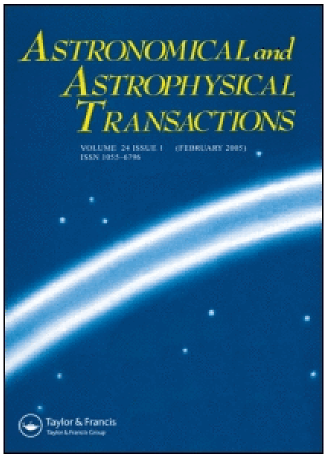

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Magnetic configurations produced by hydromagnetic dynamos

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The dynamo mechanism is a generic physical mechanism which excites large-scale magnetic fields in stars (including the Sun), planets and galaxies. The dynamo action is based on two generators, i.e. differential rotation and the so-called α effect associated with mirror asymmetry of rotating convection or turbulence. We discuss why dynamo excitation gives dynamo waves of dipole symmetry in spherical shells and steady quadrupole configurations in galactic discs. The key factor determining the magnetic configuration is the direction, which gives the maximal shear of differential rotation. Dynamos with the shear directed along the shell grow steadily while dynamos based on the shear directed across the shell result in dynamo wave configurations.

Keywords: Magnetic fields; Solar activity; Galaxies

1. Introduction

A large-scale magnetic field occurs in various celestial bodies such as stars, galaxies and planets. The origin of the magnetic field is thought to be connected with the Faraday induction effect, i.e. the dynamo mechanism. The mechanism is generic and does not depend on the fine details of a particular body; however, its manifestations look rather disjointed. The dynamo-excited magnetic configuration for a stellar convective shell is a travelling wave with dipole symmetry and an oscillating period which determines the magnetic activity cycle length while the galactic magnetic field is a stationary field with quadrupole symmetry. The variety of dynamo manifestations is determined by the specific shape of the rotation curves and the geometry of convective or turbulent shells where the dynamo action occurs. The dynamo models developed for particular celestial bodies explain the observed manifestations in terms of the above features; however, corresponding remarks are hidden in papers which address a specific subject rather than a comparative study of various celestial bodies. Here we collect the remarks together to enable an assessment of the dynamo models to be made by a reader who is interested in conclusions rather in the development of dynamo theory.

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2. Principles of dynamo action

Stellar, planetary and galactic magnetic fields are thought to be excited by Faraday electromagnetic induction, which is referred to as the dynamo mechanism and is a unique generic mechanism suggested for this aim by modern science. Possible exceptions such as presumably relic magnetic fields of Ap stars or battery effects in protogalaxies provide some useful additional alternatives to dynamo action. The induction effect in a single electric circuit leads to the suppression rather to the excitation of a magnetic field (Lenz's rule). At least two circuits or magnetic field components need to be employed to obtain magnetic field self-excitation.

Parker [1] suggested a scheme for the magnetic fields and its connections which can result in self-excitation and appears to be adequate for the majority of astrophysical applications. The magnetic field is decomposed into a toroidal component B_{T} and a poloidal component B_{P}. Quite obviously, differential rotation gives a toroidal magnetic field from the poloidal magnetic field because the field is frozen in the flow. The link between the poloidal magnetic field and the toroidal magnetic field is much more complicated. Parker connected it with cyclonic motions. A general understanding of the effect (now known as the α effect) was given many years later by Krause and Rädler [2]. The Coriolis force acting in a stratified rotating turbulence or convection destroys mirror symmetry and gives an excess of vortexes of one sign of helicity in a given hemisphere. It gives a component of the mean electromotive force $\boldsymbol{E}$ which is parallel (rather than perpendicular) to the mean magnetic field $\boldsymbol{B}$:

$$\boldsymbol{E} = \alpha \boldsymbol{B} + \cdots , \tag{1}$$

where the ellipsis $(\cdots)$ means other (standard) terms in Ohm's law. α effects close the chain of dynamo self-excitation for the magnetic field.

Parker's scheme underlies the main part of dynamo models for particular celestial bodies; however, its description in terms of partial differential equations needs some adjustment for a particular context. To be specific, we give here the corresponding equations for a thin solar shell:

$$\frac{\partial B_{\mathrm{P}}}{\partial t} = [\alpha(\theta) B_{\mathrm{T}}] + B_{\mathrm{P}}, \tag{2}$$

$$\frac{\partial B_{\mathrm{T}}}{\partial t} = -DGB_{\mathrm{P}} \cos\theta + B_{\mathrm{T}}. \tag{3}$$

Here B_{P} is the poloidal field, B_{T} is the toroidal field, θ is the colatitude ($\theta = 0$ means the solar equator) and the prime indicates the derivative according to θ. The equations are averaged over the radial extent of the shell and the radial gradient G of the rotation rate only is taken into account. $D < 0$ means that the dynamo waves, if available, propagates equatorwards. The equations are presented in a dimensionless form to isolate the dimensionless dynamo number D, which gives the intensity of dynamo action.

On the basis of the equations presented, we depart, in our analysis, from the case of the solar dynamo wave.

3. Asymptotic approach

The distinction between various manifestations of the Parker dynamo becomes clear when there is a limiting case of strong generation, *i.e.* when $|D| \gg 1$. Then the wavelength dynamo wave is expected to be small with respect to the length of the solar meridian. Then a short-wavelength *Ansatz* for equations (2) and (3) can be exploited to arrive at a dispersion relation [3] for the magnitude k of the wave vector and the complex growth rate Γ (Im Γ gives the cycle length):

$$[\Gamma_0 + k^2(\theta)]^2 = i\tilde{\alpha}(\theta)k(\theta). \tag{4}$$

Here $\tilde{\alpha} = \alpha G \cos\theta$ combines both magnetic field generators, i.e. the α effect and differential rotation, as well as the geometrical factor cos θ. The dispersion relation (4) give a dynamo wave, *i.e.* Im $\Gamma \neq 0$.

In the general case the right-hand side term of the dispersion relation is [4]

$$i\tilde{\alpha}k = i\alpha\cos\theta|\boldsymbol{k}\times''\Omega| \tag{5}$$

and the generation becomes more effective for the wave propagating along the surfaces of constant Ω (Yoshimura's rule).

Note that the short-wavelength limiting case implies that

$$(\alpha B_{\mathrm{T}})' \approx \alpha B_{\mathrm{T}}. \tag{6}$$

Of course, the solar dynamo action is quite moderate and the short-wavelength asymptotic gives only a useful hint for the solar case. A naive extrapolation of the excitation analysis to the nonlinear case means that the magnetic force suppresses somehow the magnetic field growth (Re Γ); however, the magnetic field configuration in the form of the travelling wave and the cycle length survive in the nonlinear regime. Numerical inspection of the parametric space of the problem (see, for example, [5]) isolates some parameter range where the nonlinear behaviour is more complicated (say, where standing waves occur); however, the extrapolation seems to be very instructive in general.

The dispersion relation (4) gives the travelling waves for a thin disc as well as for a thin spherical shell provided that $|D|$ is sufficiently large. If the rotation rate substantially varies with the height z on the disc's equatorial plane, the dynamo mechanism provides a wave propagating in the equatorial plane. Possibly, this magnetic field configuration occurs in some galaxies [6]. The rotation curves of galactic discs depends on the galactocentric distance r rather than on z.

In principle, Parker's dynamo can excite a dynamo wave propagating in the z direction (rather than in the r direction) provided that $|D|$ is sufficiently large. There is, however, the other option for self-excitation which seems to be responsible for galactic dynamos. Let us return to equation (6) which combines the two contributions αB_{T} and $\alpha' B_{\mathrm{T}}$ into $(\alpha B_{\mathrm{T}})'$. The second contribution, which looks negligible for solar-type dynamos, becomes interesting in the galactic context. The point is that α' becomes a maximum at the galactic equator while B_{T} is a maximum for a quadrupole configuration at the galactic equator as well. This means that

$$\alpha' B_{\mathrm{T}} \gg \alpha B_{\mathrm{T}} \tag{7}$$

up to $|D| \approx 100$. Then the dispersion relation gives a steadily growing quadrupole solution with properties relevant to galactic magnetic fields (for details of the asymptotic expansion, see [7]).

4. Dynamo action in binaries

The above discussion of dynamo-excited magnetic configurations takes into account implicitly axisymmetric magnetic field configurations in axisymmetric (on average) celestial bodies.

Of course, a deviation of, say, the rotation curve or α-effect axisymmetry results in the excitation of a non-axisymmetric magnetic configuration. The most important example here is provided by close binaries. The point, however, is that the majority of dynamo models for close binaries (see, for example, [8, 9]) include non-axisymmetry in the spatial distribution of dynamo generators only and keep Parker's scheme as it is for a single star. In this sense the models are similar to the dynamo models explaining solar active longitudes (see, for example, [10]).

In principle, some dynamo models based on schemes which are specific to close binaries could be discussed. Such schemes, which are only sporadically discussed in the literature, should include matter exchange between the companions [11] or magnetic lines leaving one star and reaching another star [6].

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