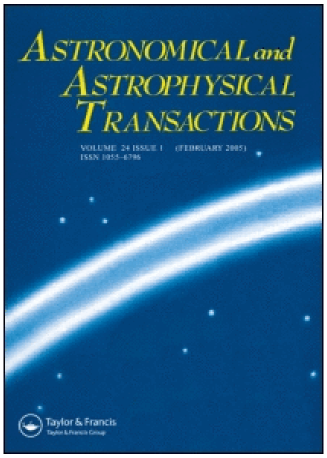

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Subdiffusion of beams through interplanetary and interstellar media

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The angular distribution of beams being propagated through a medium with random inhomogeneities is analysed. The peculiarity of this medium is that beams are trapped at random locations and random times because of wave localisation in the inhomogeneities. The mean-square deviation of the beam from its initial direction is calculated. The application of this method to the diagnostics of interplanetary and interstellar turbulent media is discussed.

Keywords: Scintillation; Turbulent plasma; Lévy flights; Subdiffusion; interstellar media

Diffusive transport plays an important role for many everyday phenomena in physics, astronomy, chemistry, biology, and engineering, where the mean-square displacement tends to a linear increase with time. Consequently, transmission of particles through a slab is characterised by the conventional Ohm's law. However, the wave nature of diffusing particles can lead to a complete halt of diffusion because of the interference of waves propagating in reciprocal multiple scattering paths, and therefore, the particles stay close to their initial place. Anderson first predicted the phenomenon (called later the Anderson localisation) in 1958 to explain the metal–insulator transition in electronic systems [1]. His localisation doctrine has opened a wide facility for scientists with various interests to develop a vast body of research. No wonder that the localisation concept may also be applied to classical wave systems [2]. Subdiffusion is a non-trivial crossover from classical diffusive regime to localisation. It accounts for the amount of time when a walker does not participate in motion. As a result, in this case, the mean-square displacement has a slower (power) increase in time.

The classical theory of scintillations assumes that individual angular increments are independent and identically Gaussian-distributed random variables. However, the pulse profiles predicted by the theory disagree with the observable results for the distant pulsars [3]. Recently Boldyrev and Gwinn [4] showed that the time broadening of the radio pulses from the distant pulsars can arise from non-Gaussian statistics of interstellar electron-density [4]. The problem requires Lévy-flights scenario. Lévy statistics contains larger-than-rare events, and its variance

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diverges. The mathematically detailed discussion of Lévy distributions is represented in the excellent book of Zolotarev [5]. Many physical (and not only physical) examples demonstrate a contribution of Lévy flights (see the comprehensive reviews [6,7] and references therein). It is interesting to observe that Lévy flights in time has a direct relationship with subdiffusion [8]. They originate from a subordination of one random process by another, and the latter is governed by a Lévy process. The subdiffusion stems from weak localisation effects.

In this connection, we will consider the beam propagation direction in a randomly inhomogeneous medium. This is an often observed phenomenon in nature (random refraction of radio waves in the ionosphere and solar corona, stellar scintillation because of interstellar inhomogeneities, etc). The classical approach to the propagation study of a beam (of light, radio waves, or sound) in random media is based on the analysis of normal diffusion processes [9,10]. But some astronomical observations [11] of the beam diffusion has an anomalous character close to subdiffusion. A recent paper [12] is devoted to the theoretical study of subdiffusion of beams in random media. The effect is caused by wave localisation at random locations and random times. This leads to traps of beams by inhomogeneities in such a medium. Therefore, the propagation of beams can contain random jerking. The approach gives a generalisation of '3/2 law' so-called in [13]. In the present paper, we intend to further explore the transport properties of the system outlined in [12].

Let a beam propagate through the medium by deflecting in random directions. Due to random inhomogeneities of refraction, the beam is trapped in some regions (weak localisation). Because of the traps, beam propagation is 'frozen' for some time. Next, the randomly deflected beam leaves the region and propagates further until it is trapped in other regions and the trapping cycle repeats. Suppose that from the point of view, trapping appearance the medium is statistically homogeneous and isotropic. The randomly winding beam path is responsible for the random refraction analysed in this study.

In our considerations, we will use the notations from [12]. The angle θ of deviation of a beam from its initial direction because of the inhomogeneities is a random variable with a probability density $V_\alpha(\theta, \sigma)$, where σ is the path travelled by the beam. The classical approach to random walks of beams assumes that the random angle jumps $\Delta\theta_i$ at points are separated by segments of equal length $\Delta\sigma_i$. In the continuous limit, the beam diffusion is described as a rotational Brownian motion. In our consideration, the random walks consist of random angle jumps $\Delta\theta_i$ alternating with random lengths $\Delta\sigma_i$ (continuous time random walk). In contrast to [12], the resulting random process is a subordination of the rotational Brownian motion by a new random process defined in [14]. The new subordinator $\overline{S}_\sigma^{(\alpha)}$ satisfies the self-similar property:

$$\overline{S}_{c\sigma}^{(\alpha)} :\overset{d}{=} c\overline{S}_\sigma^{(\alpha)}, \quad c > 0, \quad 0 < \alpha < 1,$$

where '$\overset{d}{=}$' reads 'equal in law'. When $\alpha \to 1$, then $\overline{S}_\sigma^{(\alpha)}$ becomes deterministic. Recall that the anomalous diffusion $X(\overline{S}_t^{(\alpha)})$ with such a subordinator gives a stretched exponential function of relaxation [14]. All this allows us to write the probability density $V_\alpha(\theta, \sigma)$ in the integral form

$$V_\alpha(\theta, \sigma) = \int_0^\infty V_1(\theta, \sigma x)\, \mathrm{d}g_\alpha(x)\,,$$

where the probability distribution $g_\alpha(x)$ is described by the Laplace transform

$$G_\alpha(s) = \int_0^\infty \exp\{-sx\}\, \mathrm{d}g_\alpha(x) = \exp\{-(As)^\alpha\}$$

with $s \geq 0$ and $A > 0$ [5]. This distribution is totally asymmetric, and $0 < \alpha < 1$ in view of the fact that σ_i are non-negative random variables. The probability density $V_1(\theta, \sigma)$

corresponds to the normal diffusion. Then the relations yield the mean

$$\overline{\cos\theta} = \mathrm{e}^{-2D\sigma^{\alpha}},$$

where D is constant. At a large σ, all beam directions are equiprobable. However, in contrast to normal diffusion ($\alpha = 1$), any beam has to travel a longer path σ to reach this state. This process is somewhat analogous to 'superslow' relaxation.

Following the tools developed in [13], we can find the mean square of the distance r from the starting point to the observation point reached by the beam that has travelled an intricate path of length σ through the medium:

$$\overline{r^2} = \frac{\sigma}{D_1}\int_0^{\infty} x\,\mathrm{d}g_{\alpha}(x) - \frac{1}{2D_1^2}\left(1 - \mathrm{e}^{-2D\sigma^{\alpha}}\right), \quad D_1 = \text{const.} \tag{1}$$

The first term of expression (1) tends to infinity, as the function $g_{\alpha}(x)$ is the probability distribution of a fully skewed Lévy process. Thus, the value (1) has the same trend. If the z axis of a polar coordinate system is aligned with the initial beam direction, then the mean square of the distance passed by the beam along this axis is given by the formula

$$\overline{z^2} = \frac{1}{3D_1}\left[\sigma\int_0^{\infty} x\,\mathrm{d}g_{\alpha}(x) - \frac{1}{6D_1}\left(1 - \mathrm{e}^{-6D\sigma^{\alpha}}\right)\right]. \tag{2}$$

Again, because of the first term, the mean $\overline{z^2}$ becomes infinite. Now, the mean square deviation of the beam from its initial direction can be calculated by combining equation (1) with equation (2):

$$\overline{\rho^2} = \overline{r^2} - \overline{z^2} \to \infty. \tag{3}$$

The behaviour of the means $\overline{r^2}$, $\overline{z^2}$, and $\overline{\rho^2}$ is not something strange. It is governed by Lévy flights, and the variance of the mean squares diverges. If $\alpha = 1$, then the mean squares (1)–(3) take the finite values supporting the '3/2 law' [13], as the subordinated random process has an ordinary Gaussian distribution.

In the paper [12], the physical situation is quite different. This subdiffusion has the inverse-time α-stable Lévy subordinator. Though both $\Delta\sigma_i$ and $\Delta\theta_i$ are Markov processes, the subordinated process may not preserve the Markov property [15]. The subordinator inserts long-term memory effects, and the probability density of the subordinated process obeys a fractional Fokker–Planck equation in spherical coordinates. The signature of such memory effects is just a power kernel of the fractional operator in time, and the corresponding random process is something intermediate between a purely random process and a deterministic one (see more details in [16]). All the mean squares $\overline{r^2}$, $\overline{z^2}$, and $\overline{\rho^2}$ become finite (for any value of $0 < \alpha < 1$). They increase as σ^{α} at a large σ. As a result, we arrive at a generalised '3/2 law' in the form

$$\sqrt{\overline{\rho^2}} \approx \frac{2\sqrt{2}}{\sqrt{\Gamma(3\alpha+1)}}B^{1/2}\sigma^{3\alpha/2}, \quad B = \text{const.}, \quad B\sigma^{\alpha} \ll 1. \tag{4}$$

In fact, in the case of $\alpha = 1$, the normal diffusion results in the classical '3/2 law', but for $0 < \alpha < 1$, according to equation (4), the value $\sqrt{\rho^2}$ behaves as $\sigma^{3\alpha/2}$ for a small $B\sigma^{\alpha}$. As has been shown above in (3), the process $V_{\alpha}(\theta, \sigma)$ behaves otherwise. In any case, both processes can be present in random media, but they correspond to different physical situations.

Finally, we briefly sum up our consideration. Plasma transport in the presence of turbulence depends on a wide variety of parameters such as the levels of magnetic field, the ratio between the particle Larmor radius and the turbulence correlation lengths, the turbulence anisotropy, etc [17]. Many researchers addressed this issue from both theoretical and experimental points of view [18] (and references therein), but a full understanding is still lack. The point is that the turbulence itself is a very difficult phenomenon in plasma physics. Sometimes, the wave–particle transport in turbulent plasma is characterised by anomalous, non-Gaussian features. This means that there exist regimes where wave–particle interactions depend on Lévy statistics. In particular, the density of particles can be described by fractional Fokker-Planck equations [17]. Although surprising, this possibility is implicit in the idea of wave interference resulting from multiple scattering. The probing into turbulent plasma with radio wave propagation allows one to get an additional information on the turbulence anisotropy in solar wind as well as in other astrophysical situations. The analysis gives tools to detect of weak localisation in the astrophysical plasma. The experimental difference, noticed in [11], from the '3/2 law' (more precisely, from an exponent of 3/2 in the classical power law) gives us hope.

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