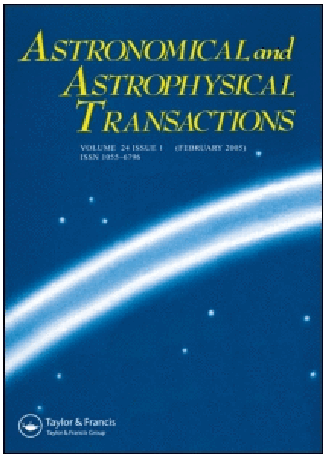

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Review of theory of interplanetary and interstellar scintillation

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Review of theory of interplanetary and interstellar scintillation

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Basic theoretical methods and solutions describing radio wave propagation in the turbulent plasma are briefly reviewed. Consideration is given to the results on scattering effects such as angular scattering, pulse broadening and spectral line spread. Also treated are phase and frequency fluctuations. Of particular concern are problems of the correlation theory of intensity fluctuations for weak and strong diffractive and refractive scintillations. Special attention is paid to the effect of refractive scintillation on a diffractive pattern.

Keywords: Scattering; Scintillation; Turbulence; Interstellar plasma; Solar wind

1. Basic equations

The propagation of a time-harmonic electromagnetic wave in a random medium containing a weak large scale index of refraction fluctuations is governed by the scalar wave equation [1]

$$\Delta U + k^2 n^2(\mathbf{r}) U = 0 \tag{1}$$

Here $k = 2\pi/\lambda$ is wave number, λ is wavelength, n is refraction index. For plasma [2] we have

$$n^2 = 1 - \left(\frac{\omega}{\omega_\mathrm{p}}\right)^{-2}, \tag{2}$$

where ω is cyclic frequency and ω_p is Lengmuir (plasma) frequency that is determined by the equation [2]

$$\omega_\mathrm{p}^2 = \left(\frac{4\pi e^2}{m}\right) N_\mathrm{e} \tag{3}$$

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Here e and m are electron charge and mass, N_e is electron density. E. Salpeter [3] reduced equations (2) and (3) to the following form

$$\delta n = 1 - n \cong \left(\frac{\lambda^2 r_e}{2\pi}\right) N_e \tag{4}$$

where r_e is electron radius.

We consider the propagation of a radio wave field in random media where integral scattering takes place only over small angles. Under this assumption, it is possible to use the parabolic (diffusive) approximation of wave equation (1) [4]. Let

$$U(\mathbf{r}) = E(\mathbf{r}) \exp(ikz) \tag{5}$$

where the z axis is directed along the mean direction of propagation. Neglecting the term $\partial^2 E/\partial z^2$ we obtain the function $E(\mathbf{r})$ satisfies the equation

$$2ik\left(\frac{\partial E}{\partial z}\right) + \Delta_\perp E + 2k^2 \delta n(\mathbf{r}) E = 0 \tag{6}$$

$$\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Here the mark $\perp$ denotes the direction perpendicular to axis z.

2. Linear phase (phase length) and phase scintillation

Neglecting the diffractive term $\Delta_\perp E$ in equation (6), we obtain a simple solution

$$E(\boldsymbol{\rho}, z) = E_0 \exp[-i S_{\text{lin}}(\boldsymbol{\rho}, z)] \tag{7}$$

where S_{lin} is linear phase (phase length)

$$S_{\text{lin}} = (\lambda r_e) \int_0^z \mathrm{d}z' N_e, \quad \boldsymbol{\rho} = (x, y). \tag{8}$$

The linear phase depends on columnar electron content that is named dispersion measure (DM) for the case of interstellar plasma. The time delay of pulse propagation through the turbulent plasma is also determined by columnar electron content:

$$t_{\text{delay}} = \left(\frac{1}{kc}\right) S_{\text{lin}} = \frac{(\lambda^2 r_e)}{(2\pi c)} \int_0^z \mathrm{d}z' N_e \tag{9}$$

We will suppose that fluctuations of S_{lin} are subject to the Central Limit Theorem and the distribution function of these fluctuations approaches the normal law for large values of distances z. For this case the main statistical characteristic of the linear phase is a structure function that is determined by the following equation

$$\begin{aligned} D_{\text{S,lin}}(\boldsymbol{\rho}, z) &= \langle [S(\boldsymbol{\rho}_1, z) - S(\boldsymbol{\rho}_1 - \boldsymbol{\rho}, z)]^2 \rangle = \int \mathrm{d}z' D(\boldsymbol{\rho}, z') \\ D(\boldsymbol{\rho}, z) &= 4\pi \lambda^2 r_e^2 \int d^2\mathbf{q}_\perp [1 - \cos(\mathbf{q}_\perp \boldsymbol{\rho})] \Phi_{\text{Ne}}(\mathbf{q}_\perp, \mathrm{q}_\parallel = 0) \end{aligned} \tag{10}$$

Here $\Phi_{\text{Ne}}(\mathbf{q})$ is the spatial power spectrum of electron density fluctuations. $D(\boldsymbol{\rho}, z)$ and $\Phi_{\text{Ne}}(\mathbf{q}_\perp, \mathrm{q}_\parallel = 0)$ are equivalent statistical characteristics of the turbulent plasma.

Another model of Levy flight statistics was proposed in [5] and is discussed in [6].

For the case of a power law spectrum

$$\Phi_{\mathrm{Ne}}(\mathbf{q}) = C_{\mathrm{Ne}}^2 q^{-\mathrm{n}}$$
$$L^{-1} < q < l^{-1}, \quad 3 < n < 4 \tag{11}$$

the structure function of a linear phase is defined by

$$D_{\mathrm{S,lin}}(\boldsymbol{\rho}) = (k\theta_0\rho)^{\mathrm{n}-2}, \quad l < \rho < L$$
$$\cong (k\theta_0 l)^{\mathrm{n}-2}\left(\frac{\rho}{l}\right)^2, \quad \rho < l \tag{12}$$

where L is the outer and l is the inner scale. For a Kolmogorov spectrum $n = 11/3$. θ_0 is a characteristic scattering angle.

3. Methods of solution taking into account diffraction effects

An exact solution of equation (6) was obtained only for the phase screen model. This model has been extensively studied by a number of authors [3, 7–9]. Assuming the field on the exit plane of a layer of irregular refractive medium to be only phase modulated, *i.e.*

$$E\mid_{z=\Delta z} = E_0 \exp[-iS_{\mathrm{lin}}(\boldsymbol{\rho}, \Delta z)] \tag{13}$$

and solving equation (6) in free space with boundary condition (13), we obtain at distance z

$$E(\boldsymbol{\rho}, z) = \left(\frac{ik}{2\pi z}\right)\int d^2\boldsymbol{\rho}' \exp\left[\frac{-ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2 - \mathrm{i}S_{\mathrm{lin}}(\boldsymbol{\rho}', \Delta z)\right]E_0(\boldsymbol{\rho}') \tag{14}$$

For extended medium solutions these were obtained by perturbation methods. Weak modulation of field or single scattering is described by the Born approximation, that corresponds to term E_1 in perturbation series for the wave field [1]

$$E = E_0 + E_1 + E_2 + \cdots \tag{15}$$

$$E_1(\boldsymbol{\rho}, z) = \left(\frac{K^2}{2\pi}\right)\int dz'\left[\frac{1}{(z-z')}\right]\int d^2\boldsymbol{\rho}' \exp\left[\frac{-ik(\boldsymbol{\rho}-\boldsymbol{\rho}')^2}{2(z-z')}\right]\Delta n(\boldsymbol{\rho}', z')E_0(\boldsymbol{\rho}') \tag{16}$$

The Born approximation is correct for the case of weak phase fluctuations: $|S| \ll 1$. The Rytov approximation corresponds to the term ψ_1 in the perturbation series for a complex phase of the wave field [1]

$$\psi = \ln\frac{E}{E_0} = \psi_0 + \psi_1 + \psi_2 + \cdots \tag{17}$$

The term ψ_1 is determined by equation (16) without factor $E_0(\boldsymbol{\rho}')$ in the integrand. Rytov approximation is correct for the case of weak amplitude fluctuations: $|\operatorname{Re}\psi_1| \ll 1$. For large values of ρ

$$\psi_1(\rho) \cong iS_{\mathrm{lin}}(\boldsymbol{\rho}) \tag{18}$$

The stochastic equation (6) can be reduced to dynamical equations for field moments if phase fluctuations are small on distance of the order of an outer scale of inhomogeneities L. These equations are correct for weak and strong scintillation.

The mutual coherence function of the second order describes the scattering effects

$$B_{\mathrm{E}}(\boldsymbol{\rho}, \Delta f, z) = \left\langle E\left(\boldsymbol{\rho}_1 - \left(\frac{1}{2}\right)\boldsymbol{\rho}, f - \left(\frac{1}{2}\right)\Delta f, z\right) E^*\left(\boldsymbol{\rho}_1 + \left(\frac{1}{2}\right)\boldsymbol{\rho}, f + \left(\frac{1}{2}\right)\Delta f, z\right)\right\rangle \tag{19}$$

where Δf is a frequency lag. It is determined by the equation [10–12]

$$\left(\frac{\partial}{\partial z}\right) B_{\mathrm{E}}(\boldsymbol{\rho}, \Delta f, z) + \left(\frac{i}{2k}\right)\left(\frac{\Delta f}{f}\right)\Delta_{\perp} B_{\mathrm{E}}(\boldsymbol{\rho}, \Delta f, z) = -\left(\frac{1}{2}\right) D(\boldsymbol{\rho}, z) B_{\mathrm{E}}(\boldsymbol{\rho}, \Delta f, z) \tag{20}$$

For the case of an initially spherical wave the task is statistically homogeneous on the sphere. To reduce equation (20) for this case we must change z to r (distance from a source) and $\boldsymbol{\rho}$ to $r\boldsymbol{\eta}$ ($\boldsymbol{\eta}$ is the angular distance between two points on the sphere) [13].

Intensity fluctuations are described by the fourth moment of the field [13,14]

$$M(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \boldsymbol{\rho}_3, \boldsymbol{\rho}_4, z) = \langle E(\boldsymbol{\rho}_1, z) E(\boldsymbol{\rho}_2, z) E^*(\boldsymbol{\rho}_3, z) E^*(\boldsymbol{\rho}_4, z)\rangle$$

For the simplest case of the initial plane waves this moment is determined by the equation [13,14]

$$\left(\frac{\partial}{\partial z}\right) M + \left(\frac{i}{k}\right)(\boldsymbol{\nabla}_{\mathbf{u}}\boldsymbol{\nabla}_{\mathbf{v}}) M + f(\mathbf{u}, \mathbf{v}) M = 0$$

$$f(\mathbf{u}, \mathbf{v}) = D(\mathbf{u}) + D(\mathbf{v}) - \left(\frac{1}{2}\right) D(\mathbf{u} + \mathbf{v}) - (D(\mathbf{u} - \mathbf{v}) \tag{21}$$

$$\mathbf{u} = (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_3) = \boldsymbol{\rho}_4 - \boldsymbol{\rho}_2)$$

$$\mathbf{v} = (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_4) = (\boldsymbol{\rho}_3 - \boldsymbol{\rho}_2)$$

To reduce equation (21) for case of the initially spherical wave we must change z to r and $\mathbf{u}$, $\mathbf{v}$ to $r\boldsymbol{\eta}$, $r\boldsymbol{\zeta}$ ($\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$ are angular distances between two points on the sphere).

If we introduce the spectral function

$$M_{\mathrm{Sp}}(\mathbf{q}, \mathbf{v}, z) = \left(\frac{1}{2\pi}\right)^2 \int d^2\mathbf{u} \exp(-i\mathbf{q}\mathbf{u}) M(\mathbf{u}, \mathbf{v}, z) \tag{22}$$

then equation (21) becomes [14]

$$\left(\frac{\partial}{\partial z}\right) M_{\mathrm{Sp}} - \left(\frac{1}{k}\right)(\mathbf{q}\boldsymbol{\nabla}_{\mathrm{v}}) M_{\mathrm{Sp}} + D(\mathbf{v}) M_{\mathrm{Sp}} = \int d^2\mathbf{q}_1 \Psi(\mathbf{q} - \mathbf{q}_1, \mathbf{v}) M_{\mathrm{Sp}}(\mathbf{q}_1, \mathbf{v}, z) \tag{23}$$

$$\Psi(\mathbf{q}, \mathbf{v}) = 4\pi(\lambda r_{\mathrm{e}})^2 \Phi_{\mathrm{Ne}}(\mathbf{q}_{\perp}, q_{\|} = 0)[1 - \cos(\mathbf{q}\mathbf{v})] \tag{24}$$

The form of the equation (23) corresponds to the non-stationary transfer equation, where z is an equivalent to time, $\mathbf{v}$ is an equivalent to a spatial coordinate and $\mathbf{q}$ is equivalent to velocity.

4. Scattering effects

4.1 *Angular scattering*

The angular scattering effect is described by the second-order coherence function $B_{\mathrm{E}}(\boldsymbol{\rho}, \Delta f = 0, z)$. This coherence function corresponds to the response of an interferometer with base ρ.

Solving equation (20) for $\Delta f = 0$ we obtain [10, 13]

$$B_{\mathrm{u}}(\boldsymbol{\rho}, z) = B_{\mathrm{E}}(\boldsymbol{\rho}, z) = B_{\mathrm{E},0}(\boldsymbol{\rho}, z) \exp\left[-\left(\frac{1}{2}\right) D_{\mathrm{S,lin}}(\boldsymbol{\rho}, z)\right] \tag{25}$$

The Fourier transform of the coherence function $B_{\mathrm{E}}(\boldsymbol{\rho}, z)$ gives the brightness distribution

$$I(\boldsymbol{\theta}) = \left(\frac{1}{2\pi}\right)^2 \int d^2\boldsymbol{\rho} \exp\left[-i(k\boldsymbol{\theta}\boldsymbol{\rho}) - \left(\frac{1}{2}\right) D_{\mathrm{S,lin}}(\boldsymbol{\rho})\right] B_{\mathrm{E},0}(\boldsymbol{\rho}, z) \tag{26}$$

4.2 *Pulse scattering*

The shape of the scattered pulse is expressed as the Fourier transform of the coherence function $B_{\mathrm{E}}(\rho = 0, \Delta f)$.

$$P(\tau) = \int d(\Delta f) \exp[i2\pi\tau(\Delta f)] B_{\mathrm{E}}(\rho = 0, \Delta f) \tag{27}$$

We must solve equation (20) for $B_{\mathrm{E}}(\boldsymbol{\rho}, \Delta f)$ and then substitute this solution in equation (27). For the case of the phase screen the shape of the scattered pulse is determined by the brightness distribution $I(\boldsymbol{\theta})$ on the exit plane of the phase screen. The time delay of pulse ray propagating in the direction $\boldsymbol{\theta}$ is equal to

$$\tau = \frac{z\theta^2}{2c} \tag{28}$$

Using this relation we obtain [15]

$$P(\tau) = 2\pi I\left(\mid \boldsymbol{\theta} \mid = \left[\frac{(2c\tau)}{z}\right)\right]^{1/2}\right), \quad \tau \geq 0 \qquad 0, \tau \leq 0 \tag{29}$$

For the case of the power law turbulent spectrum the pulse tail has a power law form

$$P(t) \propto \left(\frac{1}{\tau_{\mathrm{sc}}}\right)\left(\frac{\tau_{\mathrm{sc}}}{t}\right)^{\mathrm{n}/2}, \tau_{\mathrm{sc}} = \frac{z\theta_0^2}{2c} \tag{30}$$

If the phase structure function has a square form (12) then the pulse shape is exponential:

$$P(t) \propto \exp\left(\frac{-t}{\tau_{\mathrm{sc}}}\right)$$

Extension of the turbulent medium leads to smoothing of the leading region of pulse [16–18]. The structure of the tail region is the same for the phase screen as for the extended medium [15].

In the presence of strong angular refraction we must change $\boldsymbol{\theta}$ to $\boldsymbol{\theta}_{\mathrm{ref}} + \boldsymbol{\theta}$ in equation (28). The shape of scattered pulse becomes symmetrical [16]

$$P(\tau) = \int d(\Delta f) \exp[i2\pi\tau(\Delta f)] B_{\mathrm{E}}\left(\boldsymbol{\rho} = 2\left(\frac{\Delta f}{f}\right) z\boldsymbol{\theta}_{\mathrm{ref}}\right) \tag{31}$$

4.3 *Spectral line broadening*

Modulation of the radio wave by a moving turbulent medium leads to temporal field modulation. In the case of medium moving relatively to the line of sight with a velocity **V**, we obtain [19]

$$
\begin{aligned}
B_{\mathrm{E}}(\boldsymbol{\rho} = \mathbf{V}t) &= \langle H \rangle \exp\left[-\left(\frac{1}{2}\right) D_{\mathrm{S,lin}}(\boldsymbol{\rho} = \mathbf{V}t)\right] \\
I(\Delta f) &= \int dt \exp[-i2\pi(\Delta f)t]\langle H \rangle \exp[-D_{\mathrm{S,lin}}(t)]
\end{aligned}
\tag{32}
$$

where H is flux and $I(\Delta f)$ is flux spectral density.

5. Intensity scintillation

5.1 *Phase screen*

If diffraction effects are ignored we can obtain from equation (20) on the exit plane of the phase screen [9, 13]

$$
\begin{aligned}
M\ |_{z=\Delta z} &= H_0^2 \exp[-F(\mathbf{u}, \mathbf{v})] \\
F(\mathbf{u}, \mathbf{v}) &= \int_0^{\Delta z} dz' f(\mathbf{u}, \mathbf{v}) = D_{\mathrm{S,lin}}(\mathbf{u}) + D_{\mathrm{S,lin}}(\mathbf{v}) - \left(\frac{1}{2}\right) D_{\mathrm{S,lin}}(\mathbf{u}+\mathbf{v}) \\
&\quad - \left(\frac{1}{2}\right) D_{\mathrm{S,lin}}(\mathbf{u}-\mathbf{v}),
\end{aligned}
\tag{33}
$$

where H_0 is the initial flux. The spectral function, equation (22), is

$$
M_{\mathrm{Sp}}(\mathbf{q}, \mathbf{v}, z) = \left(\frac{1}{2\pi}\right)^2 H_0^2 \int d^2\mathbf{u} \exp[-i\mathbf{qu} - F(\mathbf{u}, \mathbf{v})] \tag{34}
$$

The solution of equation (23) in free space corresponds to substitution $\mathbf{v} \to \mathbf{v} - \mathbf{q}z/k$.

$$
M_{\mathrm{Sp}}(\mathbf{q}, \mathbf{v}, z) = \left(\frac{1}{2\pi}\right)^2 {H_0}^2 \int d^2\mathbf{u} \exp\left\{-i\mathbf{qu} - F\left[\mathbf{u}, \mathbf{v} - \left(\frac{\mathbf{q}z}{k}\right)\right]\right\} \tag{35}
$$

Scintillation regimes depend on the value of $D_{\mathrm{S,lin}}((z/k)^{1/2})$ for the case of the power law turbulent spectrum. When qz/k is small, we can expand $\exp[-F(\mathbf{u}, \mathbf{v}) + D_{\mathrm{S,lin}}(\mathbf{v} - \mathbf{q}z/k)]$ in power series and obtain the low-frequency expansion for M_{Sp}, *i.e.* [9],

$$
M_{\mathrm{Sp}} = M_{\mathrm{Sp},0} + M_{\mathrm{Sp},1} + \cdots \tag{36}
$$

$$
M_{\mathrm{Sp},0} = H_0^2 \exp[-D_{\mathrm{S,lin}}(\mathbf{v})]\delta(\mathbf{q}) \tag{37}
$$

$$
M_{\mathrm{Sp},1}(\mathbf{q}, v, z) = 4\pi(\lambda r_{\mathrm{e}})^2 \Delta z H_0^2 \mathrm{Sin}^2\left(\frac{q^2}{2kz}\right) \Phi_{\mathrm{Ne}}(\mathbf{q}, q_{\parallel} = 0) \exp\left[-D_{\mathrm{S,lin}}\left(\frac{\mathbf{v} - \mathbf{q}z}{k}\right)\right] \tag{38}
$$

5.2 *Weak scintillation*

$M_{\mathrm{Sp},1}(\mathbf{q}, z)$ corresponds to the Born and Rytov approximations [1, 13] if $D_{\mathrm{S,lin}}(\mathbf{q}z/k) \ll 1$:

$$M_{\mathrm{Sp},1}(\mathbf{q}, z) \cong M_{\mathrm{Sp},1,\mathrm{B}}(\mathbf{q}, , z) = 4\pi(\lambda r_{\mathrm{e}})^2 \Delta z H_0^2 \mathrm{Sin}^2\left(\frac{q^2}{2kz}\right)\Phi_{\mathrm{Ne}}(\mathbf{q}, q_{\parallel} = 0) \tag{39}$$

For small z equation (39) describes the scintillation spectrum for important regions of spatial frequencies. In this case the scintillation index is

$$m^2 = \frac{\langle (H - \langle H\rangle)^2\rangle}{\langle H\rangle^2} \cong m_{\mathrm{B}}^2 = \left(\frac{1}{\langle H\rangle^2}\right)\int d^2\mathbf{q} M_{\mathrm{Sp},1,\mathrm{B}}(\mathbf{q}, , z) \approx D_{\mathrm{S,lin}}\left[\left(\frac{z}{k}\right)^{1/2}\right] \ll 1 \tag{40}$$

For the large value of $|\mathbf{q}z/k|$, $F(\mathbf{u}, \mathbf{v}) - D_{\mathrm{S,lin}}(\mathbf{u}) \to 0$, and we can expand $\exp[-F(\mathbf{u}, \mathbf{v}) + D_{\mathrm{S,lin}}(\mathbf{u})]$ in power series and obtain the high-frequency series expansion for M_{Sp}, *i.e.* [9, 13],

$$M_{\mathrm{Sp}} = M_{\mathrm{Sp}}^{(0)} + M_{\mathrm{Sp}}^{(1)} + \cdots \tag{41}$$

$$M_{\mathrm{Sp}}^{(0)} = \left(\frac{1}{2\pi}\right)^2 H_0^2 \int d^2\mathbf{u} \exp[-i\mathbf{q}\mathbf{u} - D_{\mathrm{S,lin}}(\mathbf{u})] \tag{42}$$

$M_{\mathrm{Sp}}^{(0)}$ is the dominant term for large z. The equations (37) and (42) correspond to Gaussian approximation for the field statistics.

5.3 *Extended medium*

The solution of equation (23) in the form of iteration series is similar to the low-frequency expansion (36) for the phase screen case [20]. The zero term of this row is determined by equation (37). The first term is

$$\begin{gathered} M_{\mathrm{Sp},1}(\mathbf{q}, z) = 4\pi(\lambda r_{\mathrm{e}})^2 H_0^2 \int_0^z dz_1 \mathrm{Sin}^2\left[\frac{q^2}{2k(z - z')}\right]\Phi_{\mathrm{Ne}}(\mathbf{q}, q_{\parallel} = 0)\exp(-L) \\ L = \int_0^{z1} dz' D\left[\frac{\mathbf{q}z}{k(z - z_1)}\right] + \int_{z1}^{z} dz' D\left[\frac{\mathbf{q}z}{k(z - z\prime)}\right] \end{gathered} \tag{43}$$

The spatial spectrum of weak scintillation is determined by the equation [1,13]

$$M_{\mathrm{Sp}}(\mathbf{q}, z) = 4\pi(\lambda r_{\mathrm{e}})^2 H_0^2 \int_0^z dz' \mathrm{Sin}^2\left[\frac{q^2}{2k(z - z')}\right]\Phi_{\mathrm{Ne}}(\mathbf{q}, q_{\parallel} = 0) \tag{44}$$

The scintillation index is equal

$$m^2 \cong D_{\mathrm{S,lin}}\left(\left(\frac{z}{k}\right)^{1/2}\right) < 1 \tag{45}$$

and spatial scale is equal

$$b_{\mathrm{Fr}} \cong \left(\frac{z}{k}\right)^{1/2} \tag{46}$$

For large values of $D_{\mathrm{S,lin}}[(z/k)^{1/2}]$ according to relations (36) and (41) we obtain the zero term in the perturbation series for $M(\mathbf{u},\mathbf{v},z)$ that is equal to

$$M_0(\mathbf{u}, \mathbf{v}, z) = H_0^2 \exp[-D_{\mathrm{S,lin}}(\mathbf{u})] + H_0^2 \exp[-D_{\mathrm{S,lin}}(\mathbf{v})] = |B_E(\mathbf{u})|^2 + |B_E(\mathbf{v})|^2 \tag{47}$$

This relation corresponds to Gaussian statistics of field fluctuations. It can be used to express the frequency correlation function of diffractive flux fluctuation as:

$$B_{\mathrm{H}}(\Delta f) = \langle H(\boldsymbol{\rho}_1, f_1) H(\boldsymbol{\rho}_1 + \boldsymbol{\rho}, f_1 + \Delta f)\rangle - \langle H^2\rangle = |B_{\mathrm{E}}(\Delta f)|^2 \tag{48}$$

Here $B_{\mathrm{E}}(\Delta f)$ is frequency coherence function that discussed above (see equations (19), (20) and (27). According to equation (27) the characteristic frequency scale of diffractive scintillation Δf_{dif} and the characteristic pulse broadening τ_{sc} are conjugated parameters

$$2\pi \Delta f_{\mathrm{dif}} \tau_{\mathrm{sc}} \cong 1 \tag{49}$$

5.4 *The effect of a source size on scintillation characteristics*

For simplicity we consider the case of the phase screen [3, 21]. If the point source produces the diffraction pattern H_0 (ρ) on the observer plane, then the same source displaced by an angle θ will produce a similar pattern shifted by distance $z\theta$. The diffraction pattern for an extended source is given by [3,21]

$$H(\boldsymbol{\rho}, z) = \int \mathrm{d}^2\boldsymbol{\theta}\, H_0(\boldsymbol{\rho} + z\boldsymbol{\theta}) J(\boldsymbol{\theta}) \tag{50}$$

Here $J(\boldsymbol{\theta})$ is the initial brightness distribution of the source. A spatial spectrum of intensity fluctuations is determined by the following equation

$$\begin{aligned} M_{\mathrm{Sp}}(\mathbf{q}) &= \int \mathrm{d}^2\boldsymbol{\rho}\, \exp[-\mathrm{i}(\mathbf{q}\boldsymbol{\rho})] B_{\mathrm{H}}(\boldsymbol{\rho}) = M_{\mathrm{I},0}(\mathbf{q}) M_{\mathrm{J}}(z\mathbf{q}) \\ M_{\mathrm{J}}(\mathbf{q}) &= \left| \int \mathrm{d}^2\boldsymbol{\theta}\, \exp[-\mathrm{i}(z\mathbf{q}\boldsymbol{\theta})] J(\boldsymbol{\theta}) \right|^2 \end{aligned} \tag{51}$$

Using this relation we can obtain the amplitude of the source visibility function.

For extended medium the spectrum of weak scintillation is determined by equation (43), where $H_0^2 \exp(-L)$ is changed for $M_{\mathrm{J}}[(z - z')\mathbf{q}] \exp(-L)$. For the case of saturated scintillation the correlation function of diffractive scintillation is described by the expression [22]

$$\begin{aligned} B_{\mathrm{H}}(\mathbf{u}) &= M_0(\mathbf{u}, \mathbf{v} = \mathbf{0}, z) - H_0^2 = \int \mathrm{d}^2\boldsymbol{\theta}\, \exp\left[-\int_{\boldsymbol{\theta}}^{z} dz' D[\boldsymbol{\vartheta}(z - z_{\prime})])\right] J_1(\boldsymbol{\theta}) \\ J_1(\boldsymbol{\theta}) &= \int \mathrm{d}^2\boldsymbol{\theta}_1 J(\boldsymbol{\theta} - \theta_1) J(\boldsymbol{\theta}_1) \end{aligned} \tag{52}$$

For large sources with angular size $\varphi_0 \gg 1/kz\,\theta$ the shape of $B_{\mathrm{H}}(\mathbf{u})$ doesn't depend on the form of $J_1(\boldsymbol{\theta})$. The main information on the source angle size is contained in the scintillation index.

6. Effect of refractive scintillation on the diffractive pattern

We discussed above the properties of field moments for the case of full averaging on the turbulent medium statistics. The process of averaging over the statistics of the turbulent medium corresponds to temporal integration with a long integration time T. If

$$t_{\mathrm{dif}} < T < T_{\mathrm{ref}} \tag{53}$$

then the averaging is partial. We will designate such partial averaging by angular brackets with the subscript "dif".

For the case of the phase screen we can represent the partly averaged brightness distribution $\langle I(\boldsymbol{\theta})\rangle_{\mathrm{dif}}$ on the exit plane as [23]

$$\langle I(\boldsymbol{\rho}+z\boldsymbol{\theta},\boldsymbol{\theta},z=0)\rangle_{\mathrm{dif}} = F(\boldsymbol{\theta}-\boldsymbol{\theta}_{\mathrm{ref}}(\boldsymbol{\rho})) \tag{54}$$

where $F(\boldsymbol{\theta})$ is the brightness distribution of scattered irradiance and $\boldsymbol{\theta}_{\mathrm{ref}}(\boldsymbol{\rho})$ is random angular refraction due to large scale (refractive) inhomogeneities. At distance z we have

$$\langle I(\boldsymbol{\rho},\boldsymbol{\theta},z)\rangle_{\mathrm{dif}} = \langle I(\boldsymbol{\rho}+z\boldsymbol{\theta},\boldsymbol{\theta},z=0)\rangle_{\mathrm{dif}} = F(\boldsymbol{\theta}-\boldsymbol{\theta}_{\mathrm{ref}}(\boldsymbol{\rho}+z\boldsymbol{\theta})). \tag{55}$$

For small variations of $\boldsymbol{\theta}_{\mathrm{ref}}$

$$\boldsymbol{\theta}_{\mathrm{ref}}(\boldsymbol{\rho}+z\boldsymbol{\theta}) \cong \boldsymbol{\theta}_{\mathrm{ref}}(\boldsymbol{\rho}) + z\boldsymbol{\theta}\nabla_{\boldsymbol{\rho}}\boldsymbol{\theta}_{\mathrm{ref}}(\boldsymbol{\rho}) \tag{56}$$

and

$$\langle I(\boldsymbol{\rho}+z\boldsymbol{\theta},\boldsymbol{\theta},z=0)\rangle_{\mathrm{dif}} \cong F\left[\theta_{\mathrm{x}}\left(1-z\left(\frac{d\theta_{\mathrm{ref}}}{dx}\right)\right)-\boldsymbol{\theta}_{\mathrm{ref}}(\boldsymbol{\rho}),\theta_{\mathrm{y}}\right], \tag{57}$$

where the x axis is directed along $\boldsymbol{\theta}_{\mathrm{ref}}$. Using this expression we can obtain the following relations:

Flux density variations due to the refractive scintillation are determined by

$$\Delta H = z\left(\frac{d\theta_{\mathrm{ref}}}{dx}\right)\langle H\rangle \tag{58}$$

Frequency scale f_{diff} variations correlate with flux density variations

$$\Delta f_{\mathrm{diff}}\langle f_{\mathrm{diff}}\rangle \cong \frac{(\Delta H)}{\langle H\rangle} \tag{59}$$

Temporal scale variations partly correlate with flux density variations

$$\Delta t_{\mathrm{diff}}/\langle t_{\mathrm{diff}}\rangle \cong \frac{(\Delta H)\mathrm{Cos}\alpha}{\langle H\rangle} \tag{60}$$

where α is angle between $\boldsymbol{\theta}_{\mathrm{ref}}$ and velocity $\mathbf{V}$.

For extended medium, the correlation between ΔH, Δf_{diff} and Δt_{diff} is weak [24].

6.1 *Slanting features in dynamic spectrum*

For the case of phase screen in the presence of strong angular refraction a displacement angle of the source can be compensated for by the differential angular refraction. An observer can see the same point of the diffractive pattern during long time if he is moving along the characteristic line [16]

$$\Delta f = \left(\frac{df}{dt}\right)\Delta t \tag{61}$$

This effect leads to slanting features or frequency drift in the dynamical spectrum. This phenomenon can also be explained by the two-beam interference model. If a relative displacement of two beams is greater than the size of the scattering disc $R_{\mathrm{sc}} = z\theta_0$ then the interference of fields of these beams can modulate the diffraction pattern. Let us take the sum of fields in the form

$$E(\boldsymbol{\rho}, f) = E_1(\boldsymbol{\rho}, f)\exp[-iS_1(\boldsymbol{\rho}, f)] + E_2(\boldsymbol{\rho}, f)\exp[-iS_2(\boldsymbol{\rho}, f)] \tag{62}$$

Here $E_{1,2}(\boldsymbol{\rho}, f)$ is determined by diffractive irregularities and $S_{1,2}(\boldsymbol{\rho}, f)$ is an additional phase that is determined by large-scale inhomogeneities. The partly averaged correlation function of flux density fluctuations is given by

$$\begin{aligned} B_{\mathrm{H,dif}}(\boldsymbol{\rho}, \Delta f) &= \langle H(\boldsymbol{\rho}_1, f_1)H(\boldsymbol{\rho}_1 + \boldsymbol{\rho}, f_1 + \Delta f)\rangle_{\mathrm{dif}} - \langle H^2\rangle_{\mathrm{dif}} \\ &= B_{\mathrm{H,dif,1}}(\boldsymbol{\rho}, \Delta f) + B_{\mathrm{H,dif,2}}(\boldsymbol{\rho}, \Delta f) \\ &\quad + 2B_{\mathrm{E,dif,1}}(\boldsymbol{\rho}, \Delta f)B_{\mathrm{E,dif,2}}(\boldsymbol{\rho}, \Delta f)\cos[k(\boldsymbol{\rho}\theta_{\mathrm{ref}}) - \Delta f/\Delta f_{\mathrm{ref}}] \end{aligned} \tag{63}$$

Here $B_{\mathrm{E,dif,1,2}}(\boldsymbol{\rho}, \Delta f)$ is two-frequency coherence function that is discussed above and $B_{\mathrm{H,dif,1,2}}(\boldsymbol{\rho}, \Delta f)$ is a spatial-frequency flux correlation function that is determined by an equation similar to (48). The refraction angle $\boldsymbol{\theta}_{\mathrm{ref}}$ and the refraction frequency scale Δf_{ref} are given by

$$\begin{aligned} \boldsymbol{\theta}_{\mathrm{ref}} &= (1/k)\nabla_\rho[S_1(\boldsymbol{\rho}, f) - S_2(\boldsymbol{\rho}, f)] \\ \Delta f_{\mathrm{ref}} &= \left\{\left(\frac{d}{df}\right)[S_1(\boldsymbol{\rho}, f) - S_2(\boldsymbol{\rho}, f)]\right\}^{-1} \end{aligned} \tag{64}$$

6.2 *A spatial-frequency spectrum*

To analyze the effect of refractive scintillation on speckles of diffractive patterns it is convenient to use the next spectral function

$$M_{\mathrm{Sp,2}}(\boldsymbol{\theta}, \tau) = (1/2\pi)^2\int d^2\boldsymbol{\rho}\int df\,\exp[-i(k\boldsymbol{\theta}\rho) - 2\pi i\tau f]B_{\mathrm{H,dif,1}}(\boldsymbol{\rho}, \Delta f) \tag{65}$$

This spatial-secondary frequency spectrum of the diffraction pattern corresponds to measurement of the brightness distribution of the scattered pulse by antenna with a diagram that is limited by the scattering angle. Here spatial frequency $\mathbf{q}$ measured in k is equivalent to the direction $\mathbf{q}/k = \boldsymbol{\theta}$ and secondary frequency τ is equivalent to the time delay. For the case of the power law spectrum of turbulence and for the large angles $|\boldsymbol{\theta}| \gg \theta_{\mathbf{scat}}$ we can use single scattering approximation and obtain the next relation

$$M_{\mathrm{Sp,2}}(\boldsymbol{\theta}, \tau) \propto [\langle G_{\mathrm{H}}(\mathbf{r} = 0, \mathbf{r}_1)\rangle_{\mathrm{dif}}\langle G_{\mathrm{H}}(\mathbf{r}_1, \mathbf{r})\rangle_{\mathrm{dif}}]^2\Phi_{\mathrm{Ne}}\left[\mathbf{q}_\perp = k\left(\frac{z}{z_1}\right)\boldsymbol{\theta}_1(\mathbf{r}_1), q_\parallel = 0; \mathbf{r}_1\right] \tag{66}$$

Factors $\langle G_H(0, \mathbf{r}_1)\rangle_{\mathrm{dif}}$ and $\langle G_{\mathrm{H}}(\mathbf{r}_1, \mathbf{r})\rangle_{\mathrm{dif}}$ describe flux modulations due to refractive scintillations. Here $\mathbf{r}_1 = (\boldsymbol{\rho}_1, z_1)$ is the position of the scattering point and $\boldsymbol{\rho}_1, z_1$ are determined by the equations

$$\begin{aligned} \boldsymbol{\rho}_1 &= (z - z_1)\boldsymbol{\theta} \\ \tau &= \left(\frac{1}{2}\right)z\left(\frac{z}{z_1 - 1}\right)|\boldsymbol{\theta}|^2 \end{aligned} \tag{67}$$

$\theta_1(\mathbf{r}_1)$ is the local scattering angle

$$\theta_1(\mathbf{r}_1) = \boldsymbol{\theta}\left(\frac{z}{z_1}\right) + \Delta\boldsymbol{\theta}_{\mathrm{ref}}(\mathbf{r}_1) \tag{68}$$

where $\Delta\boldsymbol{\theta}_{\mathrm{ref}}(\mathbf{r}_1)$ is an additional angle determined by refractive inhomogeneities.

We see that the temporal–angular distribution of the intensity of the scattered pulse gives information on the spatial structure of the turbulence level. For the case of an extended medium this distribution is continuous with speckles determined by flux fluctuations due to refractive scintillation. For the case of phase screen, non-zero values of the function $M_{\mathrm{Sp},2}(\boldsymbol{\theta}, \tau)$ are concentrated in a surface that is determined by equation (65). Similar results can be obtained for the case of transmitting antenna.

If we measure dynamical spectrum (temporal–frequency structure of flux fluctuations) we can obtain the secondary spectrum of the dynamical spectrum [25]

$$S_2(f_{\mathrm{t}}, f_{\mathrm{f}}) = \int dt \int df \exp(-2\pi f_{\mathrm{t}} t - 2\pi f_{\mathrm{f}} \Delta f) B_{\mathrm{H,dif}}(t, \Delta f) \tag{69}$$

$M_{\mathrm{Sp},2}(\boldsymbol{\theta}, \tau)$ can be reduced to $S_2(f_{\mathrm{t}}, f_{\mathrm{f}})$ by the integration on angle component $\theta_\perp$ normal to velocity $\mathbf{V}$. The cases of an observer and source movements correspond to receiving and transmitting antennas.

For the case of a phase screen with strongly anisotropic inhomogeneities, enhanced intensity of the secondary spectrum concentrates near an arc structure that is determined by the relation [26]

$$\tau = \left(\frac{1}{2}\right) z \left(\frac{z}{z_1 - 1}\right) |\boldsymbol{\theta}|^2, \quad \theta_{\mathrm{V}} = \frac{2\pi f_{\mathrm{t}}}{kV} \tag{70}$$

where θ_{V} is the component along the direction of velocity $\mathbf{V}$.

The passing of rays through caustics can generate a similar structure. For example, if an observer (or a source) is placed near a focal line of a cylinder lens and if focusing dominates scattering we can see enhanced intensity of the secondary spectra near arc structure

$$\tau = z_{\mathrm{foc}} |\theta_{\mathrm{x}}|^2, \quad \theta_x = \frac{2\pi f_{\mathrm{t}}}{kV_{\mathrm{x}}} \tag{71}$$

where z_{foc} is the focal distance.

7. Asymmetry coefficient

The most important measured quantity is the scintillation index

$$m^2 = \langle (H - \langle H \rangle) \rangle^2 / \langle H \rangle^2 \tag{72}$$

where H is total flux density of a source. However, extragalactic sources have complex structures, consisting of a compact scintillating component and an extended non-scintillating component. Consequently, without knowing the flux density of the scintillating component H_{c}, it is not possible to determine the corresponding scintillation index, m_{c}:

$$m_{\mathrm{c}}^2 = \frac{\langle (H - \langle H \rangle)^2 \rangle}{\langle H_c \rangle^2} = m^2 \left(\frac{\langle H_{\mathrm{c}} \rangle}{\langle H \rangle}\right)^2 \tag{73}$$

It was proposed in [27,28] to overcome these difficulties using measurements of the asymmetry coefficient of the flux-fluctuation distribution

$$\gamma = \frac{\langle (H - \langle H \rangle)^3 \rangle}{\left[\langle (H - \langle H \rangle)^2 \rangle\right]^{3/2}} \tag{74}$$

As is shown in [28], γ is given by the relation

$$\gamma = Am_{\mathrm{c}} \tag{75}$$

The numerical coefficient is determined by the flux-fluctuation distribution. It depends on the shape of the turbulent spectrum and on the scintillation regime. For the Kolmogorv turbulent spectrum in the weak scintillation regime the flux-fluctuation distribution function follows a logarithmic normal law [28,29] and $A = 3$. For diffractive scintillation $A = 2$ and for refractive scintillation $A = 3$ [28].

To analyse the fluctuations on various scales it was proposed to use the asymmetry function [29]

$$\begin{aligned}
\gamma_{2,1}(t) &= \frac{-2\langle[\Delta_2(t)]^3\rangle}{\langle[\Delta_2(t)]^2\rangle\langle[\Delta_1(2t)]^2\rangle^{1/2}} \\
\Delta_1(\mathrm{t}) &= H(t_1 + t) - H(t_1) \\
\Delta_2(\mathrm{t}) &= H(t_1 + t) - 2H(t_1) + H(t_1 - t)
\end{aligned} \tag{76}$$

The function $\gamma_{2,1}(t)$ is related to the function $[D_{\mathrm{H}}(2t)]^{1/2}/\langle H_{\mathrm{c}}\rangle$ by the formula

$$\gamma_{2,1}(t) \cong \frac{A_{2,1}[D_{\mathrm{H}}(2t)]^{1/2}}{\langle H_{\mathrm{c}}\rangle} \tag{77}$$

The factor $A_{2,1}$ is weak function of t. For large $t \gg t_0$

$$\gamma_{2,1}(t) \cong 2^{1/2} \quad \gamma = 2^{1/2}Am_{\mathrm{c}} \tag{78}$$

The distribution function of small-scale flux-fluctuations of weak and diffractive scintillation follows the Rice–Nakagami distribution function [30]. Using this distribution function we obtain for a small time lag $t \ll t_0$, $A_{2,1} = 3/2$. For refractive scintillation $A_{2,1} = 3$ [30].

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