha$:

$$\tilde{h}'^{\mu\nu} = \tilde{h}^{\mu\nu} + \Delta_\xi \tilde{h}^{\mu\nu}, \qquad \varphi'^A = \varphi^A + \Delta_\xi \varphi^A. \tag{3.5}$$

Here the gauge additions generalizating (3.1) present infinite expansions with respect to an arbitrary finite vector field ξ^μ and its derivatives (see below).

If the system is invariant under some transformations, then these transformations do not change the physical situation. Here, in spite of transform u'^α, gauge transformations do not change real particle motion, only the manner of its description is changed. The fact that the background world has an auxiliary character, is directly related to the assertion made.

We should like to present the relations permitting to avoid infinite expansions in (3.5). This is possible if the coordinate transformations inducing the gauge transformations are known (Grishchuk and Petrov, 1987; Popova and Petrov, 1988). Let two solutions in the geometric formulation of *GR*, $g'_{\mu\nu}$ and $g_{\mu\nu}$, be connected by the coordinate transformation:

$$x'^\alpha = x'^\alpha(x), \tag{3.6}$$

whose reverse one is $x^\alpha = x^\alpha(x')$. Then after a transition to the field description (see (3.3)) we have:

$$\tilde{h}'^{\mu\nu}(x') + \tilde{\gamma}'^{\mu\nu}(x') = [\hat{A}\tilde{h}^{\mu\nu}(x) + \hat{A}\tilde{\gamma}^{\mu\nu}(x)]|_{x=x(x')},$$

where $\hat{A}$ is an abstract notation for the operator of the coordinate transformation (3.6). (The specific expression of $\hat{A}$ depends on the transformation properties of the variables considered. For instance, for the vector ζ^α: $\hat{A}\zeta^\alpha(x) = (\partial x'^\alpha/\partial x^\beta)\zeta^\beta(x)$.) Furthermore, requiring, that the form of the functions $\tilde{\gamma}'^{\mu\nu}$ in the coordinates x'^α is the same as that of $\tilde{\gamma}^{\mu\nu}$ in the coordinates x^α and making the substitution $x'^\alpha \equiv x^\alpha$, one obtains:

$$\tilde{h}'^{\mu\nu}(x) = \tilde{h}^{\mu\nu}(x) + [\hat{A}\tilde{h}^{\mu\nu}|_{x=x(x)} - \tilde{h}^{\mu\nu}(x) + \hat{A}\tilde{\gamma}^{\mu\nu}|_{x=x(x)} - \tilde{\gamma}^{\mu\nu}(x)]. \tag{3.7}$$

Let us define the vector field ξ^μ presenting the transformation (3.6) in the form:

$$x'^\alpha = x^\alpha + \xi^\alpha + \frac{1}{2!}\xi^\beta\xi^\alpha_{,\beta} + \frac{1}{3!}\xi^\rho(\xi^\beta\xi^\alpha_{,\beta})_{,\rho} + \cdots.$$

Then the transformation (3.7) acquires the form (3.5) for $\tilde{h}^{\mu\nu}$. In a similar way one obtains that particle trajectories are transformed according to the relation:

$$u'^\alpha(x) = u^\alpha(x) + [\hat{A}u^\alpha|_{x=x(x)} - u^\alpha(x)] = \hat{A}u^\alpha|_{x=x(x)}. \tag{3.8}$$

(This formula is correct if background material fields are absent. The flat background to be used below belongs just to this case, for more detail see in Grishchuk *et al.* (1984).) Thus instead of (3.5) we will use the formulae (3.7) and (3.8) as those for gauge transformations.

It seems attractive to describe the two solutions (2.1) and (2.5) in the field approach and to compare them. Indeed, both these cases have many similarities: (i) after the transformation (2.2) $g_{\mu\nu}$ satisfy the conditions (2.3); (ii) $g_{\mu\nu}$ do not depend on t or τ; (iii) the metrics $g_{\mu\nu}$ approach the flat metric in the spherical coordinates for $r \to \infty$; (iv) the hypersurface $r = \alpha$ is null and defines the event horizon while the hypersurface $r = -\alpha$ defines a true singularity. The difference has been already discussed, it consists in the description of a falling particle in the neighbourhood of the event horizon.

For each of these solutions it is reasonable to describe the flat background in the spherical coordinates. So the background metric can be written in the form:

$$ds^2 = c^2dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{3.9}$$

(For the solution (2.5) the substitution $\tau \equiv t$ is made.) The field indices will be numerated according to (3.9): $x^0 = ct$, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$.

Now using (3.3) and (3.9) let us define the field $h^{\mu\nu}$ for (2.1):

$$h^{00} = -1 + \frac{(1+\alpha/r)^3}{1-\alpha/r}, \qquad h^{11} = \frac{\alpha^2}{r^2}, \tag{3.10}$$

the rest $h^{\mu\nu} = 0$. In the same manner for (2.5) one obtains:

$$\begin{gathered} h'^{00} = -1 + \left(1+\frac{\alpha}{r}\right)^2\left[1 + \frac{2\alpha}{r+\alpha} + \left(\frac{2\alpha}{r+\alpha}\right)^2 + \left(\frac{2\alpha}{r+\alpha}\right)^3\right], \\ h'^{01} = -\frac{4\alpha^2}{r^2}, \qquad h'^{11} = \frac{\alpha^2}{r^2}, \end{gathered} \tag{3.11}$$

the rest $h'^{\mu\nu} = 0$. Let us note the similarities of the field configurations (3.10) and (3.11). Everywhere in the background world excluding the point $r = 0$, both fields satisfy the conditions:

$$h^{\mu\nu}{}_{;\nu} = 0, \tag{3.12}$$

where the semicolon means covariant derivatives with respect to the flat metric (3.9). This fact is a direct consequence of the fact that after the transformation (2.2) both metrics (2.1) and (2.5) satisfy the conditions (2.3). Both fields do not depend on the time coordinate t, that is, for the auxiliary background they are always and everywhere static. The existence of the following two conclusions is indebted for the fact that for each of the solutions (2.1) and (2.5) the asymptotic behaviour of $g_{\mu\nu}$ at $r \to \infty$ is the same, and the flat background is also the same. In the Minkowsky coordinates of the flat background (see for comparison in Grishchuk *et al.* (1984); Grishchuk and Petrov (1987)) both fields $h^{\mu\nu}$ and $h'^{\mu\nu}$ decrease at $r \to \infty$ no slower than $1/r$. The total energy for each of the field configurations is the same, namely: Mc^2 (as it should be for the Schwarzshild black hole, see in Grishchuk *et al.* (1984)).

For both solutions the choice of the background metric in the form (3.9) excludes the domain $-\alpha \leqslant r < 0$ from consideration in the field description. We do not know a solution where the complete space-time presented by the solution (2.1) (not its part) could be considered in unique flat background world without breaking the conditions (3.12). A search of such solutions is equivalent to a search of harmonic coordinates discussed in the end of Sec. 2. We are not interested in this question because for our purpose (to compare particle trajectories in the neighbourhood of $r = \alpha$ for solutions (2.1) and (2.5)) the peculiarity mentioned is not an obstacle.

Finally, let us discuss the trajectories of the test particles. For a free particle falling radially in the field configuration (3.10) a solution of the equations of motion has the form:

$$u^0 = \frac{r+\alpha}{r-\alpha}, \qquad u^1 = -\left(\frac{2\alpha}{r+\alpha}\right)^{1/2}, \qquad u^2 = u^3 = 0. \tag{3.13}$$

After integration of $cdt = (u^0/u^1)dr$ one obtains the equation (2.4), where now t and r are the auxiliary flat background world coordinates. The particle approaches $r = \alpha$ for an infinitely long time t. On the other hand, for the field configuration (3.11) we have:

$$\begin{aligned} u'^0 &= \frac{1}{1+\left(\frac{2\alpha}{r+\alpha}\right)^{1/2}}\left[1+\left(\frac{2\alpha}{r+\alpha}\right)^{1/2}+\frac{2\alpha}{r+\alpha}+\left(\frac{2\alpha}{r+\alpha}\right)^{3/2}+\left(\frac{2\alpha}{r+\alpha}\right)^{2}\right], \\ u'^1 &= -\left(\frac{2\alpha}{r+\alpha}\right)^{1/2}, \qquad u'^2 = u'^3 = 0. \end{aligned} \tag{3.14}$$

Now, by integrating $cdt = (u'^0/u'^1)dr$ one obtains the equation (2.6) (with the change $\tau \equiv t$), that is, unlike (3.13), the particle approaches the event horizon and penetrates under it at finite value of the parameter t.

Now let us consider the connection between the field configuration (3.10) and (3.11). For their construction the solutions (2.1) and (2.5) were taken as initial ones. However, the solutions (2.1) and (2.5) are connected by a coordinate transformation. Therefore, (see Introduction and the first half of this section) these field configurations should be connected by a gauge transformation (3.7). Indeed, this is so. Picking out one of the discussed field configurations as an initial one, using (3.7) one obtains the other one. The operator $\hat{A}$ in (3.7) is defined by the coordinate transformations given in the Appendix.

The same is valid for the vector components u^α. Each of the solutions (3.13) and (3.14) can be obtained one from another by the gauge transformation (3.8). Essentially for a test particle trajectory in the auxiliary background world this means the following. By a gauge transformation either this line is saved from a "catastrofic" discontinuity at the event horizon or, on the contrary, an initially continuous trajectory is "broken". It is pertinent to remind that gauge transformations act neither upon the coordinates, nor upon the background metric. Besides that, here they do not violate the conditions (3.12). Thus, in the exact field formulation of GR a particle trajectory in a background world can be essentially changed by a finite gauge transformations. In the case of the weak field formulation this line is changed only "slightly" (Mashhoon and Grishchuk, 1980).

In conclusion, let us note the following. Taking into account that (3.12) is satisfied, one could ascribe to the parameter t a wrong meaning of an observed variable in the flat world. Then as a result of an investigation of the field configuration (3.10) one could get the opinion that black holes are impossible since $t \to \infty$ as far as the particle approaches the Schwarzshild sphere (Logunov and Mestvirishvili, 1986). Here, without breaking the conditions (3.12), we presented a solution where the particle approaches the event horizon and crosses it without obstacles. that is, we show, that the additional conditions (3.12) do not exclude black holes.

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APPENDIX

In the Appendix we show how the new harmonic coordinates were found.

The general form of the equations for coordinate transformations conserving the conditions (2.3) is given in Fock (1959). Using the metric (2.1) as an initial example, let us write these equations for the transformations that preserve spherical symmetry (and which also do not alter the angles, that is, $\theta \to \theta$, $\varphi \to \varphi$):

$$\begin{aligned} &(r^2 - \alpha^2)\tau'' + 2r\tau' - \frac{(r+\alpha)^3}{r-\alpha}\ddot{\tau} = 0, \\ &(r^2 - \alpha^2)\rho'' + 2r\rho' - 2\rho - \frac{(r+\alpha)^3}{r-\alpha}\ddot{\rho} = 0, \end{aligned} \tag{A.1}$$

where $\tau = \tau(t, r)$, $\rho = \rho(t, r)$, $(\dot{}) = \partial/c\partial t$, $(') = \partial/\partial r$. Restricting ourselves by the requirement that the metric in the new coordinates should not depend on τ, we have $\tau = A_1 t + A_2 + R(r)$, $\rho = \rho(r)$. Then, using (A.1) one gets:

$$\begin{aligned} \tau &= A_1 t + A_2 + B_1 \ln \left| \frac{r-\alpha}{r+\alpha} \right| + B_2, \\ \rho &= C_1 r + C_2 \left(\frac{r}{2\alpha} \ln \left| \frac{r-\alpha}{r+\alpha} \right| + 1 \right), \end{aligned} \tag{A.2}$$

where A_1, A_2, B_1, B_2, C_1 and C_2 are constants. With no loss of generality we take $A_2 = B_2 = 0$. If we desire to have the Minkowsky metric at $r \to \infty$ after a transformation like (2.2), we should put $A_1 = C_1 = 1$. Following Belinfante and Garrison (1962), we choose $C_2 = 0$, since for $C_2 \neq 0$ the one to one correspondence between the physical space-time points and the harmonic coordinate points is broken even outside the event horizon. Finally, from the requirement that the particle should approach the event horizon at a finite time τ the choice $B_1 = 2\alpha/c$ follows. The mentioned logarithmic term in the geodesic equation (2.4) vanishes just in this case, that is the equation (2.4) is transformed into (2.6).

So the transformation to be found takes the final form

$$c\tau = ct + 2\alpha \ln \left| \frac{r-\alpha}{r+\alpha} \right|, \qquad r = r, \qquad \theta = \theta, \qquad \varphi = \varphi. \tag{A.3}$$

If this transformation applied to (2.1), the metric (2.5) is obtained. Vice versa, the transformation inverse to (A.3) applied to the metric (2.5), gives (2.1).

According to the point of view of Zelmanov's method of chonometric invariants (see in Zelmanov (1956); Zel'manov and Agakov (1989)), the transformation (A.3) differs from the transformations from Schwarzshild's coordinates to the Lemaitre, Kruskal-Szekeres or Novikov coordinates (Landau and Lifshitz, 1975; Misner *et al.*, 1973; Novikov and Frolov, 1986) and it is only similar to the transformation into the Finkelstein ones (Finkelstein, 1958).

The coordinates found are similar to the contracting Finkelstein coordinates. In the solution (2.5) particles falling into a black hole are described well at the event horizon, unlike those ejected from its interior. In order to construct a coordinate system, where description of the ejected particles would be convenient, one has to choose $B_1 = -2\alpha/c$ in (A.2). Then as a result one would obtain expanding coordinates. They will be also harmonic.

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