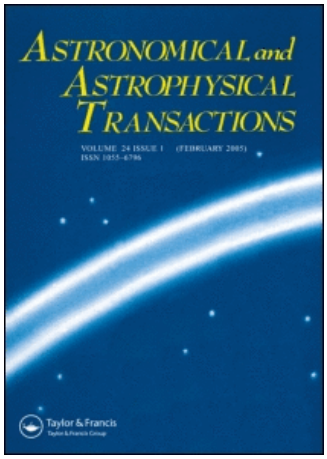

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The highly excited atom balance equation diffusion approximation

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The highly excited atom balance equation diffusion approximation

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Currently, hydrogen and carbon radio recombination lines are observed from HII and CII regions. To find the ionized gas physical conditions, the intensity interpretation analysis is carried out based on the balance equation analytical solutions. The balance equations are formulated accounting for the spontaneous and collision transition rates and photorecombination. The collision transition rate is calculated in an impact approximation applicable for the highly excited atom states with numbers $n > 50$. The balance equations are solved in a diffusion approximation for the departure coefficient flow-like functions, which depend upon the principal quantum number or that of a recombination line. The balance equation solution accuracy is determined by that of the transition rate. The diffusion equation initial condition is shown to be a function of the principal quantum number for which the non-equilibrium amplification coefficient has a maximum value. With the experimentally known state number with the maximum amplification coefficient, the ionized gas electron density can be found from the initial condition equation. The amplification coefficients analytically found are compared with those numerically determined and discussed. Interpreting the non-equilibrium line optical depths, the HII and CII region temperatures can be found, since the electron densities are calculated.

Keywords: Radio recombination lines; CII region; HII region; Highly excited atom populations

1. Introduction

The hydrogen and carbon radio recombination lines (RRLs) numbered $H_{40\alpha} - H_{42\alpha}$, $H_{92\alpha}$, $H_{165\alpha}$, $C_{201\alpha} - C_{273\alpha}$, $C_{570\alpha} - C_{686\alpha}$ and $C_{860\beta} - C_{868\beta}$ have been studied and show the kinematics of the HII and CII regions due to the Doppler line shift.

Extragalactic RRLs were first detected in 1977 [1]. The RRLs numbered $H_{92\alpha}$, $H_{166\alpha}$ were studied from 1987 to 2002 and gave information about the velocity field of an ionized gas in the starburst galaxies. These experiments motivated further interferometric observations of the compact HII regions in many galaxies. Hydrogen RRLs are observed from the galaxies Arp220, M82, M83, NGC 2146 and many others using the Very Large Array and IRAM 30 m telescope [1–5].

The lowest-frequency carbon RRLs were first detected as early as 1979 [6, 7]. In further experiments more than 50 lines were observed in the extended CII region towards Cassiopeia

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A from 1984 to 2001. The carbon centimeter RRLs numbered $C_{200\alpha} - C_{270\alpha}$ are broadened due to the Doppler effect. As follows from experiments, the lowest-frequency lines are broadened due to a combination of the Doppler, radiation and collision broadening effects. Since the Doppler linewidth component is experimentally found and the radiation-induced width component is calculated for the radiation temperature measured, the collision line width component can be determined and the electron volume density may be found theoretically by accounting for the impact approximation employed in the case $1.6 \cdot 10^5\,\mathrm{K}/(n^2 T_e) \ll 1, n > 50$. Interpreting the experimental intensity values of the radio lines numbered $C_{200\alpha} - C_{229\alpha}$, the CII region temperature can be determined, since the electron density is calculated and the CII region dimension is taken $R = 1000$pc. Current experiments are carried out with the instruments UTR-2 (Kharkiv), DKR-1000 (Puschino), GMRT (India) and VLA (USA) [6–12].

This paper aims to describe theoretically the RRL non-equilibrium intensities. The non-equilibrium intensities are functions of the principal quantum number, medium density and temperature. To calculate the density value, the collision transition rate is determined in an impact approximation and this approximation case is valid for $(1.6 \cdot 10^5\mathrm{K}/n^2 T_e) \ll 1$, where n is the principal quantum number. The balance equations are solved in a diffusion approximation, which is conditioned by the situation that the transition rates are the most probable from the nearest levels. The solutions found are applied to interpret the hydrogen and carbon RRLs observed in experiments.

The line intensity is known to be determined by the highly excited state population difference and spontaneous transition rate. The highly excited state populations are calculated with the spontaneous and collision transition rates in the case of the lines formed in an astrophysical plasma. The balance equations are studied and solved in [13–19]. There are two approaches to the balance equation solution: numerical solutions are tabulated in [16] and an analytical solution is based on the flux-like formula for the departure coefficients [18, 19]. For the analytical approach, the spontaneous and collision transition rates are written as smooth functions of the principal quantum number with strong dependence on the difference in the quantum numbers, since the principal quantum number changes. Describing the kinetic phenomena in this form, the balance equation is written as a Fokker–Planck equation [20]. To show that the principal quantum number change time is much longer than one transition time, the spontaneous transition rate is calculated as a function of the angular momentum quantum number. The transition rates for small angular momenta are as small as $(1/n^2) \to 0$. The most probable transition is that to the nearest level. The collision transition rate is calculated using the impact approximation. This approximation applicability and accuracy are discussed in [18, 19, 21, 22]. Since the steep function is $(1/\Delta n^4)$, it is evident that collisions slowly change the atom state. The balance equation in the diffusion approximation is a differential equation with the right-hand side in the form of photorecombination, and it can be integrated in closed form. The general solution can be determined by the initial condition, and the latter is determined in the form $(\partial b_n/\partial n)|_{n=n1} \sim (1/n1^2)$ and is a maximum for some state $n = n1$. The non-equilibrium factors $b_n\beta_n$ have maximum as functions of electron density and weakly depend upon temperature, the initial condition state $n1$ can be taken from the experimental data in which the maximum $b_n\beta_n$ factor is measured for the HII region of ionized gas. By using the initial condition equation the electron density can be found as $N_e \sim (n1)^{-7}$. From experiments using Cassiopeia A the maximum amplification coefficient state can be chosen as $n1 = 350 - 400$ to find the medium density [10, 11].

The calculation coefficients are compared with those already found [16]. The comparison results are shown in tables 1 and 2. The initial state $n1$ is calculated with accuracy better than 5% in table 3, but the general solution initial condition given in [16] is twice as large as the analogous value in this work. The collision diffusion coefficient is proportional to n^7, since

Table 1. The amplification coefficients for $T_e = 10^4$ K, $N_e = 10\,\mathrm{cm}^{-3}$.

n	1	2	3
50	10.2	10.5	13.9
75	22.8	25.2	38.5
100	85.2	50.0	83.1
125	169.0	79.4	139.2
150	210.0	92.0	167.0
175	112.2	77.4	143.0
200	145.2	54.3	102.5
225	107.2	36.1	69.0
250	87.5	24.2	46.5
275	65.0	16.6	32.1
300	53.7	12.0	22.7

Table 2. The amplification coefficients for $T_e = 10^4$ K, $N_e = 100\,\mathrm{cm}^{-3}$.

n	1	2	3
50	12.0	13.6	20.3
75	61.0	36.1	60.6
100	118.9	60.8	109.0
125	102.0	55.0	102.1
150	73.4	33.8	64.1
175	51.3	19.3	37.1
200	36.2	11.4	22.1
225	27.8	7.1	14.0
250	19.0	4.6	9.1
275	13.7	3.2	6.2
300	11.1	2.2	4.6

the collision cross-section is proportional n^4 [22, 23]. Then the diffusion approximation $b_n\beta_n$ factors should be proportional to $b_n\beta_n \sim (1/n^6)$ and half the analogous value given in [16] for the most highly excited levels with $n = 300$.

We show the applicability of the balance equation diffusion approximation by using M82 RRL observations. From the initial condition the HII region densities can be calculated with the initial atom state $n1 = 110 \pm 10$. It is the state on which the amplification coefficient has maximum value for the M82 gas. Since the electron densities are determined, the electron temperatures can be found from the optical depth values. The distance to M 82 is approximately 3 Mpc and the resolution angle is about $0.6''$ [4], thus estimating the HII region dimensions and calculating the RRL amplification coefficients, the region temperature is found from the generally accepted non-equilibrium optical depth formula.

The electron density is a steep function of the initial principal quantum number, its determination accuracy depends upon the initial condition state determination. The value $n1$ is

Table 3. The $n1$ parameter numerical and analytical calculation.

	1	2
$N_e = 10\,\mathrm{cm}^{-3}$, $T_e = 10^4$ K	145	150
$N_e = 100\,\mathrm{cm}^{-3}$, $T_e = 10^4$ K	105	100
$N_e = 10\,\mathrm{cm}^{-3}$, $T_e = 10^3$ K	130	125
$N_e = 100\,\mathrm{cm}^{-3}$, $T_e = 10^3$ K	90	95

assumed to be found with accuracy 10%, thus the electron density can be calculated with accuracy 50%.

The paper has five sections. Section 2 is devoted to the spontaneous and collision transition rates. In section 3 the diffusion approximation is described and the balance equation is brought to the diffusion form and solved. The initial condition is discussed and the solutions found are compared with the numerical analogous values. In section 4 the experimentally found RRL optical depths are interpreted and the HII and CII region electron densities and temperatures are calculated. Section 5 is devoted to the main conclusions.

2. The spontaneous and collision transition rates

The aim of this section is to show that the most probable spontaneous and collision transition rates are those for which the principal quantum number change is $n - n' = \Delta n = 1$. The spontaneous transition rate formula in the dipole approximation is well-known [24–26] and can be written as follows

$$A_{n,n+\Delta n} = \frac{4(\Delta n \omega_n)^3}{3\hbar c^3} e^2 r^2_{\Delta n}(n, \varepsilon),$$
$$\varepsilon = \sqrt{1 - \frac{(l + 1/2)^2}{n^2}}, \tag{1}$$

where ω_n is the transition frequency for the state with the principal and angular quantum numbers n, l; $r_{\Delta n}(n, \varepsilon)$ is the matrix element of the dipole component with the eccentricity ε; c is the velocity of light; $\hbar$ is the Planck constant; e is the electron charge.

If we assume that $l \sim n$ and due to transition the highly excited state changes $n - n' = \Delta n \sim n$, then $l' \sim 1$. Transitions with $l - l' \sim n$ are multi-pole and the most probable transition rates are proportional to the small parameter $(\Delta n \omega_n n^2/c)^{\Delta n} \sim (1/n)^n, n \gg 1$. In the other case of small l, l' $l \sim 1, l - l' = 1$ the most probable transition rate is small due to the little statistic weight of the small angular momentum number $A_{n,n+\Delta n} \sim (2l + 1/n^2)$.

It is most probable that the spontaneous transition rates are determined by the angular momentum $l \sim n$, and having summed up formula (1) with angular momentum, the spontaneous transition rate can be found

$$A_{n,n+\Delta n} = \frac{0.789 \cdot 10^{10}}{n^3 \Delta n}(s^{-1}), \quad \Delta n = 1, \tag{2}$$

where the values are determined in formula (1).

The highly excited atom has a large cross-section, which is proportional to $n^4, n \gg 1$ and thus the collision transition rates determine the non-equilibrium factors in the balance equations. Collisions are studied in the model of a highly excited atom and scattering electron with the interaction potential in a dipole approximation. This model cross-section is described in the impact approximation with an accuracy that is determined by the condition $(1.6 \cdot 10^5 \mathrm{K}/n^2 T_e) \ll 1, n \gg 1$. The scattering problem is considered in [17, 18, 21–23, 27]. The collision transition rate calculation is as follows

$$a_{\Delta n}(n, \varepsilon) = \frac{1}{\hbar} \int_{-\infty}^{\infty} dt V_{n,n+\Delta n} \exp(i \Delta n \omega_n t),$$
$$V_{\Delta n}(n, \varepsilon) = e^2 \frac{r_{\Delta n}(n, \varepsilon)}{(\rho^2 + v^2 t^2)} [\cos\theta \cos\theta' - \sin\theta \sin\theta' (\cos\phi - \sin\phi)],$$

$$\sigma_{\Delta n}(n,\varepsilon) = 2\pi \int_{\rho_0}^{\infty} \rho d\rho a_{\Delta n}^2(n,\varepsilon),$$

$$W_{\Delta n}^C(n) = \sum_{l=1}^{n} \frac{2l+1}{n^2} < v\sigma_{\Delta n}(n,\varepsilon) >,$$

$$W_{\Delta n}^C(n) = N_e \frac{4}{3\sqrt{2}\pi} 1.18 \left(\frac{m}{kT_e}\right)^{1/2} \left(\frac{\hbar}{m}\right)^2 n^4 \left[\ln\left(\frac{n^2 T_e}{1.6\cdot 10^5 \mathrm{K}}\right) + 1/2\right](s^{-1}),$$

$$\Delta n = 1, \frac{1.6\cdot 10^5 \mathrm{K}}{n^2 T_e} \ll 1, \tag{3}$$

where $a_{\Delta n}(n,\varepsilon)$ is the first-order perturbation theory coefficient; $V_{\Delta n}(n,\varepsilon)$ is the dipole potential with the atom radius $r_{\Delta n}(n,\varepsilon)$; $\theta\theta'\phi$ are the angular coordinates of the atom and scattering electron; ε is the atom eccentricity determined in formula (1); ρ, v are the target distance and scattering electron velocity; $\sigma_{\Delta n}(n,\varepsilon)$ is the atom cross-section; $W_{\Delta n}^C(n)$ is the collision transition rate summed up with the angular quantum number; N_e, T_e are the electron density and temperature, correspondingly; m is the electron mass; k is the Boltzmann constant.

The collision transition rate is calculated in the perturbation theory approximation, since in this approximation the coefficients $a_{\Delta n}(n,\varepsilon)$ are found in the case in which the atom electron velocity is smaller than that of the scattering electron.

Formulas (2) and (3) are written with semi-classical accuracy. For the highly excited states with the numbers $n = 50 - 100$ the Gaunt correction formula is $g_{n,n+\Delta n} = 1 - 0.1728(2^{2/3}/n^{2/3}\Delta n^{2/3})$ and is equal to 0.97. Thus the semi-classical matrix element approximation is carried out with good accuracy [15].

3. The balance equation diffusion approximation

The non-equilibrium population factors are known to be determined by the spontaneous and collision transition rates [13–19]. The highly excited electron source is photorecombination, since this process is not compensated by photoionization in the optically thin medium [26]. The balance equations can be written as follows

$$\sum_{\Delta n=1}^{\infty} A_{n+\Delta n,n} b_{n+\Delta n} \exp\left(\frac{E_n - E_{n+\Delta n}}{kT_e}\right) - b_n \sum_{\Delta n=1}^{\infty} A_{n,n-\Delta n}$$

$$+ \sum_{\Delta n=-\infty}^{\infty} W_{|\Delta n|}^C (n+\Delta n) b_{n+\Delta n} \exp\left(\frac{E_n - E_{n+\Delta n}}{kT_e}\right)$$

$$- b_n \sum_{\Delta n=-\infty}^{\infty} W_{|\Delta n|}^C(n) = W_n^{rec} S_n,\ W_n^{rec} = N_e \frac{64}{3}\sqrt{\frac{\pi}{3}} \frac{a_0 \hbar}{m} \alpha^3$$

$$\left(\frac{1.6\cdot 10^5 \mathrm{K}}{n^2 T_e}\right)^{3/2} \left[\ln\left(\frac{n^2 T_e}{1.6\cdot 10^5 \mathrm{K}}\right) + 0.5772\right],\ S_n = \frac{(T_e/1.6\cdot 10^5 \mathrm{K})^{3/2}}{4\pi^{3/2} a_0^3 N_e n^3},$$

$$N_n = b_n N_n^*,\ a_0 = 0.529\cdot 10^{-8}(\mathrm{cm}),\ \alpha = \frac{1}{137}, \tag{4}$$

where b_n is the departure factor from the LTE population value N_n^*; W_n^{rec} is the photorecombination rate calculated using Kramer's formula. The other parameters are described in formulas (2) and (3).

Since the spontaneous and collision transition rates are smooth functions of principal quantum number, they are expanded in a Taylor series, as can be shown by an example of the spontaneous transition rate as follows

$$A_{n+\Delta n,n}b_{n+\Delta n} \approx A_{n,\Delta n}b_n + \Delta n\frac{\partial}{\partial n}(A_{n,\Delta n}b_n) + \frac{(\Delta n)^2}{2}\frac{\partial^2}{\partial n^2}(A_{n,\Delta n}b_n) + \cdots,$$

$$B_n = \sum_{\Delta n=-\infty}^{\infty} \Delta n A_{n,\Delta n},\ D_n = \sum_{\Delta n=-\infty}^{\infty} \frac{(\Delta n)^2}{2} A_{n,\Delta n},$$

$$B_n = \frac{3.2\cdot 10^5 \mathrm{K}}{n^3 T_e} D_n,\ \frac{1.6\cdot 10^5}{n^2 T_e} \ll 1, \tag{5}$$

where the parameters are described in formulas (1) and (2).

Balance equation (4) is transformed to the diffusion approximation equation and solved in closed form as follows:

$$\frac{\partial}{\partial n}\left(D_n \frac{\partial b_n}{\partial n}\right) = W_n^{rec} S_n,$$

$$\frac{\partial b_n}{\partial n} = \left(\frac{w}{5n^2} + Cw\right)\frac{2}{1+2D_n^C},$$

$$w = \frac{64}{3}\sqrt{\frac{\pi}{3}}\frac{a_0\hbar}{m}\alpha^3 \frac{1}{4\pi^{3/2}a_0^3 0.789\cdot 10^{10}},$$

$$D_n^C = \sum_{-\infty}^{\infty} \frac{(\Delta n)^2}{2} W_{|\Delta n|}^C(n) \frac{n^3}{0.789\cdot 10^{10}}, \tag{6}$$

where C is the constant of the general solution, the other parameters are described in formulas (1) and (2).

The general solution constant can be determined in the form $\sim (1/n1^2)$, since the partial solution is proportional to $(1/n^2)$. The state $n1$ can be determined from the condition in which the spontaneous transition rate is equal to that of collision. For the state $n1$ the non-equilibrium amplification coefficients $b_n\beta_n$ are maximum. In this paper let us calculate $C = 1$. To calculate the number $n1$, the initial condition equation can be written for the maximum value of the $(\partial b_n/\partial n)$-factor as follows

$$2D_n^C|_{n=n1} = 1, \tag{7}$$

where D_n^C is found in formula (6)

There are well-known numerical solutions in [16]. Let us compare the numerical solution and that using formula (6).

We present the values calculated using formula (6) in tables 1–3.

In tables 1 and 2 $b_n\beta_n$-factors are given for the different densities: column 1 givesthe values found in [16]; in column 2 the values are calculated using formula (6) with constant $C = 1$; in column 3 the values are determined with constant $C = 2$.

In column 1, results from [16] are detailed for the different temperatures and densities. In column 2 the initial condition state numbers are calculated using formula (7).

It is evident that the initial condition state numbers chosen coincide. The insignificant difference for $n1$ is calculated with accuracy 5%. The amplification coefficient difference can be explained by the C constant choice. If we take $C = 2$, the maximum coefficients found analytically differ from those found numerically by 10%. Some difference for $n = 300$ may signify that in the numerical paper the collision transition rate is chosen in some other form.

4. The observed data interpretation

Currently, the hydrogen recombination lines are observed from the HII regions in extragalactic sources. These data are found by using the Very Large Array and IRAM 30 m telescope. The line intensities and widths are represented in [1–5]. Let us interpret the recombination line data for M82. The data represented in [4] are described in table 4.

Assuming the initial condition state number $n1 = 110$, it is easy to calculate the electron densities in the HII regions marked E_{2a}, E_{1a} in table 4 and accounting for formula (7). The region dimension is determined by taking into account that the source distance is $R_S \approx 3$Mpc and angular dimension is $0.6''$. The HII region temperatures may be calculated by the well-known formula in [28] as follows

$$\tau_L = 0.19 \cdot 10^{-3} \left(\frac{10^4\text{K}}{T_e}\right)^{5/2} N_e^2 R \frac{1}{\sqrt{\pi}\Delta\nu_L} b_n\beta_n, \tag{8}$$

where τ_L is the line optical depth; R is the region dimension; $\Delta\nu_L$ is the Doppler centimeter line width; $b_n\beta_n$ is the amplification coefficient. The other parameters are described in formulas (3), (6) and (7).

The calculation data are represented in table 5.

The parameters in table 5 are described in table 4 and formula (8).

There are five HII region data which are shown in [4] and used for interpretation. To find the electron densities, the E1a and E2a regions are chosen. The other region data of the line optical depths differ from those shown here by less than 20%. The average HII region electron densities calculated are found $N_e \approx 100\text{cm}^{-3}$for the five regions. The HII region temperatures are calculated with a better accuracy than that of the electron densities. In table 5 the amplification coefficients are ≈ 100, in [4] the same coefficients are smaller. Thus the electron temperature estimations differ. Since the region dimension is determined inaccurately, then the electron temperature will be found with inaccuracy greater than 100%. This inaccuracy is principal and the M82 electron temperatures cannot be calculated with higher accuracy.

The carbon RRLs in emission can be interpreted by using formulas (6,7). The experimental intensity and width values of the radio lines numbered $C_{200\alpha} - C_{229\alpha}$ are detailed in table 6, the CII region temperature may be calculated by formula (8), if the CII region dimension is taken $R = 1000$ pc, this choice is determined by the extremely low electron density and the necessity to get a high value for the $b_n\beta_n$-factors. The carbon observation experiments were carried out with the instruments UTR-2 (Kharkiv), DKR-1000 (Puschino), GMRT (India) and VLA (USA) [6–12].

Table 4. The RRL observation data.

	Coordinates (1950)	Peak Flux (mJy)	Width $\Delta\nu_L$(kms^{-1})
E_{2a}	$69°55'08''9^h51^m46^s$	0.57 ± 0.03	121 ± 37
E_{1a}	$69°54'00''9^h51^m42^s$	0.87 ± 0.07	90 ± 18

Table 5. The HII region data.

	$n1$	N_e(cm^{-3})	$b_n\beta_n$	R	T_e(10^4K)
E_{2a}	110	75	70	8.7	3.6
E_{1a}	110	75	70	8.7	4.4

Table 6. The carbon RRL observation data.

	$\tau_L(10^{-4})$	$\Delta\nu_L(\mathrm{kms}^{-1})$
G00-00	2.1	20.5
Cas A	3.8	4.1

Table 7. The CII region data.

	$n1$	$N_e(\mathrm{cm}^{-3})$	$b_n\beta_n$	R	$T_e(\mathrm{K})$
G00-00	400	0.016	10	1000	300
Cas A	350	0.04	10	1000	500

The parameters in tables 6 and 7 are described in formula (8).

The density and temperature data in tables 5 and 7 strongly depend upon the initial condition state number choice, which is written in the form to get high values for the $b_n\beta_n$-factors. From calculations using formulas (6) and (7) the beta lines numbered $H_{110\beta} - H_{112\beta}$ and $C_{350\beta} - C_{400\beta}$ are those with amplification coefficient higher than that of the alpha lines. These lines may be observed as follows from the alpha line optical depth values and thus it can be proved that the initial condition state number is chosen accurately and near its real value.

5. Conclusions

The velocity fields and physical conditions have been studied by ionized gas RRL observations. In this paper the non-equilibrium amplification coefficients are calculated to interpret the densities and temperatures of the compact HII regions and extremely large CII regions. To explain the non-equilibrium line emission, the balance equations are formulated and solved analytically in the diffusion approximation.

The spontaneous and collision transition rates determine the highly excited atom populations. With this condition the balance equations were studied numerically in [13–16, 29, 30]. There is an analytical approach to the balance equation solution based on the diffusion description of the population flows. The balance solutions are described analytically in [17–19, 26]. The diffusion approximation is based on the condition that the most probable transition rates are those which change the state n on n' with $\Delta n = 1, n \gg 1$. With this goal the radius matrix elements are calculated in section 2. It is shown that the most probable transitions are those with the angular momentum $l \approx n$, the principal quantum numbers change by the rule $\Delta n = 1$ as follows from formula (2).

Since the collision transitions determine the populations on the most highly excited levels, the impact approximation collision cross-sections with the transition rates are given by formula (3) in which the accuracy of the calculation method is determined by the condition for the temperature and level number $(1.6 \cdot 10^5\mathrm{K}/n^2T_e) \ll 1$.

Since the first diffusion coefficient is much smaller than the second $B_n = (3.2 \cdot 10^5\mathrm{K}/n^3T_e)D_n$, the diffusion approximation equation has a particularly simple form and its solution is the power function of principal quantum number by formula (6). The electron source is photorecombination, which is not compensated by ionization in the optically thin plasma. Thus the differential equation has the partial and general solutions. To chose the general solution, it is shown that the general solution should be written in the form $\sim (1/n1^2)$, where $n1$ is the initial condition state number for which the spontaneous transition rates are equal to those of the collision. The number $n1$ is calculated by equation 7.

The numerical approach is compared with analytical solution. In table 3 the initial condition state number is represented and differs from analogous value with the accuracy more than 5%. In tables 1 and 2 the amplification coefficients $b_n\beta_n$ are written with those of a numerical value in [16]. The analytical coefficients differ from those of the numerical value with accuracy 10% for the maximum amplification coefficients. A remarkable difference can be seen for $n = 300$. The analytical solution is much smaller due to the power function of electron flow for the most highly excited levels $b_n\beta_n \sim (1/n^6)$, as follows from formula (6).

Currently, hydrogen RRLs are observed from the HII regions in extragalactic sources. These data are observed by using the Very Large Array and IRAM 30 m telescope. The $H_{42\alpha}$, $H_{53\alpha}$ and $H_{92\alpha}$ line intensities and widths are detailed in [1–5]. We interpret the RRL data for M82. The data represented in [4] is described in table 4. In table 5 the electron densities and temperatures are represented for the ionized gas of the HII regions studied.

The carbon observation experiments are carried out with the instruments UTR-2 (Kharkiv), DKR-1000 (Puschino), GMRT (India) and VLA (USA) [6–12]. The carbon intensities of RRLs in emission can be interpreted by using formulas (6) and (7). The experimental intensity and width values of the radio lines numbered $C_{200\alpha} - C_{229\alpha}$ are given in table 6, the CII region temperature may be calculated by formula (8), since the CII region dimension is taken as $R = 1000$ pc, the extremely low electron densities and high temperatures are represented in table 7. According to the centimeter line intensity data the CII regions have extremely low electron densities and very large dimensions.

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