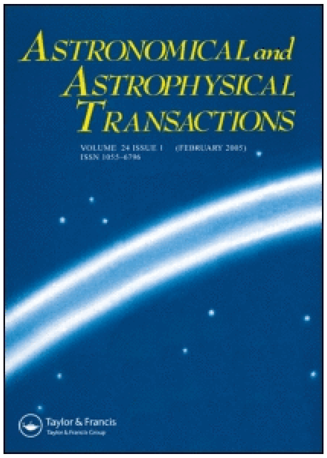

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Apsidal motion in eclipsing binary systems

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A review of work devoted to the problem of apsidal motion in eclipsing binary systems (EBSs) is given. It is shown that recent theoretical models for stars in the main sequence in general are in agreement with the observed apsidal motion velocities. However, the stars of types I–III show more central condensation than predicted by the theoretical models. It is revealed that the observed rates of the stellar photospheric rotation poorly reflect the real rates of axial rotation of greater part of stellar mass (core) and that cores of components of the EBS rotate, on average, more rapidly than their photospheres.

Keywords: Eclipsing binary systems; Apsidal motion; Star rotation

In this paper, which was pesented at the conference dedicated to the centenary of the famous Russian astrophysicist D.Ya. Martynov, the recent state of affairs concerning the solution of the problem of apsidal motion in eclipsing binary systems (EBSs) (Martynov had devoted a significant part of his life to the investigation of EBSs) is reported. This problem is as follows.

The tidal and rotational distortions of the stellar configuration induce the gravitational quadruple moments that cause the revolution of the orbit of close binary systems. Therefore the longitude of the periastron secularly advances if the orbit is eccentric.

Figure 1 displays the relative orbit of the primary component of an EBS. Because of the eccentricity of the orbit at the longitude ω of the periastron, from this figure the distance from primary minimum (denoted Min I) to the secondary minimum (denoted Min II) is more than from Min II to Min I and, owing to the different rates in the orbit, the widths of the minima (eclipses) differ from each other. When the orbit revolves, the form of the light curve changes and Min II oscillates cyclically about the orbital phase 0.5 with a period equal to that of apsidal motion.

Figure 2 shows clearly the changes in the duration of eclipses (the widths of minima) and separation of minima during about 60 years of observations for the eclipsing system RU Mon. Dubiago and Martynov [2] in 1929 discovered the apsidal motion in this system. The complete cycle of the orbit revolution in this system occurs during $U_{\rm obs} = 348$ years [3].

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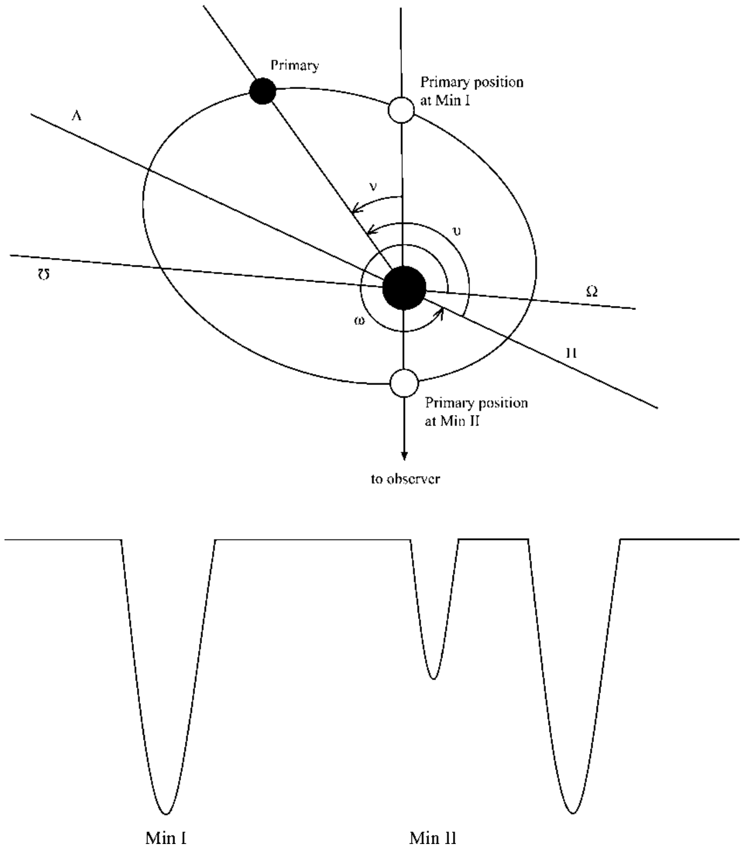

Figure 1. A schematic diagram of the relative orbit of the primary component of an EBS and of its light curve.

From the equations of motion it is easy to obtain the theoretical relations for the times T_1 and T_2 of the maximal phases of the eclipses of Min I and Min II respectively [4]:

$$T_{1,2} = T^0_{1,2} + EP_{\rm s} + \pi_{1,2}, \tag{1}$$

where

$$\pi_{1,2} = \mp 2\frac{P}{2\pi} e\cos\omega + \frac{P}{2\pi}e^2\frac{1+2(1-e^2)^{1/2}}{[1+(1-e^2)^{1/2}]^2}\sin(2\omega)$$
$$\pm\frac{2}{3}\frac{P}{2\pi}e^3\frac{1+3(1-e^2)^{1/2}}{[1+(1-e^2)^{1/2}]^3}\cos(3\omega), \quad \omega=\omega_0+\omega_{\rm obs}E, \quad \omega_{\rm obs}=2\pi\frac{P}{U_{\rm obs}}. \tag{2}$$

Equation (2) is correct for the orbital inclination $i = \pi/2$. If $i < \pi/2$, the small corrections appear as described in detail by Martynov [5]. Now by comparison, from the observed times of minima with their theoretical values, one can determine all the parameters in equations (1) and (2) and specifically the apsidal period $U_{\rm obs}$.

However, it is not always so easy. The pure apsidal motion is usually distorted by the other factors. The most typical example is the system RU Mon. Figure 3 shows the differences between the observed times O of minima and the theoretical values C computed by using linear elements. The dashed curves are the theoretical curves representing the apsidal motion in accordance with equation (2). From an analysis of the apparent deviations of the observed

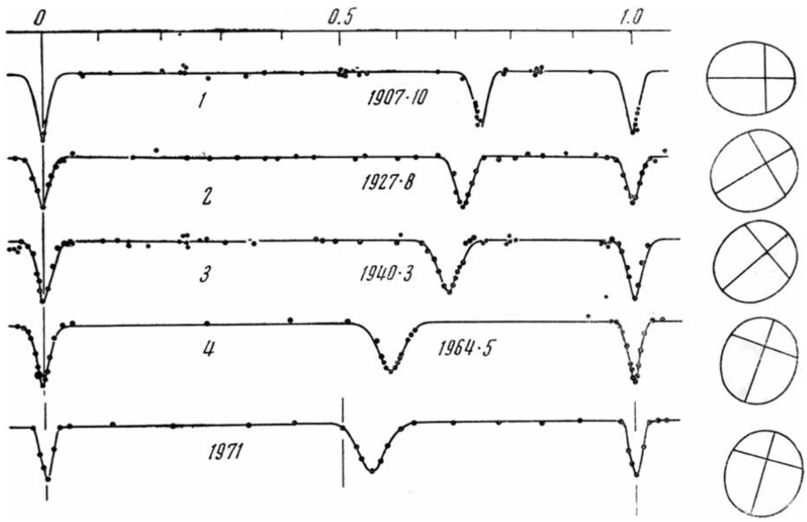

Figure 2. The progressive change in the light curve of RU Mon from 1907 to 1971 caused by the apsidal motion. On the right the positions of the orbit relative to the observer below are shown. This figure is taken from the paper by Martynov [1].

points from the theoretical points due to apsidal motion the presence of the third body was discovered in this system [3]. It should be noted that the presence of the third and the fourth components in the systems with apsidal motion is not an exception and such components are discovered in almost half of the systems that have been investigated well.

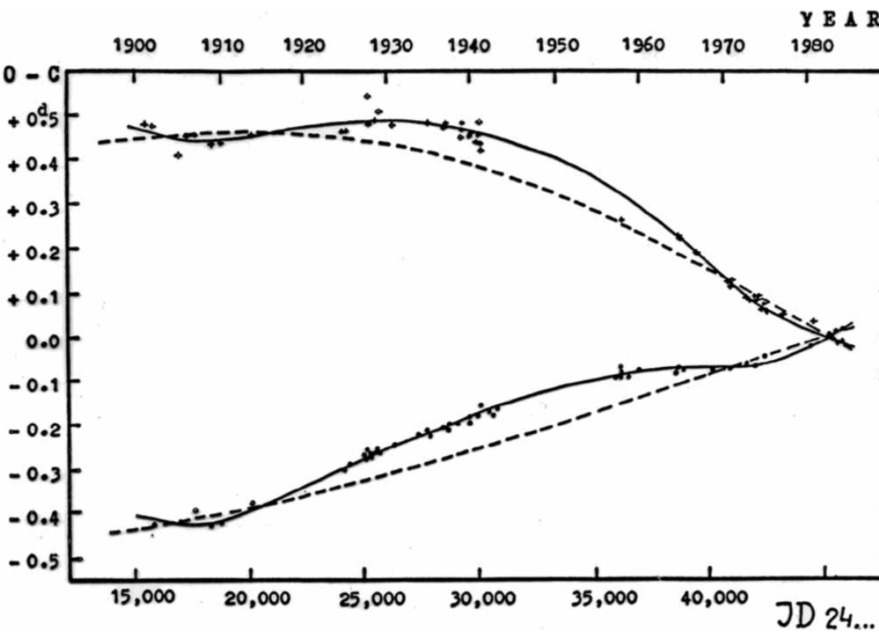

Figure 3. The differences between the observed times O of the minima of RU Mon and the theoretical values C computed by using linear elements, versus the Julian date (JD). The full circles represent the primary minimum, and the crosses the secondary minimum. The dashed curves are the theoretical curves representing the apsidal motion only; the solid curves are the theoretical curves representing both apsidal motion and the light equation due to the third body.

The apsidal periods for some EBSs are equal to thousands and tens of thousands of years and for these we cannot find the observed times of minima if only a quarter of the period is available for using equations (1) and (2) and for determining U_{obs}, ω, e, etc. In this case the parameters to be found can be determined from the combined analysis of several (two at least) of the light curves corresponding to different epochs for times separated as much as possible. The method employed for this analysis has been described by Martynov and Khaliullin [6] and Khaliullin [7].

So we can find U_{obs}, ω, e and the relative radii $r_1 = R_1/a$ and $r_2 = R_2/a$ of components from the analysis of the times of Min I and Min II and the light curves of the system in investigation. What else can we do?

Russel [8], Chandrasekhar [9], Cowling [10] and Sterne [11] elaborated the theory of the apsidal motion in binary systems due to the tidal and rotational distortions of the components. Martynov [4] and Kopal [12, 13] made great contributions to the solution of the problem of apsidal motion. The external potential of the perturbed stars can be expressed in terms of the apsidal motion parameters k_j, which were introduced by means of the relations

$$k_j = \frac{j + 1 - \eta_j(R)}{2[j + \eta_j(R)]}, \tag{3}$$

where the functions $\eta_j(r)$ are equal to zero in the centre of the star ($r = 0$) and are determined by means of the following differential equation:

$$r\frac{\mathrm{d}\eta_j}{\mathrm{d}r} + 6\frac{\rho(r)}{\bar{\rho}(r)}(\eta_j + 1) + \eta_j(\eta_j - 1) = j(j + 1). \tag{4}$$

Here the subscript j determines the order of the parameter, r is the mean radius of a given equipotential, $\rho(r)$ is the mass density at a distance r from the centre, $\bar{\rho}(r)$ is the mean density within a sphere of radius r, R is the radius of the hole star and therefore $\eta_j(R)$ is the value of the function η_j on the surface of the star. Equation (4) is frequently called the Radau equation and is usually solved by the Runge–Kutta method for the mass density distribution $\rho(r)$ obtained from the model computations of the stellar internal structure. The follow simple formula can be used for rough computations:

$$k_j = \frac{3(j + 2)}{2j + 1}\frac{1}{R^{2j+1}}\int_0^R \frac{\rho(r)}{\bar{\rho}(r)} r^{2j}\, \mathrm{d}r. \tag{5}$$

The last relation demonstrates the physical meanings of the widely used parameters k_j. Because of the close connection with $\rho(r)$, the parameters k_j are frequently called the internal structure constants. Only the only second-order apsidal parameters k_2 now have practical importance. By using k_2 the apsidal motion theory leads to the simple relation

$$\frac{P}{U_{\mathrm{cl}}} = C_1 k_{2,1} + C_2 k_{2,2}. \tag{6}$$

Here P is the orbital period and U_{cl} is the apsidal motion period due to tidal and rotational distortions of the components. The subscript of C_i and the second subscript of $k_{2,i}$ indicate that they belong to the primary ($i = 1$) or secondary ($i = 2$) component. The coefficients C_i

are determined by the geometrical and physical characteristics of the binary systems:

$$C_i = \left(\frac{R_i}{a}\right)^5 \left[\frac{M_{3-i}}{M_i} 15 f(e) + \left(\frac{\omega_{\mathrm{r},i}}{\omega_{\mathrm{k}}}\right)^2 \left(1 + \frac{M_{3-i}}{M_i}\right) g(e)\right], \tag{7}$$

where

$$f(e) = \left(1 + \frac{3}{2}e^2 + \frac{1}{8}e^4\right) \frac{1}{(1-e^2)^5}, \quad g(e) = (1-e^2)^{-2}. \tag{8}$$

Here $\omega_{\mathrm{r},i}/\omega_{\mathrm{k}}$ is the ratio of the angular velocity of the star's rotation to the average angular orbital velocity. In these relations the directions of the vectors of axial and orbital rotation are assumed to coincide. Kopal [13] and Shakura [14] considered the causes of departure from this condition and the effects that arise from that.

The orbit of an EBS revolves also owing to the relativistic effects, and the period U_{rel} of this motion is expressed as [15]

$$\frac{P}{U_{\mathrm{rel}}} = 6.37 \times 10^{-6} \frac{M_1 + M_2}{a(1-e^2)}. \tag{9}$$

Here the masses M_1 and M_2 of the components and the semimajor axis $a = a_1 + a_2$ of the relative orbit are in solar units. Both effects (classical and relativistic) act in the same direction and we observe the total result only:

$$\frac{P}{U_{\mathrm{obs}}} = \frac{P}{U_{\mathrm{cl}}} + \frac{P}{U_{\mathrm{rel}}}. \tag{10}$$

Therefore to determine the parameters k_2 and, correspondingly, P/U_{cl}, we should take into account the relativistic term in apsidal motion which sometimes gives a considerable contribution. However, in the middle of the twentieth century the correctness of the general relativity conclusions when used with binary systems with comparable masses was called into question [16]. Therefore, at the beginning of the 1970s, we, together with D.Ya. Martynov, made a list of EBSs in order to investigate the general relativistic 2 effects and began the photometric investigations of these stars. The results of investigations on this topic carried out by many astronomers have been presented in the papers by Gimenez [17], Khaliullin [7], Khaliullin and Khaliullina [18] and others. It follows from these extensive investigations that there are no serious observational reasons now to doubt the correctness of equation (9) for the relativistic term in apsidal motion. The observations of most systems showed agreement with this theoretical formula. Therefore it is just used to correct the observations for the relativistic term with the purpose of determining the value of P/U_{cl}.

The considerable anomalies in the apsidal motion observed for DI Her [6], AS Cam [19] and some others are probably due to perturbations resulting from a third body [20–25]. It is quite possible that in some cases such anomalies may arise because the directions of the vectors of axial and orbital rotation do not coincide [14].

About 70 eclipsing binary systems with observed apsidal motion are known. Catalogues of these systems which are more or less complete have been presented in the papers by Claret and Gimenez [26], Claret [22], Petrova and Orlov [27–29], Claret and Willems [30] and Young and Arnett [31]. For many of these systems the masses, radii, other physical characteristics and orbital parameters are known; on the basis of this, we can compute the observed apsidal parameters k_2^{obs}, which can be compared with the theoretical values k_2^{th}.

The theoretical values of k_2^{th} for the distribution $\rho(r)$, given by the model computations of the stellar internal structure, have been published in a number of papers. A detailed survey of papers devoted to constructing theoretical models of stars and their evolution for 1988 has been given in the monograph by Masevich and Tutukov [32]. Many papers about this problem are published every year, but only a few of these contain the internal structure constants or the distribution $\rho(r)$ needed for their computation. Chandrasekhar [9] used the polytropic models to calculate these parameters. The values of k_2 for more realistic stellar models within the main sequence, based on the nuclear source of energy, were derived for the first time by Schwarzchild [33] and then by Mathis [34], Semeniuk and Paczynski [35], Stothers [36], Hejlesen [37–39], Jeffery [40] and others. Starting with these pioneering studies it becomes clear that real stars are more centrally condensed than predicted by theoretical models for the zero-age main-sequence stars. Numerically this has been expressed by the fact that k_2^{th} was on average about three times $\bar{k}_2^{\mathrm{obs}}$. For the first time it seemed that the observed differences can be fully explained by evolutionary effects since it was known that during the burning of hydrogen in the core in the initial stage of stellar evolution (within the main sequence) the values of k_2 decreases as a rule. However, then the other basic stellar parameters will vary (in particular, L and R), and theory should satisfy simultaneously all the parameters M, L, R and k_2. Furthermore the theoretical models for EBSs should satisfy the natural requirements of the same age and initial chemical composition of the components. Hitherto the theory of the stellar internal structure and evolution has been unable to describe satisfactorily the observed values; taking into account the evolutionary effects decreased the systematic differences between k_2^{obs} and k_2^{th} from three to one and half times. Taking into account stellar axial rotation and variations (within reasonable limits) in the initial chemical composition, the mixing length in convective suns and the core overshooting by computing the stellar models somewhat improves the situation [41] but cannot remove the systematic differences completely. The new tables of opacities published by Rogers and Iglesias [42] and Iglesias and Rogers [43] changed the state of the problem appreciably. In these new tables the opacities are increased by up to two to three times in comparison with recently used data because more attention was paid to the absorption in spectral lines and because the approximation equations of atomic conditions were improved. Using the new tables of opacities, Claret and Gimenez [44–46] and Claret [47–50] calculated a wide series of stellar models for different masses, chemical compositions

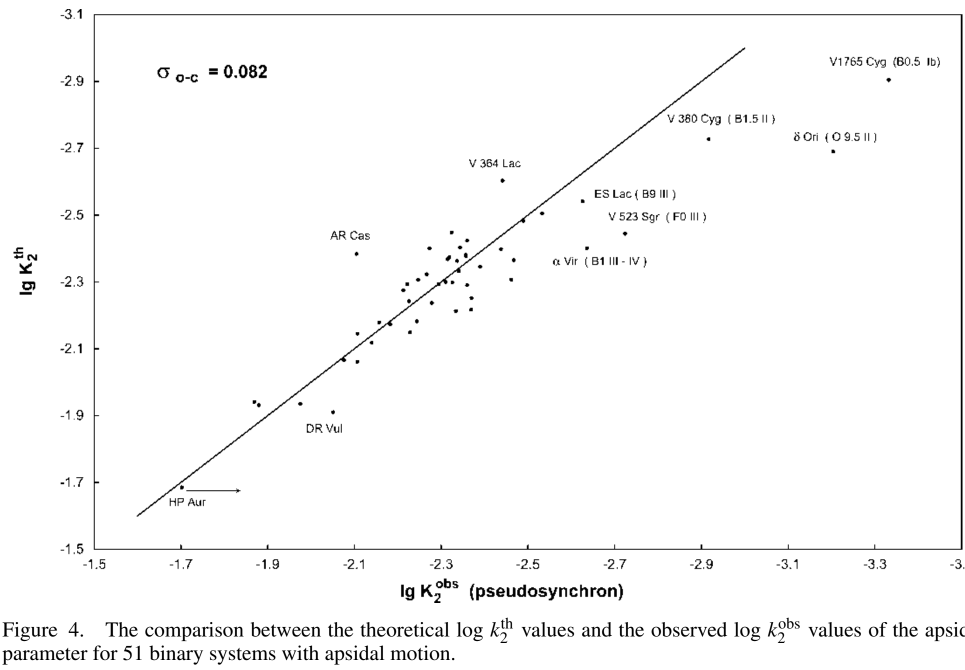

Figure 4. The comparison between the theoretical log k_2^{th} values and the observed log k_2^{obs} values of the apsidal parameter for 51 binary systems with apsidal motion.

and different stages of evolution (up to the first phase of the carbon burning) and obtained the apsidal parameters k_2 for all the models.

Figure 4 shows a comparison between k_2^{obs} and k_2^{th} in accordance with the state of observed and theoretical data at present (2006). It can be seen in this figure that stars of types I–III show a more central condensation than predicted by the theoretical models. However, it is true that there are only six such systems now, and the accuracy of their parameter determination is not very high. At the same time there are no systematic differences between k_2^{obs} and k_2^{th} for the majority of stars—for the main-sequence stars (of IV–V luminosity types). These results corroborate the conclusions of the earlier papers by Claret and Gimenez [26] and Khaliullin [7], but a random scattering is observed with $\sigma(\log k_2^{\text{obs}}) \approx 0.082$ (excluding the six stars mentioned above of I–III luminosity types) that noticeably exceeds the errors of determination of apsidal parameters. In order to clarify the probable causes of this scattering we selected 20 systems with highly accurate values of mass, radii and $V_{\text{r}} \sin i$ for both components [18]. For these stars we computed the observed values of k_2^{obs} for two suppositions.

(i) The values $\omega_{\text{r},i}/\omega_{\text{k}}$ in equation (7) are determined from condition of pseudosynchronization of axial and orbital rotation in the periastron: $(\omega_{\text{r},i}/\omega_{\text{k}})^2 = (1+e)/(1-e)^3$.
(ii) The values of $\omega_{\text{r},i}/\omega_{\text{k}}$ are determined from the observed values $V_{\text{r},i} \sin i$.

The results of this comparison are shown in figure 5. It appears that using the measured values of $V_{\text{r},i} \sin i$ not only did not improve the situation but also made it significant worse(!). This result was highly surprising to us. Together with analysis of other data it leads to the

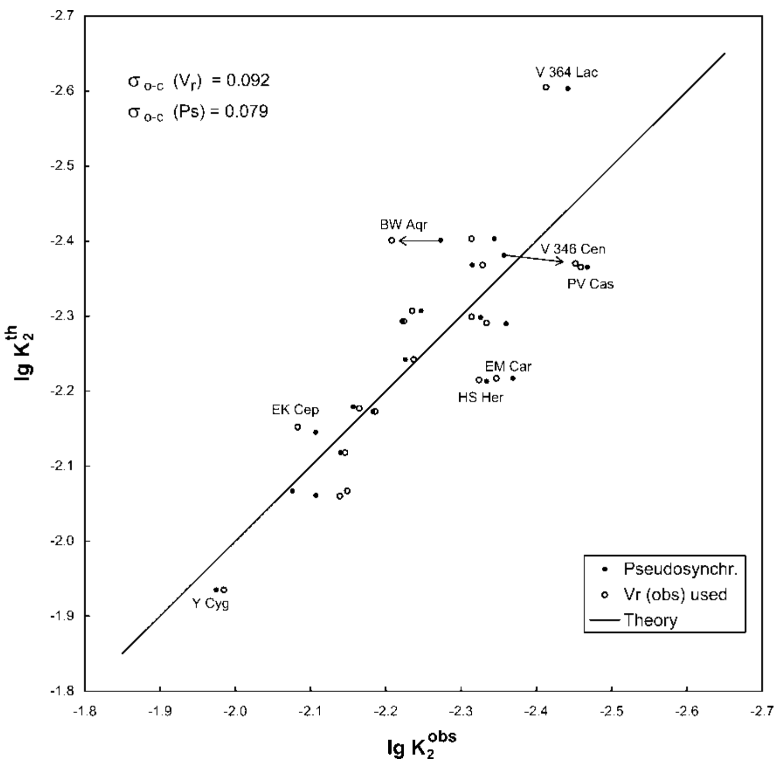

Figure 5. The comparison between the theoretical apsidal parameters k_2^{th} and the observed values k_2^{obs} obtained by using equations (6) and (7) for two different suppositions about the angular velocities $\omega_{\text{r},i}$ of axial rotation of components: (i) The values $\omega_{\text{r},i}/\omega_{\text{k}}$ are computed according to pseudosynchronization of axial and orbital rotation in the periastron (points). (ii) The values of $\omega_{\text{r},i}/\omega_{\text{k}}$ are computed by using the observed values $V_{\text{r},i} \sin i$ (open circles).

conclusion that the observed rates of the stellar photospheric rotation poorly reflect the rates of axial rotation of the greater part of stellar mass (core) and that cores of the components of EBSs rotate, on average, more quickly than their photospheres, *i.e.* differential rotation is observed. Here differential rotation means rotation as a function of the radius. Thus, statistical analysis of the observed data of apsidal motion leads to the same conclusion about the presence of differential rotation of components as obtained by Yildiz [51, 52] from the analysis of two eclipsing systems with eccentric orbits: EK Cep and PV Cas. Note that the conclusion by Yildiz was based on somewhat other physical premises and observed data.

Summarizing, we can say that EBSs with an eccentric orbit are an inexhaustible source of information about stellar interiors. The development of the theory of the stellar internal structure and evolution are closely connected with the theory and observations of apsidal motion in binary systems. The apsidal parameter k_2 together with the other fundamental stellar characteristics (mass, luminosity and radius) give rather difficult observational limits for the theory. Therefore it seems reasonable that a wide international programme should be organized for the investigation of eclipsing variable stars with an eccentric orbit, in which it is possible to measure, together with M, R and L, the apsidal motion rate and, therefore, to determine the parameter k_2.

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