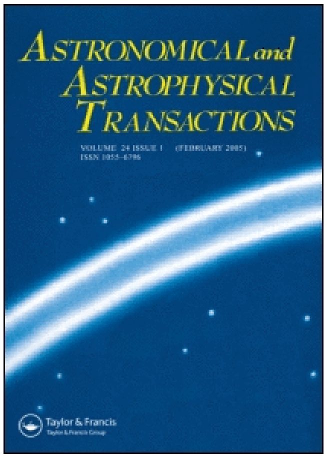

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Influence of additional dimensions on the dynamics of the expansion of the Universe

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We consider the possibility of the influences of additional dimensions on the dynamics of the expansion of our Universe. It is shown that this influence is expressed only through the anisotropy parameter of additional dimensions. In the isotropic (in relation to additional dimensions) case the dynamics of our world remains constant. All calculations were carried out for a single-component ideal fluid with power diagonal metrics. The dynamic characteristics of the scale factor have allowed us to draw a conclusion about the physical nature of the initial substance.

Keywords: Additional dimensions; Cosmology; Dark matter; Expansion of the Universe; Multi-dimensional gravitation; Universe

1. Introduction

The general theory of relativity has opened up new ways of studying the properties of the world in space scales [1]. These emerging opportunities were specified for the first time by Albert Einstein in 1917. Cosmological decisions allow us to consider the world as a whole. When studying the Universe on large scales [2, 3] it is necessary to digress from local inhomogeneities [4], connected with the distribution of substance in stars and stellar systems. The density of weights is taken to mean the average density in areas of space whose sizes considerably exceed the distances between galaxies. The isotropic cosmological model was described for the first time by A. Friedmann in 1922 [5–7], based on the assumption of the isotropy and uniformity of the distribution of substances in space. The basic property of this model is the non-stationarity of the Universe [8, 9]. After the discovery in spectra of far galaxies of red shift [10], the expansion of the Universe is considered to be a fundamental fact [11] and can serve as a starting point for the construction of new cosmologies or studying properties of the Universe in the future, i.e. for the description of its evolution it is necessary to know about the properties of the substance filling it. Initially this substance was thought of as an usual substance, and the choice of this or that model depended on the average density of this substance in the Universe. The discovery of dark matter [12, 13], and then the accelerated expansion [14, 15], have essentially

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expanded our opportunities in the choice of filling substance. The properties of our world were earlier investigated within the framework of the model of the single-component ideal fluid [16], necessary for the maintenance of the accelerated expansion in three-dimensional isotropic space under conditions of the existence of additional dimensions. It was proved that in the case of isotropy additional dimensions (without dependence from their number), do not appear to be any influence on expansion of the Universe. In this work the general case is considered in relation to additional dimensions [17–19]. It is proved that the influence of additional dimensions in our world is expressed only through the parameter of their anisotropy. The dynamic characteristics of the Universe are also studied, depending on the parameters of the equations of state. Though additional dimensions have not yet been found, their existence does not contradict both observant given, and to modern theoretical constructions [20, 21]. Our three-dimensional world can be considered as a projection of multidimensions – the so-called 'brane world' [22–24]. Now it is considered [25, 26] that spaces of additional measurements can be compact ($r \simeq 10^{-17}$ cm) or flat. This multidimensional model enables us to resolve the well-known paradox: 'if the god can all, whether he can destroy'. To destroy itself to the god there is no necessity. It is enough for him to get the properties making him not observable in our three-dimensional world. As in the case of isotropy of additional dimensions (without dependence on its number), and in its full absence, the dynamics of the expansion of the Universe remains constant. These models can be compared to the concepts of pantheism (God is nature) and atheism (God is not present). It is enough for God to create the world and further to not interfere in any way with its development. Only an infringement of the balance of additional dimensions (the occurrence of their anisotropy) allows Him to interfere with the dynamics of our Universe. To change the properties of our three-dimensional world, it is enough 'to tighten up' the additional dimensions only a little. As the well-known English science fiction writer Arthur Clarke has said, 'the world is subject to physical laws, God can stop them or turn back' [27].

The picture of *The Offering of Abraham* by Rembrandt Harmenz van Rijn (1635) (figure 1) symbolically and figuratively reflects the mechanism of the influence of additional dimensions considered in this article (in this case an angel) on the dynamics of our world. Note that the angel can be seen, or can be invisible, but at separate moments is capable of influencing us without dependence from our will, including physically. To understand, and realise the mechanism of the influence of additional dimensions, it is useful to consider the offered picture closely in the beginning.

2. Description of the model

As in previous work [28], the Universe is set in the framework of cosmological models describing the evolution of N one-dimensional spaces at present as a single-component ideal fluid [29].

The metrics of model are

$$ds^2 = -dt_s^2 + \sum_{i=1}^{N} a_i^2(t_s)\varepsilon^i (dy^i)^2,$$

where t_s is the synchronous time, $\varepsilon^i = \pm 1$, $i = 1, \ldots, N$. The range of definition of the metric is $M = (t_-, t_+) \times R^N$.

The energy–momentum tensor

$$T = \mathrm{diag}(-\rho(t), p_1(t), \ldots, p_N(t))$$

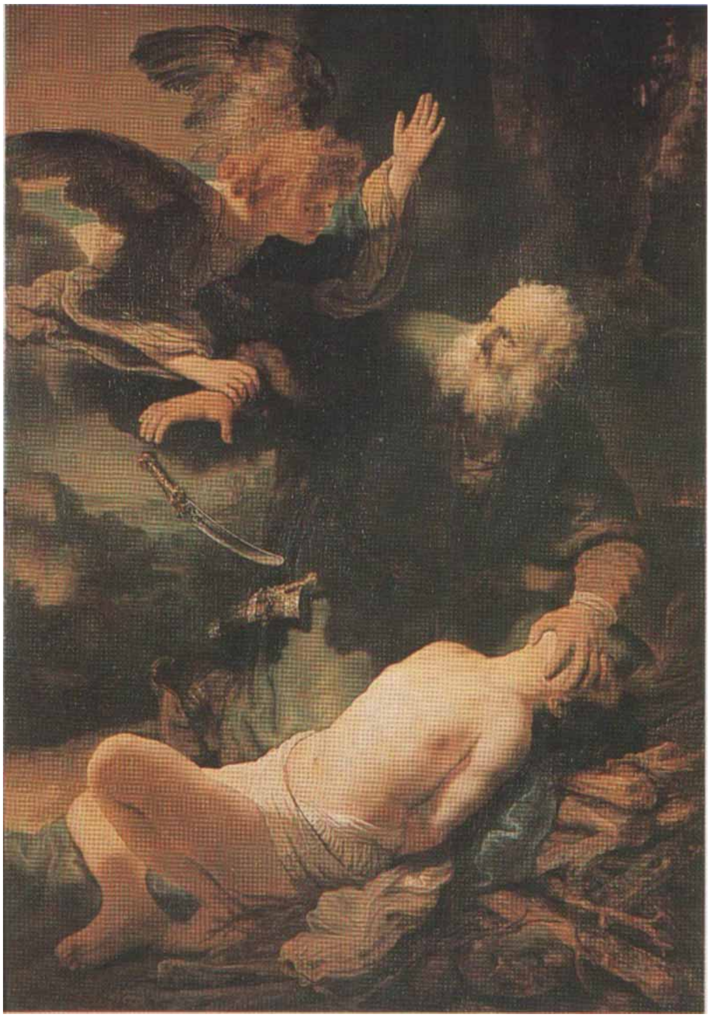

Figure 1. Rembrandt Harmenz van Rijn: *The Offering of Abraham* (1635). The Hermitage, St Petersburg.

describes an anisotropic fluid, where ρ is the density, and p_i is the pressure in ith one-dimensional space.

The pressure in each one-dimensional space is taken to be proportional to density. The scale factor looks like

$$a_i = a_i(t_s) = A_i t^{\nu_i}$$

where

$$\nu_i = 2u^i / \langle u^\Lambda - u, u\rangle$$

and $u^\Lambda = 2$ corresponds to a component of the Λ-term ($p = -\rho$).

$$u^i = \sum_{j=1}^{N} G^{ij} u_j$$

$$\langle u^{\Lambda} - u, u\rangle = \sum_{i,j=1}^{N} G^{ij}(2 - u_i)u_j$$

$$G^{ij} = \delta_{ij} + \frac{1}{2 - D},$$

where D is the total number of dimensions (including time),

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j. \end{cases}$$

3. Anisotropy of additional dimensions

We consider our Universe as isotropic three-dimensional space $u_1 = u_2 = u_3 = u$, $p = (1 - u)\rho$, with additional dimensions $u_4, u_5, \ldots$.

The purpose of this section is to deduce the dependence of the scale factor $a(t)$ of our Universe which is expressed through the exponents ν_1, ν_2, ν_3 from the equations of the state of additional dimensions.

As a starting point we take the condition of the accelerated expansion that is a logical continuation obtained before results [28] and gives to calculations a certain physical actuality. In section 4 the obtained result will be generalized on all possible scripts of the dynamic evolution of the Universe.

Let n be the number of additional dimensions; then $D = 1 + 3 + n$, $N = 3 + n$. As we have already said, the opportunity of influencing the dynamics of the Universe in the isotropic, in relation to additional dimensions, case was considered earlier. We shall now consider the anisotropic case. The parameters of the equations of the condition of internal spaces will be completely any.

From section 2 we deduce:

$$u^i = u_i + \frac{1}{2 - D}\sum_{j=1}^{N} u_j.$$

As is known, the world of our Universe is three-dimensional. Generalization of the equations of the general theory of relativity three-dimensional to the N-dimensional case is possible. But all the matter is that in the world of other dimension stably could not exist both atoms, and planetary systems [30]. Us studying real cosmological scripts interests, instead of the simple description of geometrical images such as N-dimensional sphere. Therefore we at once separate three dimensions which are responsible for dynamics of expansion of our Universe in the described N-dimensional model. We shall call the remaining $N - 3$ spatial dimensions 'additional'.

In the absence of additional dimensions dynamics of the Universe depends only on a kind of filling substance which is expressed through the equation of state $p = (1 - u)\rho = \alpha\rho$ (where p is the pressure, and ρ is the density). For an ideal gas $\alpha = 2/3$, for radiation $\alpha = 1/3$, for space dust $\alpha = 0$, and for the physical vacuum $\alpha = -1$ [32].

We consider our three-dimensional space to be isotropic, i.e. $u_1 = u_2 = u_3 = u$.

In the remaining spatial (additional) dimensions preliminary conditions of model, and as well as in our three-dimensional are imposed only, and in the generalized n-dimensional space we deal with the diagonal power metrics. The distinction between our spatial and additional dimensions is expressed only through parameters of the equation of state $u_4 \ldots u_N$. In our case they are completely any, i.e. $u_4 \neq \cdots \neq u_N$. Also it is necessary to remember the constancy of physical constants (which within the framework of the given article does not call into question) depends on the properties of additional dimensions.

Let us first find the sum $\sum_{i=4}^{N} u^i$, responsible for additional dimensions:

$$\sum_{i=4}^{N} \frac{1}{2-D} \sum_{j=1}^{N} u_j = \sum_{i=4}^{N} \frac{1}{2-D} \left(3u + \sum_{j=4}^{N} u_j \right) = \frac{N-3}{1-N} \left(3u + \sum_{j=4}^{N} u_j \right)$$

$$\sum_{i=4}^{N} u^i = \sum_{i=4}^{N} u_i + \frac{N-3}{1-N} \left(3u + \sum_{j=4}^{N} u_j \right).$$

From this

$$\sum_{i=4}^{N} u^i = \frac{1-N+N-3}{1-N} \cdot \sum_{i=4}^{N} u_i + 3u\frac{N-3}{1-N} = 3u\frac{N-3}{1-N} - \frac{2}{1-N} \sum_{i=4}^{N} u_i.$$

Having substituted in the derived expression for the condition of the constancy of the gravitational constant $G(t)$, finally we obtain:

$$G(t) = const \Longrightarrow \sum_{i=4}^{N} \nu_i(t) = 0 \Longrightarrow \sum_{i=4}^{N} u^i = 0$$

$$3u(N-3) = 2\sum_{i=4}^{N} u_i, \quad N-3 = n$$

$$\sum_{i=4}^{N} u_i = \frac{3}{2}nu \quad \forall u_4 \ldots u_N.$$

Thus, the required sum of parameters of the equations of state of additional dimensions $\sum_{i=4}^{N} u^i$ is expressed only through the equation of the condition of our isotropic space. The derived expression will not depend on the additional dimensions.

Let us add to the found expression the sum of the parameters of the equations of the condition of our three-dimensional space. We obtain

$$\sum_{i=1}^{N} u_i = 3u + \frac{3}{2}nu = \frac{3}{2}u(2+n)$$

$$u^i = u_i - \frac{1}{n+2} \sum_{j=1}^{N} u_j \quad [2-D = 2-4-n = -(n+2)]$$

$$u^1 = u^2 = u^3 = u - \frac{3u}{n+2} - \frac{1}{n+2} \sum_{j=1}^{N} u_j = u\left(1 - \frac{3u}{n+2}\right) - \frac{3}{2}\frac{nu}{(n+2)} = -\frac{1}{2}u$$

As we can see, u^1, u^2, u^3 are also expressed only through the parameter of the equation of state of our space u.

Let us now find $\langle 2-u, u\rangle$:

$$\langle 2-u, u\rangle = \sum_{i=1}^{N} u_i \cdot \frac{2-\sum_{i=1}^{N} u_i}{2-D} - \sum_{i=1}^{N} u_i^2$$

$$\sum_{i=1}^{N} u_i = 3u + \sum_{i=4}^{N} u_i = 3u + \frac{3}{2}\cdot nu = \frac{3}{2}\cdot u(2+n) \quad [2-D=-(n+2)].$$

Then

$$\langle 2-u, u\rangle = \left(\frac{3}{2}+\frac{9}{4}n\right)u^2 - 3u - \sum_{i=4}^{N} u_i^2.$$

For the special case $n=2$ (two independent additional dimensions)

$$\frac{3}{2}+\frac{9}{4}n = 6, \quad \langle 2-u, u\rangle = 6u^2 - 3u - (v_1^2+v_2^2) \quad [u_4 = v_1, u_5 = v_2].$$

The found expression for $\langle 2-u, u\rangle$ except for u still includes the sum $\sum_{i=4}^{N} u_i^2$, through which the influence of additional dimensions on our Universe is expressed.

Let us now enter the parameter of anisotropic additional dimensions Δ, which will allow us to exclude the sum which has appeared in our expression:

$$\Delta = \sum_{i=4}^{N} u_i^2 - \frac{1}{n}\left(\sum_{i=4}^{N} u_i\right)^2$$

$$\sum_{i=4}^{N} u_i^2 = \Delta + \frac{1}{n}\left(\sum_{i=4}^{N} u_i\right)^2 = \Delta + \frac{1}{n}\cdot\frac{9}{4}n^2u^2 = \Delta + \frac{9}{4}nu^2.$$

From this

$$\langle 2-u, u\rangle = \frac{3}{2}u^2 + \frac{9}{4}nu^2 - 3u - \Delta - \frac{9}{4}nu^2 = \frac{3}{2}u^2 + -3u - \Delta$$

Thus, we have proved that the influence of additional dimensions on the dynamics of our three-dimensional space (within the framework of the considered model) is expressed only through the parameter of their anisotropy Δ:

$$\nu_{1,2,3} = \frac{2u^{1,2,3}}{\langle 2-u, u\rangle} = \frac{-u}{(3/2)u^2 - 3u - \Delta} \quad \forall\, n.$$

The condition of the accelerated expansion (see also section 4) $\nu_{1,2,3} > 1$ will result in the following expression:

$$\frac{-u}{(3/2)u^2-3u-\Delta} > 1 \Longrightarrow \frac{u}{(3/2)u^2-3u-\Delta} < -1 \Longrightarrow \frac{u+(3/2)u^2-3u-\Delta}{(3/2)u^2-3u-\Delta} < 0.$$

Thus,

$$\frac{(3/2)u^2-2u-\Delta}{(3/2)u^2-3u-\Delta} < 0$$

$$u \in \left(\frac{2}{3}\left(1-\sqrt{1+\frac{3}{2}\Delta}\right),\ 1-\sqrt{1+\frac{2}{3}\Delta}\right) \bigcup \left(\frac{2}{3}\left(1+\sqrt{1+\frac{3}{2}\Delta}\right),\ 1+\sqrt{1+\frac{2}{3}\Delta}\right).$$

When $\Delta = 0$ we shall obtain $u \neq 0$, $u \in (4/3, 2)$.

For the equation of the condition

$$p = (1-u)\rho, \quad p = \alpha\rho, \quad \alpha = 1-u$$

$$\alpha \in \left(-\sqrt{1+\frac{2}{3}\Delta}, \quad +\frac{1}{3}-\frac{2}{3}\sqrt{1+\frac{3}{2}\Delta}\right) \bigcup \left(+\sqrt{1+\frac{2}{3}\Delta}, \quad +\frac{1}{3}+\frac{2}{3}\sqrt{1+\frac{3}{2}\Delta}\right).$$

When $\Delta = 0$ we obtain $\alpha \in (-1, -(1/3))$, which will be coordinated to the results obtained earlier [31, 32].

4. Dynamics of the Universe

Obviously, except for the accelerated expansion, there are other cases of dynamic development of the Universe. We shall first consider the isotropic case, in relation to additional dimensions.

The scale factor of the power metrics is $a_i \sim t_i^{\nu}$. For isotropic three-dimensional space $\nu_1 = \nu_2 = \nu_3 = \nu$, $a_1 = a_2 = a_3 = a = t^{\nu}$; $\dot{a} = \nu \cdot t^{\nu-1}$, $\ddot{a} = \nu(\nu-1)t^{\nu-1}$; $\dot{a} = 0$ for $\nu = 0$; $\ddot{a} = 0$ for $\nu = 0, 1$.

As a result the following scripts for the evolution of our Universe are found:

1. $\nu < 0 \rightarrow \ddot{a} > 0, \dot{a} < 0$ – accelerated compression.
2. $0 < \nu < 1 \rightarrow \ddot{a} > 0, \dot{a} < 0$ – slowed down expansion.
3. $\nu > 1 \rightarrow \ddot{a} < 0, \dot{a} > 0$ – accelerated expansion.
4. $\nu = 0 \rightarrow \ddot{a} = 0, \dot{a} \neq 0$ – uniform expansion.
5. $\nu = 1 \rightarrow \dot{a} = 0$ – 'stop'.

For the isotropic case, in relation to additional dimensions, ($u_4 = \cdots = u_n = v$, $\nu = 2/3(2-u)$) we obtain:

1. $\nu < 0$ accelerated compression $u > 2, \alpha < -1$
2. $0 < \nu < 1$ slowed down expansion $u < 4/3, u \neq 0, \alpha > 1/3, \alpha \neq 1$
3. $\nu > 1$ accelerated expansion $4/3 < u < 2, -1 < \alpha < -1/3$
4. $\nu = 0$ uniform expansion $u = 4/3, \alpha = -1/3$
5. $\nu = 1$ 'stop' $u = 0, \alpha = 1$

The obtained ranges of dynamic evolution of the Universe are fair both in isotropy of additional dimensions, and at full of them absence. Let us generalize the received results on any configuration of additional dimensions ($u_4 \neq \cdots \neq u_n$).

$$\nu = \frac{-u}{(3/2)u^2 - 3u^2 - \Delta} \quad \Delta = \sum_{i=4}^{N} u_i^2 - \frac{1}{n}\left(\sum_{i=4}^{N} u_i\right)^2.$$

Let us consider the numerical u and α, which result in different scenarios of dynamic evolution of the Universe.

1. Accelerated compression $\nu < 0$

$$u \in \left(1-\sqrt{1+\frac{2}{3}\Delta}, \quad 0\right) \cup \left(1+\sqrt{1+\frac{2}{3}\Delta}, \quad +\infty\right)$$

$\Delta = 0 \longrightarrow u > 2, u \neq 0$

$$\alpha \in \left(-\infty, \quad -\sqrt{1+\frac{2}{3}\Delta}\right) \cup \left(1, \quad \sqrt{1+\frac{2}{3}\Delta}\right)$$

$\Delta = 0 \longrightarrow \alpha < -1, \alpha \neq 1$

2. Slowed down expansion $0 < \nu < 1$

$$u \in \left(-\infty, \quad \frac{2}{3}\left(1-\sqrt{1+\frac{3}{2}\Delta}\right)\right) \cup \left(0, \quad \frac{2}{3}\left(1+\sqrt{1+\frac{3}{2}\Delta}\right)\right)$$

$\Delta = 0 \longrightarrow u < 4/3, u \neq 0$

$$\alpha \in \left(\frac{1}{3}-\frac{2}{3}\sqrt{1+\frac{3}{2}\Delta}, \quad 1\right) \cup \left(\frac{1}{3}+\frac{2}{3}\sqrt{1+\frac{3}{2}\Delta}, \quad +\infty\right)$$

$\Delta = 0 \longrightarrow \alpha > -(1/3), \alpha \neq 1$

3. Accelerated expansion $\nu > 1$

$$u \in \left(\frac{2}{3}\left(1-\sqrt{1+\frac{3}{2}\Delta}\right), \ 1-\sqrt{1+\frac{2}{3}\Delta}\right) \cup \left(\frac{2}{3}\left(1+\sqrt{1+\frac{3}{2}\Delta}\right), \ 1+\sqrt{1+\frac{2}{3}\Delta}\right)$$

$\Delta = 0 \longrightarrow u \in (4/3, 2)$

$$\alpha \in \left(-\sqrt{1+\frac{2}{3}\Delta}, \quad \frac{1}{3}-\frac{2}{3}\sqrt{1+\frac{3}{2}\Delta}\right) \cup \left(\sqrt{1+\frac{2}{3}\Delta}, \quad \frac{1}{3}+\frac{2}{3}\sqrt{1+\frac{3}{2}\Delta}\right)$$

$\Delta = 0 \longrightarrow \alpha \in (-1, -1/3)$

4. Uniform expansion $\nu = 0$

$$u = \frac{2}{3}\left(1+\sqrt{1+\frac{3}{2}\Delta}\right), \quad u = 1+\sqrt{1+\frac{2}{3}\Delta} \quad \Delta = 0 \longrightarrow u = \frac{4}{3}, \quad u = 2$$

$$\alpha = \frac{1}{3}-\frac{2}{3}\sqrt{1+\frac{3}{2}\Delta}, \quad \alpha = -\sqrt{1+\frac{2}{3}\Delta} \quad \Delta = 0 \longrightarrow \alpha = -\frac{1}{3}, \quad \alpha = -1$$

5. ‘Stop’ $\nu = 1$

$$u = \frac{2}{3}\left(1-\sqrt{1+\frac{3}{2}\Delta}\right), \quad u = 1-\sqrt{1+\frac{2}{3}\Delta} \quad \Delta = 0 \longrightarrow u = 0$$

$$\alpha = \frac{1}{3}+\frac{2}{3}\sqrt{1+\frac{3}{2}\Delta}, \quad \alpha = \sqrt{1+\frac{2}{3}\Delta} \quad \Delta = 0 \longrightarrow \alpha = 1$$

Different opportunities of dynamic development of the Universe are represented in figure 2.

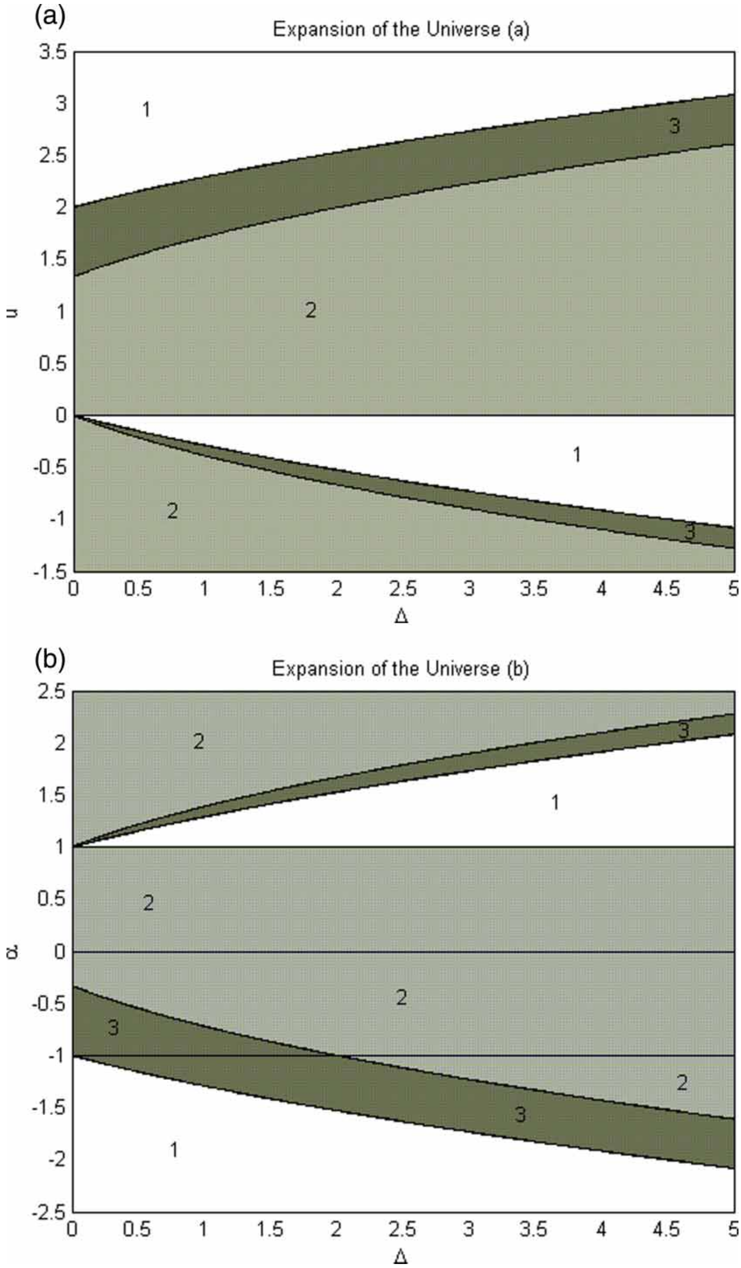

Figure 2. Dependence of the dynamics of the expansion of the Universe from a parameter of anisotropy in additional dimensions Δ and from parameters of the equation of state of a filling substance u (a) and α (b). 1 – Accelerated compression, 2 – Slowed down expansion, 3 – Accelerated expansion.

The rough development of the physics which has occurred on the boundary of the nineteenth and twentieth centuries [33] has allowed us to look at the achievements of the ancient Greek thinkers in a new fashion. The theoretical and experimental proof of the discrete structure of the substance has revived the doctrine Democritus about atoms [34]. It is possible to say that the ancient Greek atom ($\alpha\tau o\mu$) became the best known physical term of the twentieth century. On the other hand, after Einstein created the general theory of a relativity there was an actual revival in cosmology – the ancient doctrine about the cosmos ($\kappa o\sigma\mu o\varsigma$), as about finite, sensible object [35]. The boundary of the twentieth and twenty-first centuries was been marked by no less significant discoveries in physics and astronomy. The detection of dark matter and the accelerated expansion of the Universe allow us to say that for the dynamics of the Universe the usual substance is not determining. In this connection also it is pertinent to recollect ancient models that will allow us to avoid terminological confusion and, probably, will throw light on the further development of science [36, 37] for a designation of dark matter or dark energy, in may opinion, term of Greek thinkers of the Miletus School Anaximander (sixth century up to AD) apeiron [38] – unlimitary quite approaches.

5. Conclusions

1. The opportunity of the influence of additional dimensions on the dynamics of the expansion of the Universe is considered.
2. It is proved that the influence of the additional dimensions on our three-dimensional world in an obvious kind is expressed only through the parameter of their anisotropy.
3. Possible scripts of dynamic evolution of the Universe (which are investigated within the framework of model of a single-component ideal liquid) depend on parameters of the equation of the condition of the filling substance for isotropic and anisotropic (in relation to additional dimensions) cases.
4. An attempt at a metaphysical interpretation of the additional dimensions is made.

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