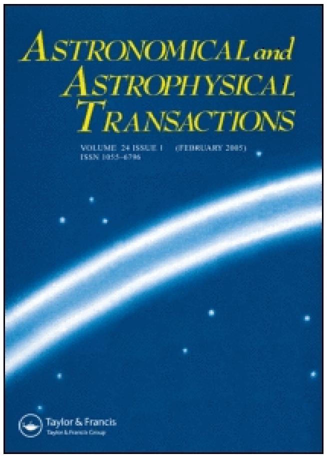

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Possibilities of the analysis of brightness distributions for the components of eclipsing variables from high-precision photometry data

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We carried out numerical experiments on the evaluation of the possibilities of obtaining information about the brightness distributions for the components of eclipsing variables from the high-precision photometry data expected to be obtained from the planned satellites COROT and Kepler. We examined a simple model of an eclipsing binary with spherical components on circular orbits and a linear law of limb darkening. The solutions of light curves have been obtained both by fitting a nonlinear model by a number of parameters including the limb-darkening coefficients and by solving the ill-posed inverse problem of restoration of brightness distributions across the discs of stars using *a priori* information about the form of these functions. The obtained estimates show that, if the observational accuracy is 10^{-4}, then the limb-darkening coefficients can be found with a relative error of approximately 0.01. The brightness distributions across the discs of components can be restored to nearly the same accuracy as well.

Keywords: Stars; Eclipsing variables; Brightness distributions

1. Introduction

A study of eclipsing variable stars is at present the basic information source about the sizes of stars and brightness distributions across their discs. Information about brightness distributions is especially important since it allows us to test the models of stellar atmospheres independent of spectral analysis data. The analysis of the light curve of an eclipsing binary is a classical problem of astrophysics. The methods of solution of this problem are detailed and connected with the names of notable researchers: H.N. Russell, D.Ya. Martynov, V.P. Tsesevich, J.E. Merrill, Z. Kopal, etc. At present there are two basic approaches to the solution of the problem: by fitting a nonlinear model by the known laws of limb darkening for the components of the eclipsing variable and by solving the ill-posed inverse problem of the restoration of brightness distributions across the discs of stars. However, the precision of ground-based photometry

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(with a relative error ε greater than 10^{-3} in flux measurements) substantially limits the accuracy of the obtained results.

The planned launches of the special satellites COROT and Kepler make it possible to expect a considerable increase in the precision of the photometry of bright objects (up to $\varepsilon = 10^{-5}$). Under these conditions, new possibilities for solving the classical problem of investigating the eclipsing variable stars have opened up [1].

The purpose of our paper is to estimate the possibilities for determining the geometrical parameters and brightness distributions for the components of the eclipsing variables from high-precision photometry data both by the model fitting method and by the method of restoration of brightness distributions.

2. The model light curve

It is known that, in the case of spherical components with a linear law of limb darkening, the problem of calculating a light curve for an eclipsing variable has an exact solution, the error being determined by the precision of the numerical estimates of one-dimensional definite integrals. Therefore, for further analysis we chose a simple model of an eclipsing variable with the following parameters: the angle of orbital inclination, $i = 89°.0$; the radius of the first component in units of the orbital radius, $r_1 = 0.30$; the luminosity of the first component, $L_1 = 0.30$; the limb-darkening coefficient of the first component, $x_1 = 0.50$; the radius of the second component, $r_2 = 0.20$; the luminosity of the second component, $L_2 = 0.70$; the limb-darkening coefficient of the second component, $x_2 = 0.30$. The luminosities of the components are connected by the equation $L_1 + L_2 = 1$.

It is known that the light losses in the minima $1 - l_i$ are described by the phase functions

$$(1 - l)_i = \alpha^i(p, k)(1 - l_A), \tag{1}$$

where $1 - l_A$ is the light loss at the moment of the internal contact of the discs, $k = r_2/r_1$, $\Delta = r_1(1 + kp)$ and Δ is the distance between the centres of the stellar discs in units of the orbital radius. These phase functions, in turn, can be expressed via the main phases for the occultations, $\alpha'(p, k)$, and for the transits, $\alpha''(p, k)$ [2]. The last two main phases are estimated numerically via calculations of one-dimensional definite integrals which depend on the parameters p and k.

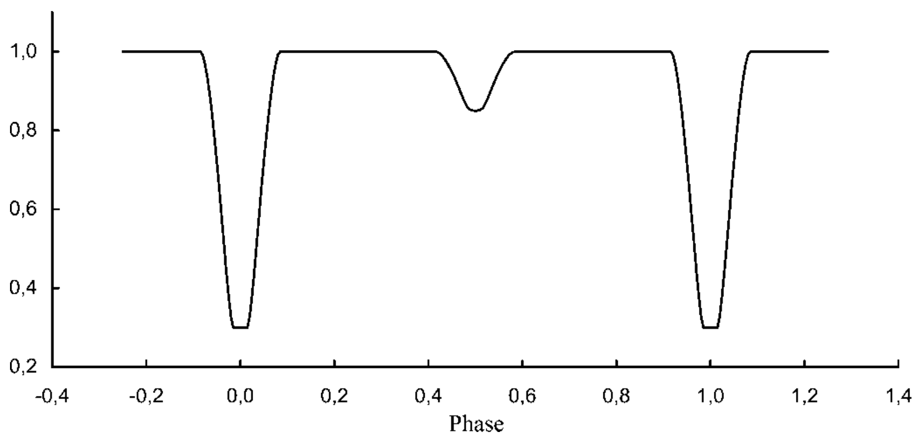

Figure 1. The calculated light curve for the chosen model of the eclipsing variable.

We have calculated the definite integrals of the phase functions by application of the Gauss–Kronrod algorithm using the subroutine DQAGE from the SLATEC FORTRAN program library. The precision of calculation of the light curve $l(i, r_1, L_1, x_1, r_2, L_2, x_2, \theta)$, where θ is the light phase ($0 \leqslant \theta \leqslant 1$), was adopted to be $\varepsilon = 10^{-12}$. The calculated light curve for our model of the eclipsing variable is shown in figure 1. The primary minimum corresponds to the total eclipse, and the secondary minimum to the annular eclipse.

3. The model fitting

One of the basic approaches to the analysis of the light curves of eclipsing variables is nonlinear-model fitting by the known laws of limb darkening for its components. As the input data we took our model light curve perturbed by the influence of random noise with different values of ε. The dispersion of noise was assumed to be constant over the magnitude scale. Gaussian pseudorandom numbers Δl_i with a zero mean and a standard deviation equal to unity are used to generate the noise. Thus, the samples of the perturbed light curve can be written as $l_i^{\mathrm{o}} = l_i^{\mathrm{c}}(1 + \varepsilon\Delta l_i)$, where l_i^{c} are the samples of the calculated model light curve.

For both the primary minimum and the secondary minimum we considered $N = 100$ equidistant samples of the perturbed light curve. The search for the optimal values of the model parameters leads to the solution of nonlinear minimization problems. In the case of the primary minimum,

$$\sum_{j=1}^{N}[l^{\mathrm{o}}(\theta_j) - l^{\mathrm{c}}(i, r_1, r_2, L_2, x_2, \theta_j)]^2 = \text{minimum} \tag{2}$$

and, in the case of the secondary minimum,

$$\sum_{j=1}^{N}[l^{\mathrm{o}}(\theta_j) - l^{\mathrm{c}}(i, r_1, L_1, x_1, r_2, \theta_j)]^2 = \text{minimum.} \tag{3}$$

We have used the DNLS1 subroutine to solve these problems and also the subroutine from the SLATEC library, which minimizes the sum of squares of nonlinear functions by a modification of the Levenberg–Marquardt algorithm [3].

The light curve depends nonlinearly on a number of parameters. This nonlinearity can lead to the presence of local minima of the residual. To check this possibility we carried out a number of numerical experiments on the solution of the problems of minimization (2) and (3) with different initial values of the model parameters. In these experiments, the initial values differed from the precise values upto two times, being half the precise values or less. In all

Table 1. The average values of the model parameters and their standard deviations obtained by the model fitting from the primary minimum for different values of relative error of the light curve.

Parameter	Precise value	Average value of the model parameter for the following relative error values		
		$\varepsilon = 10^{-5}$	$\varepsilon = 10^{-4}$	$\varepsilon = 10^{-3}$
i	$89^\circ.0$	$88^\circ.9992 \pm 0^\circ.0034$	$89^\circ.001 \pm 0^\circ.035$	$89^\circ.09 \pm 0^\circ.41$
r_2	0.20	0.1999989 ± 0.0000081	0.200002 ± 0.000085	0.20006 ± 0.00073
L_2	0.70	$0.70000000 \pm 0.00000072$	0.7000002 ± 0.0000074	0.700018 ± 0.000078
x_2	0.30	0.29998 ± 0.00024	0.3001 ± 0.0027	0.302 ± 0.023
r_1	0.30	0.3000007 ± 0.0000028	0.300000 ± 0.000030	0.30000 ± 0.00028

Table 2. The average values of the model parameters and their standard deviations obtained by the model fitting from the secondary minimum for different values of relative error of the light curve.

Parameter	Precise value	Average value of the model parameter for the following relative error values		
		$\varepsilon = 10^{-5}$	$\varepsilon = 10^{-4}$	$\varepsilon = 10^{-3}$
i	89°.0	88°.994 ± 0°.018	88°.99 ± 0°.15	88°.69 ± 0°.99
r_1	0.30	0.300 001 ± 0.000 029	0.300 00 ± 0.000 26	0.300 2 ± 0.002 8
L_1	0.30	0.300 025 ± 0.000 085	0.300 05 ± 0.000 74	0.303 5 ± 0.006 9
x_1	0.50	0.499 89 ± 0.000 55	0.499 5 ± 0.005 2	0.487 ± 0.056
r_2	0.20	0.199 996 ± 0.000 017	0.200 00 ± 0.000 16	0.199 5 ± 0.001 5

cases the solution converged well to the precise value and, therefore, the local minima were not discovered.

We carried out the model fitting to perturbed light curves for three values of $\varepsilon = 10^{-5}$, 10^{-4} and 10^{-3}. In each case, 100 curves were examined with different realizations of the random noise. The obtained average values of the model parameters and their standard deviations are given in tables 1 and 2. As can be seen from these tables, the geometrical parameters and the limb-darkening coefficients are evaluated very accurately from the high-precision photometry data. As a whole, the standard deviation in estimating a model parameter decreases linearly with decrease in the relative error in recording the light curve. If the observational accuracy of the space photometry is 10^{-4} (higher than the precision of ground-based photometry by one order of magnitude), then the limb-darkening coefficients can be found with a relative error of approximately 0.01.

4. The restoration of brightness distributions

An alternative approach to the analysis of the light curves of eclipsing variables, namely the restoration of brightness distributions across the stellar discs without rigid model constraints on the form of these functions, was developed by Cherepashchuk *et al.* [4]. Let $I(\xi)$ $(0 \leqslant \xi \leqslant r_1)$ and $I(\rho)$ $(0 \leqslant \rho \leqslant r_2)$ be the brightness distributions across the discs of the first and the second components, respectively. It can be shown that the light losses in the first minimum and the second minimum are described by the integral equations

$$1 - l_1(\Delta) = \int_0^{r_1} K_1(\xi, \Delta, r_2) I(\xi) \mathrm{d}\xi \tag{4}$$

and

$$1 - l_2(\Delta) = \int_0^{r_2} K_2(\rho, \Delta, r_1) I(\rho) \mathrm{d}\rho. \tag{5}$$

Expressions for the kernels of these integral equations can be found in the paper by Cherepashchuk *et al.* [4] and the monograph by Tsesevich *et al.* [2].

Equations (4) and (5) are integral equations of Fredholm's first kind. The solution of these integral equations is an ill-posed problem in Hadamard's sense and requires utilization of *a priori* information about the sought function. It is possible to assume that for the majority of stars with thin photospheres the brightness distributions are non-negative, monotonically non-increasing convex-upward functions. It is known that the sets of functions of these types are compact. The search for the solution of an ill-posed problem on the compact set of functions gives a unique and stable result [5]. This *a priori* information is qualitative and imposes no rigid model constraints on the form of the brightness distribution. Nevertheless, this guarantees that the obtained brightness distributions will approach their exact values while the errors in recording the observed light curve approach zero, with the exception of the points of discontinuity of the functions [5, 6]. The use of a large amount of *a priori* information about the possible form of the brightness distribution in accordance with the physics of the phenomenon enables us to achieve a solution with a high degree of stability against the effects of random noise.

Thus, as the solution of our problem can be taken in the form of non-negative, monotonically non-increasing convex-upward functions $I(\xi)$ and $I(\rho)$ that minimize the following functionals, the norms in the L_2 function space are

$$\Phi_1[I(\xi), r_1, r_2, i] = \left\| \int_0^{r_1} K_1(\xi, \Delta, r_2) I(\xi) \mathrm{d}\xi - [1 - l_1(\Delta)] \right\|_{L_2} \tag{6}$$

and

$$\Phi_2[I(\rho), r_1, r_2, i] = \left\| \int_0^{r_2} K_2(\rho, \Delta, r_1) I(\rho) \mathrm{d}\rho - [1 - l_2(\Delta)] \right\|_{L_2}. \tag{7}$$

The problem of restoration of the brightness distributions depends also on three free parameters: i, r_1 and r_2. Their values can be found by minimizing the summary residual.

We carried out a numerical experiment on the restoration of brightness distributions from our perturbed model light curve for the value $\varepsilon = 10^{-4}$. The special estimates showed that the choice of the number of points, $M = 1001$, of a uniform grid along a radius with the integration for Simpson's formula makes it possible to ensure a relative error in the calculation of integrals (4) and (5) below 10^{-6}. These grids were used later to minimize the functionals. We minimized the summary residual for both minima,

$$\Phi_1[I(\xi), r_1, r_2, i] + \Phi_2[I(\rho), r_1, r_2, i] = \text{minimum}, \tag{8}$$

and searched for the global minimum of the residual sum of squares by varying three geometrical parameters: i, r_1 and r_2. To minimize equations (6) and (7) on the compact set of non-negative, monotonically non-increasing convex-upward functions for various values of the geometrical parameters we used a modified version of the PTISR code written in FORTRAN

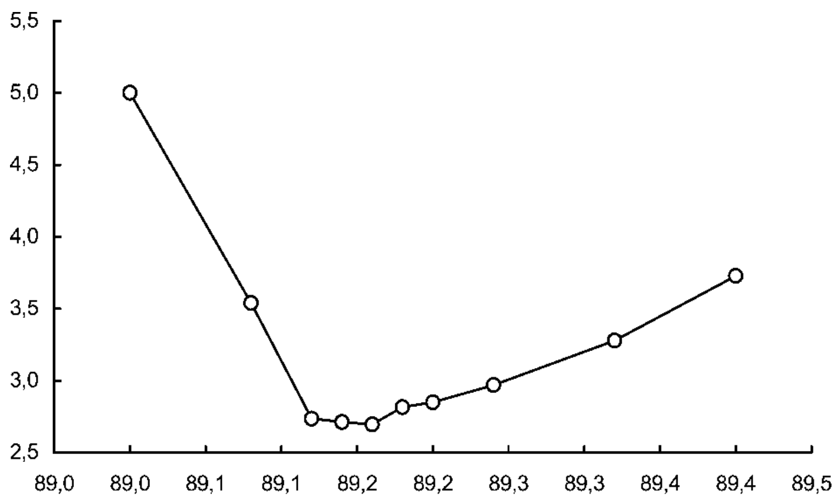

Figure 2. The dependence of the summary residual (in units of 10^{-9}) on the orbital inclination angle i (in degrees) for optimal values of the radii r_1 and r_2.

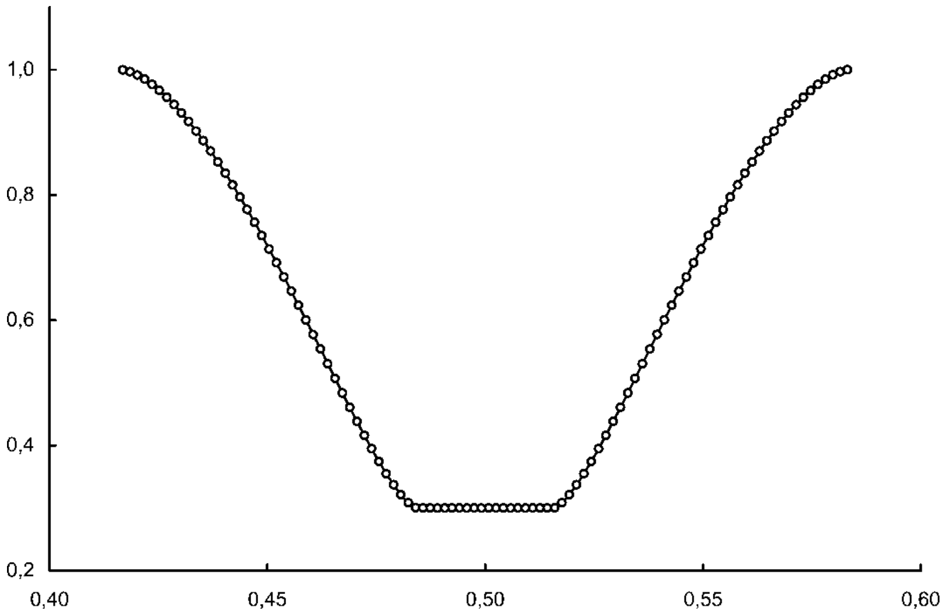

Figure 3. The samples of the perturbed light curve for the primary minimum (open circles) and the light curve corresponding to the restored brightness distribution (solid curve).

[5, 6]. This code minimizes a residual by the method of projection of the conjugate gradients on the selected set of functions. To reduce the effect of round-off errors, we transformed all the real variables used in the PTISR and its auxiliary subroutines into double-precision variables with 16 significant digits in their floating-point mantissas. Zero initial approximations for brightness distributions were used in all cases with the minimization of equations (7) and (8).

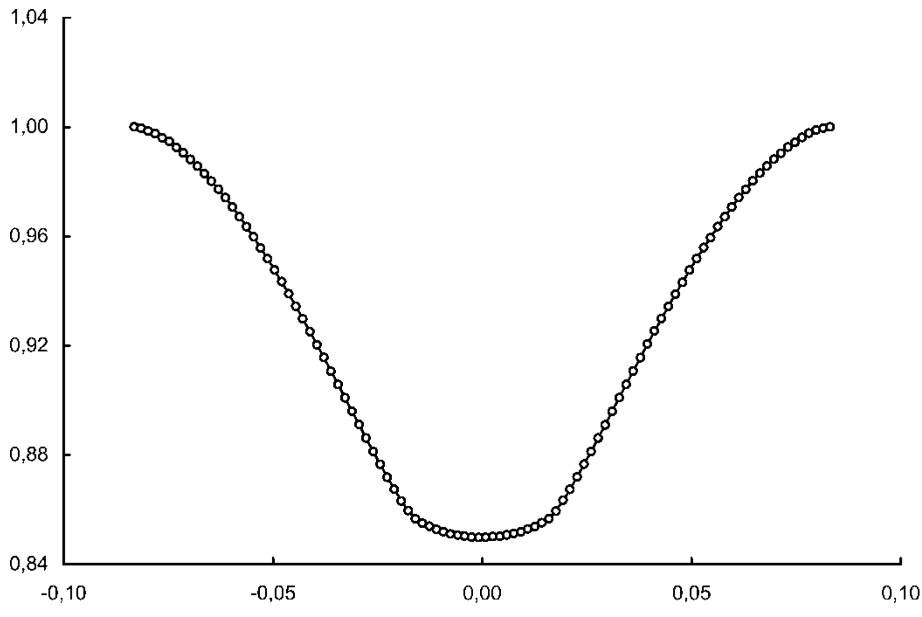

Figure 4. The samples of the perturbed light curve for the secondary minimum (open circles) and the light curve corresponding to the restored brightness distribution (solid curve).

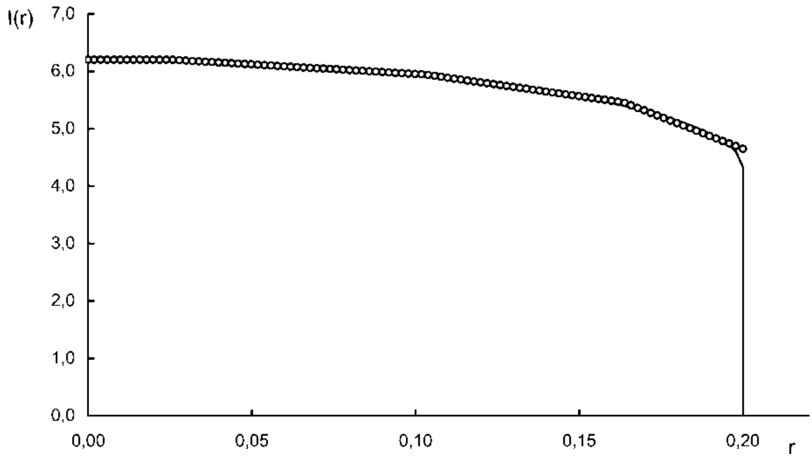

Figure 5. The samples of the restored brightness distribution for the second component (open circles) and the precise distribution (solid curve).

The numerical experiments showed that the global minimum of equation (8) can indeed be found. Figure 2 presents the summary residual depending on the orbital inclination angle i for optimal values of the radii r_1 and r_2. The values of the geometrical parameters corresponding to the global minimum of the residual are as follows: $i = 89°.16 \pm 0°.02$, $r_1 = 0.299\,90 \pm 0.000\,02$ and $r_2 = 0.200\,05 \pm 0.000\,02$. The values of the errors are formal and equal to the steps of the grids used in carrying out the variation in the geometrical parameters.

The samples of the perturbed light curve are indicated by open circles for the primary minimum in figure 3, and for the secondary minimum in figure 4. The solid curves in these figures show the light curves, which correspond to the restored brightness distributions. The samples of the restored brightness distributions are indicated by open circles in figures 5 and 6, where also the solid curves show the precise distributions. To avoid superposition, every tenth sample is shown. As can be seen from figures 5 and 6, the accuracy of restoration

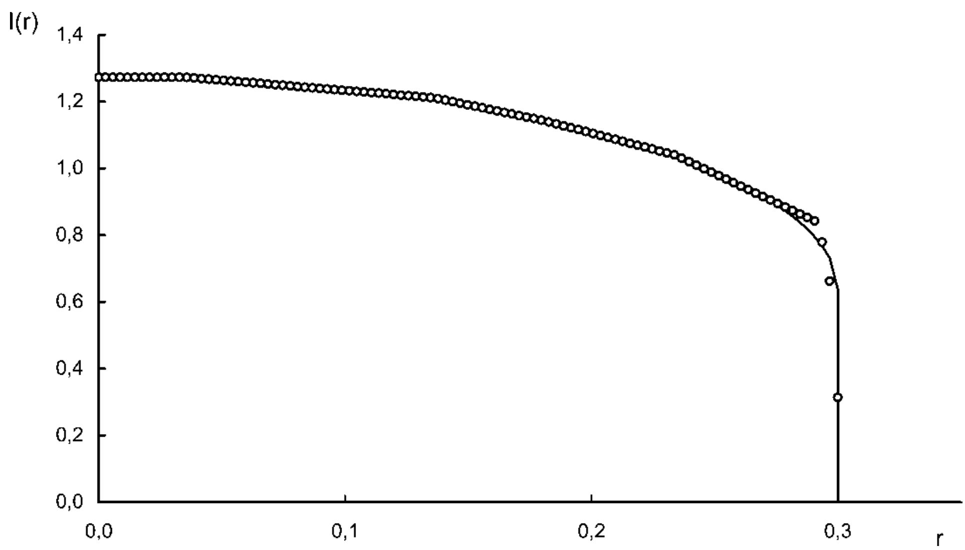

Figure 6. The samples of the restored brightness distribution for the first component (open circles) and the precise distribution (solid curve).

of the brightness distribution proves to be sufficiently high over almost the entire disc of star. Unfortunately, on the edges of discs, at the points of discontinuity of a function, the solutions are noticeably different from the precise values. This is explained by the absence of the convergence of solution of an ill-posed problem at the points of discontinuity of a function.

5. Conclusion

We carried out numerical experiments to evaluate the possibilities of obtaining information about the brightness distributions of the components of eclipsing variables from the high-precision photometry data expected to be obtained from the planned satellites COROT and Kepler. We investigated both approaches to the analysis of light curves: by fitting a nonlinear model by changing the number of parameters including the limb-darkening coefficients, and by solving the ill-posed inverse problem of restoration of brightness distributions across the discs of stars using *a priori* information about the form of these functions. It is shown that in both cases the analysis of high-precision space photometry data makes it possible to obtain results that agree well.

The standard deviations in the estimates of the model parameters decrease linearly with a decrease in the relative error of recording the light curve. If the observational accuracy of the space photometry is 10^{-4} (higher than the precision of ground-based photometry by one order of magnitude), then the limb-darkening coefficients can be found with a relative error of approximately 0.01. This accuracy will make it possible to distinguish the linear law of limb darkening easily from the nonlinear law and to use for its estimate the fitting of more complex models of brightness distributions.

The accuracy of restoration of the brightness distribution without rigid model constraints on the form of this function proves to be sufficiently high over almost the entire disc of star. Unfortunately, on the edges of discs, at the points of discontinuity of a function, the solutions are noticeably different from the precise values. The geometrical parameters of an eclipsing variable, found by the search for the global minimum of a residual in the case of restoration of brightness distributions, also prove to be close to the precise values.

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