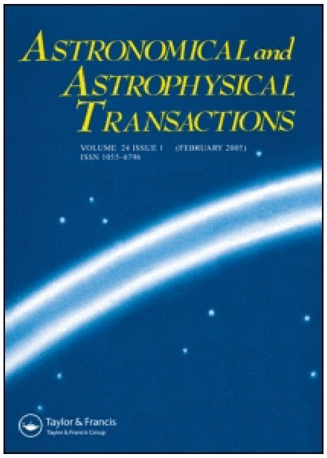

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 August 2007
To cite this Article: Nasonova, L. P. and Chujkova, N. A. (2007) 'Anomalies of an external and internal gravitational field of upper Earth layers in square law approximation', Astronomical & Astrophysical Transactions, 26:4, 361 - 369
To link to this article: DOI: 10.1080/10556790701592134
URL: http://dx.doi.org/10.1080/10556790701592134

Astronomical and Astrophysical Transactions
Vol. 26, Nos. 4–5, August–October 2007, 361–369

Anomalies of an external and internal gravitational field of upper Earth layers in square law approximation

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(Received 2 July 2007)

The explicit form for the quadratic contribution to the gravitational field from the dipole distributed anomalous masses is found. The anomalous masses are represented in the form of layers of variable height, arranged relative to the reference ellipsoid. The solution is reduced to the mathematical problem of finding an expression for the coefficients of expansion in terms of spherical harmonics of the square of any function that can be presented as a finite series of spherical harmonics, in terms of the coefficients of this initial series. The formulas have been calculated using mathematical modelling of symbol computation using computer algebra packages. The results obtained are illustrated using the example of the contribution from relief masses and density jumps on the Mohorovicic (Moho) discontinuity.

Keywords: Gravitational field of the Earth; Mohorovicic (Moho) discontinuity; Relief masses and density jumps ; Isostatic equilibrium; Anomalous masses

1. Introduction

The study of the global density structure of the Earth has shown that the lateral distribution of anomalous, i.e. not corresponding to the hydrostatic equilibrium, masses, basically has dipole character. That is, the peaks of relief correspond to the extrema of height of Moho surface (M) with opposite sign. Also, the anomalous masses of tops and bottoms of the Earth crust often correspond to the tops and bottoms of the upper Earth mantle with opposite sign [1, 2]. Consider the problem of linear contribution to the external or internal gravitational field from the laterally distributed anomalous masses. A linear relation [3] can be established between the coefficients of expansion ($\bar{C}_{nm}$, $\bar{D}_{nm}$) of gravity potential (i.e. force function taken with opposite sign), in terms of spherical functions, and those of the anomalous masses represented in form of a simple spherical layer ($\bar{a}_{nm}$, $\bar{b}_{nm}$):

$$\begin{Bmatrix} \bar{C}_{nm} \\ \bar{D}_{nm} \end{Bmatrix} = \frac{3}{2n+1} \left(\frac{R}{R_0} \right)^3 \frac{\Delta\sigma}{\bar{\sigma}} \begin{Bmatrix} \bar{a}_{nm} \\ \bar{b}_{nm} \end{Bmatrix},$$

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Astronomical and Astrophysical Transactions
ISSN 1055-6796 print/ISSN 1476-3540 online © 2007 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/10556790701592134

where R, $\Delta\sigma$, R_0, $\bar{\sigma}$ are mean radii and densities of a simple Earth layer and all Earth, respectively. Here the following representation for the external and internal gravity potential of an Earth layer is used:

$$V_e(r,\varphi,\lambda) = \frac{fM_0}{r}\sum_{n=1}^{N}\left(\frac{R}{r}\right)^n \bar{Y}_n(\varphi,\lambda), \quad V_i(r,\varphi,\lambda) = \frac{fM_0}{R}\sum_{n=1}^{N}\left(\frac{r}{R}\right)^n \bar{Y}_n(\varphi,\lambda),$$

$$\bar{Y}_n(\varphi,\lambda) = \sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{D}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin(\varphi)).$$

In these formulas $\bar{P}_{nm}(\sin(\varphi))$ are Caula – normalized spherical harmonics (Legendre's associated functions), $\{\bar{a}_{nm}, \bar{b}_{nm}\} = \{\bar{a}_{nm}, \bar{b}_{nm}\}_1$ are expansion coefficients of relative heights of the layer $h = H(\varphi,\lambda)/R$, in terms of surface harmonics.

In the linear approach the contributions of two dipole distributed simple layers with close heights are mutually compensated. However, in reality the anomalous masses are not simple spherical layers, but they are distributed within their height relative to the reference ellipsoid. In this case a quadratic approach, as shown in [4], the coefficients of the external gravity field satisfy the relation:

$$\begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix} = \begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix}_1 + \frac{n+2}{2}\begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix}_2 + \alpha(n+2)\begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix}_3, \tag{1}$$

and for the internal field:

$$\begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix} = \begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix}_1 - \frac{n-1}{2}\begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix}_2 + \alpha(n+2)\begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix}_3, \tag{2}$$

where the term in brackets $\{\ \}_2$ with lower index 2 corresponds to expansion coefficients of function $(h)^2$, and the term $\{\ \}_3$ to the expansion coefficients of function $h\bar{P}_2(\sin(\varphi))$, here $\alpha = (2/3)e$, e is the oblateness of the reference ellipsoid. It follows from (1) and (2) that the quadratic contribution $\{\ \}_2$ to dipoles, unlike linear $\{\ \}_1$ and ellipsoidal $\{\ \}_3$ contributions, is not compensated but is summed.

In this paper we present a method that establishes the analytical relations allowing one to express expansion coefficients $\{\bar{a}_{nm}, \bar{b}_{nm}\}_2$, $\{\bar{a}_{nm}, \bar{b}_{nm}\}_3$ through the linear terms $\{\bar{a}_{nm}, \bar{b}_{nm}\}_1$, and we present numerical results. These formulas have been calculated using mathematical modelling of symbol computation using computer algebra packages.

2. Problem Setting

The original presentation of some function $h = H(\varphi,\lambda)/R$ as a series of spherical functions of order $n \le N$ has the form:

$$h(\varphi,\lambda) = \sum_{n=1}^{N}\left(\sum_{m=0}^{n}\left(\bar{a}_{nm}\cos(m\lambda) + \bar{b}_{nm}\sin(m\lambda)\right)\bar{P}_{nm}(\sin\varphi)\right). \tag{3}$$

A similar expansion for h^2 can be presented as:

$$h^2(\varphi,\lambda) = \sum_{n=0}^{2N}\left(\sum_{m=0}^{n}(\{\bar{a}_{nm}\}_2\cos(m\lambda) + \{\bar{b}_{nm}\}_2\sin(m\lambda))\bar{P}_{nm}(\sin\varphi)\right). \tag{4}$$

For Legendres' associated functions $\bar{P}_{nm}(\sin(\varphi))$, normalized according to Kaula, we use the formula

$$\bar{P}_{n,m}(x) = K_{n,m} P_{n,m}(x), \quad K_{n,m} = \sqrt{\frac{\epsilon_m (2n+1)(n-m)!}{(n+m)!}}, \quad \begin{matrix} \epsilon_0 = 1 \\ \epsilon_{m>0} = 2 \end{matrix}. \tag{5}$$

The problem consists in expressing the expansion (4) coefficients $\{\bar{a}_{nm}\}_2$, $\{\bar{b}_{nm}\}_2$, through the coefficients $\{\bar{a}_{nm}\}_1 = \bar{a}_{nm}$ of the initial series for function $h(\varphi, \lambda)$. In order to solve this problem we have developed two different techniques. The first one consists in obtaining directly the decomposition of the product of finite series of elementary surface harmonics by symbolic integration. The second method is essentially based upon the Clebch–Gordon series [5]. We present below both methods in detail.

3. Expansion in Terms of Surface Harmonics

Consider the system of elementary surface harmonics $Y_{n,s}(x, \lambda)$, similar to functions $Y_{n,s}(\Theta, \lambda)$, $n = 0..2N$, $s = 0..2n$, from [3]:

$$Y_{n,s}(x, \lambda) = \bar{P}_{n,m}(x)\cos(m\lambda), \quad (s \le n, m = s),$$
$$Y_{n,s}(x, \lambda) = \bar{P}_{n,m}(x)\sin(m\lambda), \quad (s \ge (n+1), m = s - n).$$

The system of functions $Y_{n,s}(x, \lambda)$ forms an orthogonal system in the closed domain $-1 \le x \le 1$, $0 \le \lambda \le 2\pi$. Then expansion (4) can be written as:

$$h^2(\varphi, \lambda) = \sum_{n=0}^{2N} \sum_{s=0}^{2n} h_{n,s} Y_{n,s}, \quad \begin{matrix} h_{n,s\le n} = \{\bar{a}_{n,s}\}_2, \\ h_{n,s>n} = \{\bar{b}_{n,s-n}\}_2 \end{matrix}. \tag{6}$$

The coefficients $h_{n,s}$ of the expansion in terms of elementary spherical functions $Y_{n,s}$ are calculated by integration:

$$h_{n,s} = \frac{1}{Norm_{n,s}} \int_{x=-1}^{1} \int_{\lambda=0}^{2\pi} h^2 Y_{n,s} d\lambda dx, \quad x = \sin(\varphi). \tag{7}$$

where $Norm_{n,s} = 4\pi$ is the norm of functions $Y_{n,s}(x, \lambda)$.

Here in calculation (7) it is necessary to substitute in the expression instead of h^2, the square of the initial expression (3) for h in symbolic form, and to execute integration for any fixed value N.

In this way we obtain ane expression for the coefficients $\{\bar{a}_{n,m}\}_2$, $\{\bar{b}_{n,m}\}_2$ for $N \le 5$. In order not to exceed the operative memory, for greater values of N it is preferable to use a step-by-step method. Then, increasing N in the initial series by one, we do not have to recalculate all the items in coefficients $\{\bar{a}_{n,m}\}_2$, $\{\bar{b}_{n,m}\}_2$, only those taking into account the contribution of the additional terms in series (3) appearing due to the increase of N.

Notice that at such transition from N to $N + 1$ on each step the matrix of possible coefficients $\{\bar{a}_{nm}\}_2$, $\{\bar{b}_{nm}\}_2$, grows due to the index values $n = 2N + 1$, $n = 2N + 2$. This algorithm is essentially based upon the Clebch-Gordon's series [5].

The Clebch–Gordon decomposition is given in [5] for the product of two Legendres' associated functions $P_{n,m}(x) * P_{k,l}(x)$ with a different set of indexes in the form of a finite sum of functions $P_{n_1,m+l}(x)$ with numerical coefficients. A similar expansion for normalized functions has the form:

$$\bar{P}_{n,m}(x)\bar{P}_{k,l}(x) = \sum_{n_1=max(|n-k|,m+l)}^{n+k} \bar{S}_{n,m,k,l,n_1}\bar{P}_{n_1,m+l}(x), \tag{8}$$

where $\bar{S}_{n,m,k,l,n_1}$ will be called the Clebch–Gordon coefficients. The coefficients $\bar{S}_{n,m,k,l,n_1}$ can be obtained by direct integration as in (7):

$$\bar{S}_{n,m,k,l,n_1} = \frac{1}{Norm_{n_1,m+1}} \int_{x=-1}^{1} \int_{\lambda=0}^{2\pi} \bar{P}_{n,m}(x)\bar{P}_{k,l}(x)\cos((m+l)\lambda)Y_{n_1,m+l}d\lambda dx.$$

In the present work we also use another method to calculate the coefficients $\bar{S}_{n,m,k,l,n_1}$, which is more economical in numerical computation and uses less execution time. Using the known relation $P_{n,m}(x) = (1-x^2)^{m/2}(d^m/dx^m P_n(x))$, we obtain from the formula (8), using (5), a polynomial with respect to x, identically equal to zero for $-1 < x < 1$:

$$K_{n,m}\frac{d^m}{dx^m}P_n(x)K_{k,l}\frac{d^l}{dx^l}P_k(x) - \sum_{n_1=max(|n-k|,m+l)}^{n+k} \bar{S}_{n,m,k,l,n_1}K_{n_1,m+l}\frac{d^{m+l}}{dx^{m+l}}P_{n_1}(x) = 0.$$

Equating to zero the coefficients of this polynomial corresponding to the different powers of x, we get a system of ordinary linear equations on $\bar{S}_{n,m,k,l,n_1}$.

4. Expressions for the coefficients $\{\bar{a}_{nm}, \bar{b}_{nm}\}_2$, $\{\bar{a}_{nm}, \bar{b}_{nm}\}_3$

Let us denote through $h_{n,s}^{(N)}$ the coefficients of expansion (6), when initial expansion (3) for function h has been executed up to order N (the same for $N+1$). Then it follows from (7) $h_{n,s}^{(N+1)} = h_{n,s}^{(N)} + dh_{n,s}^{(N)}$, here at passage from N to $N+1$ we have to add to the factors $h_{n,s}^{(N)}$ the components $dh_{n,s}^{(N)}$, which can be expressed using (8):

1) for $s = 0$, (additives at calculation of coefficients $\{\bar{a}_{n,0}\}_2$):

$$\begin{aligned} dh_{n,0}^{N} = {} & \frac{1}{2}\int_{-1}^{+1} \bar{P}_{n,0}^2 dx \sum_{k=max(1,|N+1-n|)}^{N+1} \delta_k \bar{S}_{N+1,0,k,0,n}\bar{a}_{N+1,0}\bar{a}_{k,0} \\ & + \frac{1}{2}\sum_{p=0}^{2N+2}\sum_{j=0}^{[p/2]} \int_{-1}^{+1} \bar{P}_{p,2j}\bar{P}_{n,0}dx \sum_{k=max(1,j,|N+1-p|)}^{N+1} \delta_k \bar{S}_{N+1,j,k,j,p} \\ & \times (\bar{a}_{N+1,j}\bar{a}_{k,j} + \bar{b}_{N+1,j}\bar{b}_{k,j}), \end{aligned} \tag{9}$$

2) for $1 \le s \le n$, (additives at calculation of coefficients $\{\bar{a}_{n,m}\}_2$, $m = s$):

$$\begin{aligned} dh_{n,s}^{N} = {} & \frac{1}{4}\int_{-1}^{+1} \bar{P}_{n,s}^2 dx \\ & \times \sum_{l=\max(0,s-N-1)}^{\min(s,N+1)} \sum_{k=\max(1,s-l,|n-N-1|)}^{N+1} \delta_k \bar{S}_{N+1,l,k,s-l,n}(\bar{a}_{N+1,l}\bar{a}_{k,s-l} - \bar{b}_{N+1,l}\bar{b}_{k,s-l}) \end{aligned}$$

$$+\frac{1}{4}\sum_{j=0}^{\min([(2N+2-s/2)],\max(0,N+1-s))}\sum_{p=\max(s,s+2j)}^{2N+2}\int_{-1}^{+1}\bar{P}_{p,s+2j}\cdot\bar{P}_{n,s}dx$$
$$\times\left\{\sum_{k=\max(1,s+j,|N+1-p|)}^{N+1}\delta_k\bar{S}_{N+1,j,k,s+j,p}(\bar{a}_{N+1,j}\bar{a}_{k,s+j}+\bar{b}_{N+1,j}\bar{b}_{k,s+j})\right.$$
$$\left.+\left(\sum_{k=\max(1,j,|N+1-p|)}^{N+1}\delta_k\bar{S}_{N+1,j+s,k,j,p}(\bar{a}_{N+1,j+s}\bar{a}_{k,j}+\bar{b}_{N+1,j+s}\bar{b}_{k,j}\right)\right\}. \quad (10)$$

3) for $n < s \leq 2n$, (additional items at calculation of $\{\bar{b}_{n,m}\}_2$, $m=\bar{s}=s-n$):

$$dh^N_{n,s}=\frac{1}{4}\int_{-1}^{+1}\bar{P}^2_{n,\bar{s}}dx$$
$$\times\sum_{l=\max(0,\bar{s}-N-1)}^{\min(\bar{s},N+1)}\sum_{k=\max(1,\bar{s}-l,|n-N-1|)}^{N+1}\delta_k\bar{S}_{N+1,l,k,\bar{s}-l,n}(\bar{a}_{N+1,l}\bar{b}_{k,\bar{s}-l}+\bar{b}_{N+1,l}\bar{a}_{k,\bar{s}-l})$$
$$+\frac{1}{4}\sum_{j=0}^{\min([2N+2-\bar{s}/2],\max(0,N+1-\bar{s}))}\sum_{p=\max(\bar{s},\bar{s}+2j)}^{2N+2}\int_{-1}^{+1}\bar{P}_{p,\bar{s}+2j}\cdot\bar{P}_{n,\bar{s}}dx$$
$$\times\left\{\sum_{k=\max(1,\bar{s}+j,|N+1-p|)}^{N+1}\delta_k\bar{S}_{N+1,j,k,\bar{s}+j,p}(\bar{a}_{N+1,j}\bar{b}_{k,\bar{s}+j}-\bar{b}_{N+1,j}\bar{a}_{k,\bar{s}+j})\right.$$
$$\left.+\left(\sum_{k=\max(1,j,|N+1-p|)}^{N+1}\delta_k\bar{S}_{N+1,j+\bar{s},k,j,p}(-\bar{a}_{N+1,j+\bar{s}}\bar{b}_{k,j}+\bar{b}_{N+1,j+\bar{s}}\bar{a}_{k,j}\right)\right\}. \quad (11)$$

In these formulas we use the following notation: $\delta_k=\begin{cases}1, k\leq N\\ \frac{1}{2}, k=N+1\end{cases}$, $[p/2]=(p-p \bmod 2)/2$, here $[p/2]$ is the integral value, i.e. the integer nearest to the positive number $p/2$. In order to obtain definitive expressions for $h_{n,s}$, $(n=0..2N_{max},\ s=0..2n)$, it is necessary to sum up all the components $dh^{(N)}_{n,s}$, obtained through the passage from N to $N+1$, starting from the minimum value $N_0=[(n-1)/2]$, for which the coefficient $h_{n,s}$ appears, up to $N_{max}-1$, where N_{max} is the maximum order for initial expansion (3). Notice that for $n=0$, the formula (9) is reduced to a known result:

$$\{\bar{a}_{00}\}_2=\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\bar{a}^2_{nm}+\bar{b}^2_{nm}\right)=\iint(h_s)^2d\lambda d\sin\varphi.$$

Similarly, using (8), we get the coefficients $\{\bar{a}_{nm},\bar{b}_{nm}\}_3$, $n=0..N+2$, $m=0..\min(n,N)$, of the expansion of function $h(\varphi,\lambda)\bar{P}_2(\sin(\varphi))$, which enter in (1),(2):

$$\{\bar{a}_{n,m}\}_3=\sum_{k=\max(1,m,|n-2|)}^{\min(n+2,N)}\bar{S}_{2,0,k,m,n}\bar{a}_{k,m};\quad\{\bar{b}_{n,m}\}_3=\sum_{k=\max(1,m,|n-2|)}^{\min(n+2,N)}\bar{S}_{2,0,k,m,n}\bar{b}_{k,m}. \quad (12)$$

The absolute values of numerical coefficients in formulas (9)–(12) do not exceed 2.3.

We see that taking into account the squared terms from expansion of the relief surface of degree N brings an additional contribution to the harmonics of the potential of degree

$n = 0 \div 2N$, and the magnitude of this contribution increases with growth of n. The contribution to the null harmonic outlines the difference between the average radius of an isometric sphere and the average radius of a reference relief surface.

5. Numerical Results and Conclusions

In this paper we have presented a method for finding analytical expressions for the coefficients of expansion in terms of surface harmonics, of the square of a function with a couple of arguments ($x = \sin\varphi$; λ), written as a finite series of elementary spherical functions, through the coefficients of initial expansion. In order to implement this technique and get numerical results we developed a recurrent (with respect to the series order N) algorithm that we do not present here. The advantage of the approach, compared to direct integration (7), is that the former can be used efficiently for fast numerical computations for any value of order N. The latter one can be implemented only for small N, as, starting from $N = 6$, it rapidly leads to huge numerical operations exceeding the computer memory.

We obtained the final analytical formulas adapted for direct numerical computations for all significant terms up to $N_{max} = 9$. It allows one to estimate the contribution from the square terms to the Stokes' constant degrees $0 \div 18$. The results are illustrated using the example of the contribution from square terms of relief masses and density jumps on the (M) surface to the gravitational field of the Earth.

Figure 1 illustrates the dependence on the degree of expansion n (on abscissa) of the relative mean quadratic contribution from relief masses, density jumps on (M), and the summary contribution respectively:

$$\Delta\bar{V}_n = \frac{\bar{V}_n - (\bar{V}_n)_1}{(\bar{V}_n)_1} = \frac{\sqrt{D_n} - \sqrt{(D_n)_1}}{\sqrt{(D_n)_1}},$$

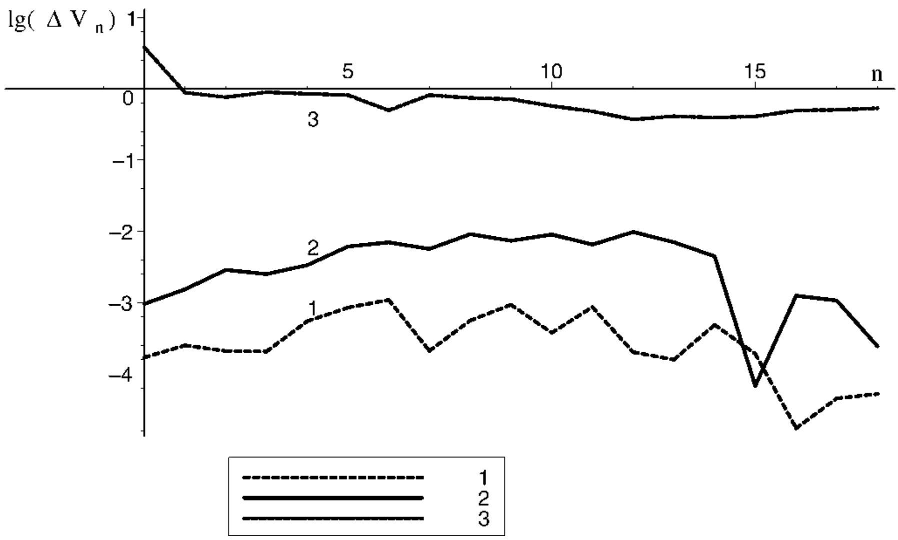

Figure 1. Dependence of relative mean quadratic contribution $\Delta\bar{V}_n$ from quadratic terms to exterior gravity potential on the series degree n.
1. for masses of the relief;
2. for jump density on (M) surface boundary;
3. for summary (relief + density jump) contribution.

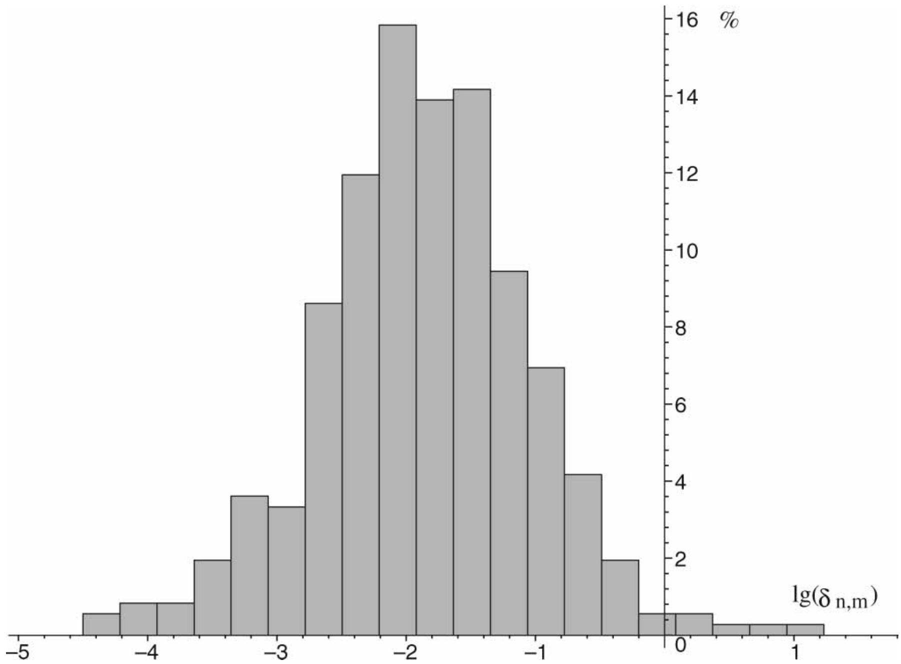

Figure 2. Histogram of distribution of the relative contribution to Stokes constants from the sum of quadratic terms for relief masses and density jumps on (M).

where $D_n = \sum_{m=0}^{n}(\bar{a}_{nm}^2 + \bar{b}_{nm}^2)$, here index 1 corresponding to linear approximation. We can see on figure 1, that the contribution from the square terms of the jump on (M) is greater by an order than the contribution from the relief. Also, the total contribution is approximately of the same order as the linear contribution for harmonics $n = 1 \div 9$, and slightly decreases (about twice) only for harmonics corresponding to the relief's incomplete isostatic compensation on (M) $(n = 6, 10 \div 18)$.

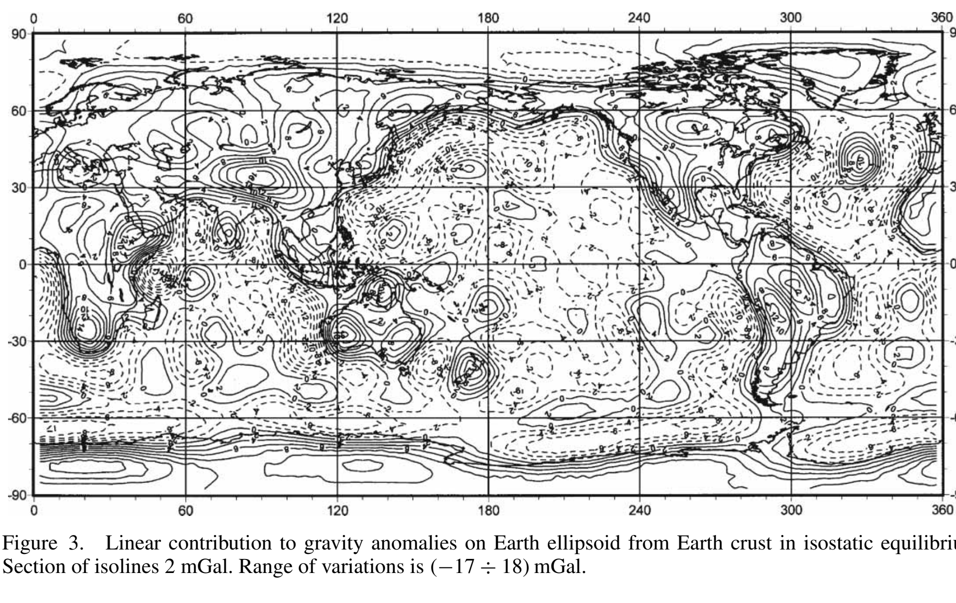

Figure 3. Linear contribution to gravity anomalies on Earth ellipsoid from Earth crust in isostatic equilibrium. Section of isolines 2 mGal. Range of variations is $(-17 \div 18)$ mGal.

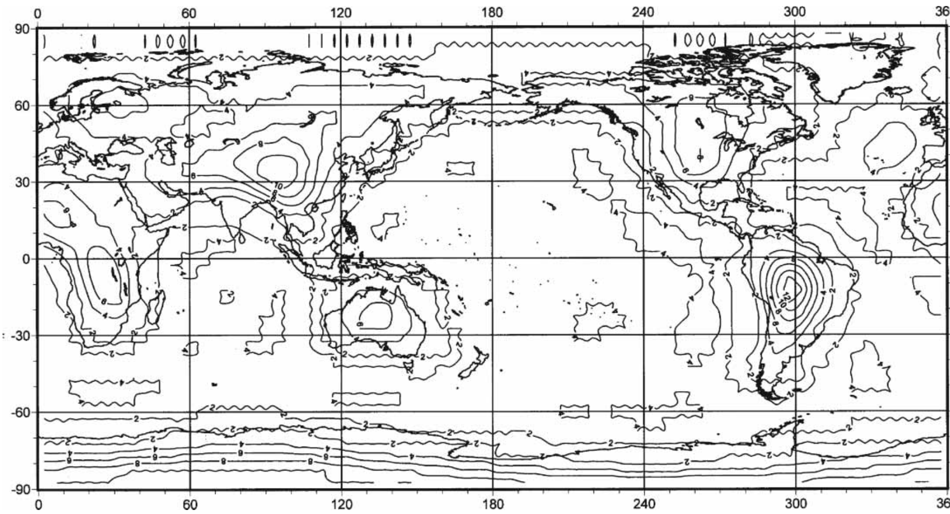

Figure 4. Quadratic terms contribution to gravity anomalies on Earth ellipsoid from relief masses and density jumps on (M). Section of isolines 2 mGal. Mean value 3.3 mGal, peak value 13 mGal.

Comparing formulas (1) and (2), it follows that contributions from square terms to external and internal gravity fields for $n > 1$ have approximately the same absolute values and opposite signs (for potential). Note that for gravity force the contibution of the squared terms to external and internal force have the same signs and differ only by factors $(n+1)(n+2)$ and $n(n-1)$ for external and internal fields, respectively. The contributions from linear terms and the terms taking into account the elliptic structure have opposite signs for external and internal gravity force, (unlike the potential, for which these contributions have the same sign).

Figure 2 presents a diagram of the distribution of relative contributions to Stokes constants from the sum of square terms for relief masses (r) and density jump on (M): $\delta_{nm} = \left|(n+2/2)\left\{\begin{matrix} a^r_{nm}+K a^M_{nm} \\ b^r_{nm}+Kb^M_{nm}\end{matrix}\right\}_2 \Big/ \left\{\begin{matrix} a^r_{nm}+Ka^M_{nm} \\ b^r_{nm}+Kb^M_{nm}\end{matrix}\right\}_1\right|$, where $K \approx 0.115$. It is apparent from figure 2,

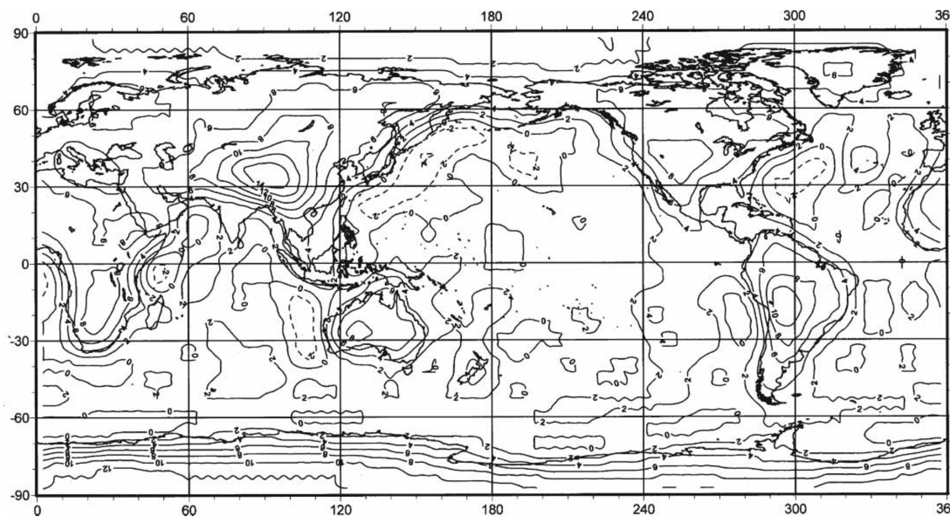

Figure 5. Summary contribution (linear and quadratic) to gravity anomalies at 500 km height from Earth crust in isostatic equilibrium. Section of isolines 2 mGal. Range of variations is $(-4 \div 16)$ mGal. Mean value 2.8 mGal, peak value for quadratic contribution 7 mGal.

that for several coefficients, the square contribution is greater than the linear contribution ($\delta_{nm} > 1$) (for $\bar{C}_{7,1}$, $\bar{C}_{11,1}$, $\bar{D}_{11,6}$, $\bar{C}_{12,0}$, $\bar{C}_{13,6}$,), and mostly ($\approx 57\%$) $\delta_{nm} > 0.01$, which is essential at the modern accuracy of Stokes' constants.

Figures 3 and 4 present maps of the total contribution to anomalies of the external gravity force on the terrestrial ellipsoid using the linear approximation (figure 3), and the quadratic approximation (figure 4), (expansion of degree $N = 18$, the ellipsoidal term is absent). It is apparent from the figures that the linear contribution is basically correlated with the heights of relief or with depths of (M), i.e. is positive for continents and negative for oceans. The quadratic terms' contribution is correlated with squares h_M^2, h_r^2, i.e. is positive everywhere. Besides, the order of magnitude can be compared to the linear contribution.

Therefore, taking into account only the linear approximation to interpret gravity anomalies may lead to incorrect estimation of the contribution of the crust boundaries to the Earth gravity field, and to incorrect estimation of the degree of correlation of these boundaries with gravity anomalies.

The contribution of squared terms is especially remarkable in the satellite zone, where for some regions its absolute value can exceed the linear contribution. Figure 5 presents a map of the total contribution of linear and squared terms to gravity force anomalies at a height of ≈ 500 km, taking as reference an ellipsoid similar to the terrestrial ellipsoid. Comparing figures 3 and 5 we see that for some regions the sum and linear contributions have opposite signs, whicht can considerably distort the interpretation of satellite data.

Acknowledgements

The present work has been partially supported by the Russian Foundation for Basic Research (project code 04-02-16681).

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