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Modification of the characteristic gravitational constants

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In the educational and scientific literature the numerical values of gravitational constants are seen as only approximately correct. The numerical values are different in work by various researchers, as also are the formulae and definitions of constants employed. In this paper, on the basis of Newton's laws and Kepler's laws we prove that it is necessary to modify the characteristic gravitational constants and their definitions. The formula for the geocentric gravitational constant of the satellites Kosmos *N* and the Moon are calculated.

Keywords: Gravitational constants; Earth; Moon; Kosmos *N*

1. Introduction

In [1], it has been proved that coefficient f of proportionality in the formula

$$F = f\frac{Mm}{\rho^2} \tag{1}$$

has not only one numerical value. On carefully reading Newton's *Philosophiae Naturalis Principia Mathematica* [2] (book I, theorems IV and; book II, theorems VII–XXII; book III, theorems I, II, III, etc.), it can be concluded that Newton's coefficients of proportionality are not identical. The following researchers used different numerical values for the gravitation constant in their experiments ($f = (6.6720 \pm 0.0041) \times 10^{-11}$): Pierre Bouguer (1740), Henry Caverdish (1798), Eötvös (1896), Heyl (1930), Zachradnices (1932), Heyl and Chrzanowski (1934) and Chertov (see p. 268 of [3]). The list of astronomical constants in the preface to the *Astronomical Yearbook for 1999* (see p. 650 of [4]) does not include a constant called the universal gravitational constant. The constant $G = 6.672 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}s^{-2}}$ is called *the Cavendish gravitational constant* and, in the *Astronomical Almanac for the Year 2000* (see p. k5 of [5]), it is called the *gravitational constant* and is equal to $6.672 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}s^{-2}}$.

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Some workers called it '*some constant value*' (see, for example, p. 598 of [6]) or *the universal constant G of gravitation* (see p. 378 of [7]), and $k^2 = 6.67 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-11}\,\mathrm{s}^{-2}$ (see p. 39 of [8]). In the system of astronomical constants given in [4, 5], for example, the astronomical constants are listed as follows: defining the Gaussian gravitational constant as

$$\kappa = 0.017\,202\,098\,95, \tag{2}$$

the (primary) geocentric gravitational constant

$$G_{\mathrm{E}} = 3.986\,005 \times 10^{14}\,\mathrm{m}^3\,\mathrm{s}^{-2} \tag{3}$$

and the (derived) heliocentric gravitational constant

$$G_{\mathrm{S}} = 1.327\,124\,38 \times 10^{20}\,\mathrm{m}^3\,\mathrm{s}^{-2}. \tag{4}$$

In that list also Kepler's constant

$$K = \frac{a^3}{T^2}, \quad \dim K = L^3 T^{-2} \tag{5}$$

is not included. On the other hand, in classical studies (see, for example, pp. 388–396 of [9], pp. 415–417 of [10] and p. 61 of [11]), we found the formulae for the following: the Gauss constant

$$\mu = \frac{4\pi^2 a^3}{T^2}, \tag{6}$$

the gravitational constants

$$f = \frac{4\pi^2 a^3}{MT^2} \tag{7}$$

and

$$f = \frac{4\pi^2 a^3}{(M+m)T^2}, \tag{8}$$

the geocentric gravitational constant

$$G_{\mathrm{E}} = f M_{\mathrm{E}} \tag{9}$$

and the heliocentric gravitational constant

$$G_{\mathrm{S}} = f M_{\mathrm{S}}. \tag{10}$$

It is noticeable that the value from equations (8) and (9) has a dominant role in determining the values in equations (10) and (11), although it it not unique. Thus it is necessary to define first the exact expression for the factor f of proportionality in equation (1). If we compare equation (1) with the expression (see p. 598 of [6])

$$\varepsilon \frac{mm'}{r^2}, \tag{11}$$

where ε is 'some constant value', which is independent of m and r, we can see that the task is not trivial.

2. The factors of proportionality of gravity

In order to understand clearly the gravitational constant, which is present in equation (1), we start from Newton's second and third laws:

$$m_1\ddot{\boldsymbol{r}}_1 = \boldsymbol{F}_1, \tag{12}$$

$$m_2\ddot{\boldsymbol{r}}_2 = \boldsymbol{F}_2, \tag{13}$$

$$\boldsymbol{F}_1 = -\boldsymbol{F}_2 \tag{14}$$

and

$$\boldsymbol{r}_2 - \boldsymbol{r}_1 = \boldsymbol{\rho} \longrightarrow \ddot{\boldsymbol{r}}_2 - \ddot{\boldsymbol{r}}_1 = \ddot{\boldsymbol{\rho}}, \tag{15}$$

where $\boldsymbol{\rho} = \ddot{\boldsymbol{r}}_2 - \ddot{\boldsymbol{r}}_1$ is the vector of the distance between the bodies as material points of masses m_1 and m_2 and $\ddot{\boldsymbol{r}}_2$ are the position vectors $\ddot{\boldsymbol{r}} = \mathrm{d}\boldsymbol{r}/\mathrm{d}t^2$ and $|\boldsymbol{e}_\rho| = 1$. This is the general axiomatic basis of our task. For condition (1) the differential equations of motion (12) and (13) can be written in the forms

$$m_1\ddot{\boldsymbol{r}}_1 = f\frac{m_1 m_2}{\rho^2}\boldsymbol{e}_\rho, \tag{16}$$

$$m_2\ddot{\boldsymbol{r}}_2 = -f\frac{m_1 m_2}{\rho^2}\boldsymbol{e}_\rho \tag{17}$$

and

$$\ddot{\boldsymbol{r}}_2 - \ddot{\boldsymbol{r}}_1 = \ddot{\boldsymbol{\rho}}. \tag{18}$$

By substituting the accelerations $\ddot{\boldsymbol{r}}_1$ and $\ddot{\boldsymbol{r}}_2$ from the equations of motion (16) and (17) into equation (18) it follows that

$$\ddot{\boldsymbol{\rho}} = f\frac{m_1 + m_2}{\rho^2}\dot{\boldsymbol{e}}_\rho,$$

or

$$f = \frac{\rho^2}{m_1 + m_2}\ddot{\boldsymbol{\rho}} \cdot \boldsymbol{e}_\rho. \tag{19}$$

We can write the scalar product in the following way:

$$\ddot{\boldsymbol{\rho}} \cdot \boldsymbol{e}_\rho = \frac{\mathrm{d}\dot{\boldsymbol{\rho}}}{\mathrm{d}t} \cdot \boldsymbol{e}_\rho = \frac{\mathrm{d}\dot{\boldsymbol{\rho}}}{\mathrm{d}t} \cdot \frac{\boldsymbol{\rho}}{\rho} = \frac{1}{\rho}\left(\frac{\mathrm{d}}{\mathrm{d}t}(\dot{\boldsymbol{\rho}} \cdot \boldsymbol{\rho}) - \dot{\rho}^2\right).$$

Also,

$$\begin{aligned}
\dot{\boldsymbol{r}}_2 - \dot{\boldsymbol{r}}_1 = \boldsymbol{v}_{\mathrm{or}} &= \dot{\rho}\boldsymbol{e}_\rho + \rho\dot{\theta}\boldsymbol{e}_n, \quad \boldsymbol{e}_\rho \perp \boldsymbol{e}_n,\\
\dot{\boldsymbol{\rho}} \cdot \dot{\rho} &= v_{\mathrm{or}}^2,\\
\dot{\boldsymbol{\rho}} \cdot \rho &= \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\rho} \cdot \boldsymbol{\rho}) - \boldsymbol{\rho} \cdot \boldsymbol{\rho} = 2\rho\dot{\rho} - \boldsymbol{\rho} \cdot \boldsymbol{\rho},\\
\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\rho} \cdot \boldsymbol{\rho}) &= \dot{\rho}^2 + \rho\ddot{\rho},\\
\ddot{\boldsymbol{\rho}} \cdot \boldsymbol{e}_\rho &= \frac{1}{\rho}(\dot{\rho}^2 + \rho\ddot{\rho} - v_{\mathrm{or}}^2).
\end{aligned}$$

It follows that

$$f = \frac{\rho(\dot{\rho}^2 + \rho\ddot{\rho} - v_{\mathrm{or}}^2)}{m_1 + m_2}. \tag{20}$$

This formula shows that the factor f has the dimensions of length, mass and time, *i.e.*

$$\dim f = \mathrm{L}^3\mathrm{M}^{-1}\mathrm{T}^{-2}; \tag{21}$$

the numerical value N of f is $N_f = N\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$.

Example 1: If two bodies of mass m_1 and m_2 are uniformly moving around themselves at an angular velocity $\dot{\theta} = 2\pi/T$ and at a constant distance $\rho = a =$ constant, from equation (20) it follows that

$$f = \frac{a^3}{m_1 + m_2}\dot{\theta}^2 = \frac{4\pi^2 a^3}{(m_1 + m_2)T^2}, \tag{22}$$

as $\dot{\rho} = 0$, $\ddot{\rho} = 0$, $v_{\mathrm{or}}^2 = \rho^2\dot{\theta}^2 = (4\pi^2/T^2)\,a^2$.

Example 2: At a distance $\rho = 1/2gt^2$ ($g =$ constant) the two bodies are in motion. In this case when $v_{\mathrm{or}}^2 = \dot{\rho}^2$, it is obvious that

$$f = \frac{g\rho^2}{m_1 + m_2}. \tag{23}$$

3. Keplerian motion

In this case, planar motion, for which Kepler's laws are applicable, is described by

$$\rho = \frac{p}{1 + e\cos\theta}, \tag{24}$$

$$\rho^2\dot{\theta} = C = \text{constant}, \tag{25}$$

$$\frac{a^3}{T^2} = K = \text{constant}. \tag{26}$$

Then equation (20) reduces to

$$f = \frac{\rho(\dot{\rho}^2 + \rho\ddot{\rho} - v_{\mathrm{or}}^2)}{m_1 + m_2} = \frac{\rho^2(\ddot{\rho} - \rho\dot{\theta}^2)}{m_1 + m_2} \tag{27}$$

because $v_{\mathrm{or}}^2 = \dot{\rho}^2 + \rho^2\dot{\theta}^2$. On the basis of equation (25),

$$\dot{\theta} = \frac{2ab\pi}{T},$$

and, on the basis of equation (26),

$$\ddot{\rho} = C^2\frac{e\cos\theta}{p} = C^2\frac{p - \rho}{p\rho^3}.$$

Therefore, we obtain

$$f = \frac{4\pi^2 a^3}{(m_1 + m_2)T^2}. \tag{28}$$

For the motion of the planets in the Solar System, in which m_1 is the Sun's mass equal to $\mathrm{M_S}$ and m_2 is the planet's mass equal to m, f differs from planet to planet. Consider the Earth, as an example according to Allen's work in 1963 (see pp. 16–19 of [12]): $m_{\mathrm{E}} = 5.97 \times 10^{24}$ kg, $T = 0.317 \times 10^8$ s, $a = 1.50 \times 10^{11}$ m and $M_{\mathrm{S}} = 1.9891 \times 10^{24}$ kg. It follows that

$$f = \frac{4\pi^2(1.5 \times 10^{11})^3}{(19\,891 + 0.0597) \times 10^{26} \cdot (0.317 \times 10^8)^2} = 6.665\,873\,5 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}.$$

According to the above, the formulae for the gravitational constants in equations (16) and (17) for ith planet of the Solar System can be written in the form

$$f = \frac{4\pi^2 a_i^3}{(M + m_i)T_i^2}. \tag{29}$$

4. The heliocentric constant of gravity

The force F_{S} of gravity of the Sun of mass M_{S} which acts on the Earth of mass m_{E} can be written in the form

$$F_{\mathrm{S}} = -\frac{4\pi^2 a_{\mathrm{F}}^3}{(M_{\mathrm{S}} + m_{\mathrm{E}})T_{\mathrm{E}}^2}\frac{M_{\mathrm{S}} m_{\mathrm{E}}}{\rho^2} \tag{30}$$

or

$$\frac{F_{\mathrm{S}}}{m_{\mathrm{E}}} = -\frac{4\pi^2 a_{\mathrm{E}}^3}{T_{\mathrm{E}}^2}\frac{M_{\mathrm{S}}}{M_{\mathrm{S}} + m_{\mathrm{E}}}\frac{1}{\rho^2} = -\mu\varepsilon\frac{1}{\rho} = \frac{G_{\mathrm{SE}}}{\rho^2}, \tag{31}$$

where

$$\mu = \frac{4\pi^2 a_{\mathrm{E}}^3}{T_{\mathrm{E}}^2},$$

$$\varepsilon_{\mathrm{SE}} = \frac{M_{\mathrm{S}}}{M_{\mathrm{S}} + m_{\mathrm{E}}},$$

$$G_{\mathrm{S}} = \mu\varepsilon_{\mathrm{SE}}$$

The Sun–Earth corrective factor $\varepsilon_{\mathrm{SE}}$ is

$$\varepsilon_{\mathrm{SE}} = 0.999\,997\,134, \tag{32}$$

and the heliocentric constant of gravity will be

$$G_{\mathrm{SE}} = \mu = \frac{4\pi^2 a_{\mathrm{E}}^3}{T_{\mathrm{E}}^2}\frac{M_{\mathrm{S}}}{M_{\mathrm{S}} + m_{\mathrm{E}}} = \mu\varepsilon_{\mathrm{SE}} = 1.325\,900\,786 \times 10^{20}\ \mathrm{m^3\,s^{-2}}. \tag{33}$$

Analogously to the above, the heliocentric constant of gravity for any other ith planet will have the form

$$G_{\mathrm{S}_i} = \frac{4\pi^2 a_i^3}{T_i^2}\frac{M_{\mathrm{S}}}{M_{\mathrm{S}} + m_i} = \mu_{\mathrm{S}i}\varepsilon_{\mathrm{S}i}. \tag{34}$$

According to equation (30), the heliocentric force acting on the ith planet is equal to

$$F_{\mathrm{S}i} = \mu_{\mathrm{S}i}\varepsilon_{\mathrm{S}i}\frac{m_i}{\rho_{\mathrm{S}i}^2}. \tag{35}$$

5. The geocentric constant of gravity

For a body of mass m which is in motion around the Earth of mass M_{E}, the formula for the force F_{E} that is attracting its satellite of mass m is of the same form as equation (30), i.e.

$$F_{\mathrm{E}} = -\frac{4\pi^2 a^3}{(M_{\mathrm{E}}+m)T^2}\frac{M_{\mathrm{E}}m}{M_{\mathrm{E}}+m}, \tag{36}$$

where T is a time that it takes to rotate once around the Earth. In the same manner, just as in the previous analysis of the planet's motion around Sun, we can conclude that

$$\frac{F_{\mathrm{E}}}{m} = -\frac{4\pi^2 a^3}{T^2}\frac{M_{\mathrm{E}}}{M_{\mathrm{E}}+m}\frac{1}{\rho^2} = -\mu\varepsilon\frac{1}{\rho^2} = \frac{G_{\mathrm{E}}}{\rho^2}, \tag{37}$$

where

$$\mu = \frac{4\pi^2 a^3}{T^2} \tag{38}$$

is the Gauss constant,

$$\varepsilon = \frac{M_{\mathrm{E}}}{M_{\mathrm{E}}+m} \tag{39}$$

is the corrective factor for the satellite of the Earth, and

$$G_{\mathrm{E}i} = \frac{4\pi^2 a_i^3}{T_i^2}\frac{M_{\mathrm{E}}}{M_{\mathrm{E}}+m_i} = \mu_i\varepsilon_{\mathbf{E}i} \tag{40}$$

is the geocentric constant of gravity for the ith satellite. The corrective factor $\varepsilon_{\mathbf{Ei}}$ depends on the mass of the satellite.

For example, let us use this value for the published data on the satellites Kosmos N (see pp. 270–274 of [13]). Let us assume that the masses of the observed satellites are $m_i \leq 1000$ kg. We wish to calculate Kepler's constant $K = a_i^3/T_i^2$, the Gauss constant μ_i and the geocentric

Table 1. Values of Kepler's constant and the geocentric constant of gravity for the satellites Kosmos N.

Satellite	Kepler's constant (10^{14} m^3 s^{-2})	Geocentric constant (10^{14} m^3 s^{-2})
Kosmos 1	10 154.746 86	4.008 933
Kosmos 11	10 145.094 70	4.005 365
Kosmos 21	10 153.680 54	4.008 515
Kosmos 31	10 164.186 61	4.012 660
Kosmos 41	10 114.695 84	3.993121
Kosmos 51	10 150.196 65	4.007 137
Kosmos 61	10 153.194 52	4.008 320
Kosmos 71	10 126.515 73	3.997 788
Kosmos 81	99 90.137 00	3.943 948
Kosmos 91	10 153.551 49	4.008 447
Kosmos 101	10 169.700 97	4.014 837
Kosmos 127	10 126.769 04	3.997 888
Average	10 133.539 16	4.000 579

constant G_{Ei}. Before that, let us determine the corrective factor (38); as the mass of the Earth is $M_E = 5.97 \times 10^{24}$ kg, and the equatorial radius of the Earth is $R = 6\,378\,140$ m,

$$\varepsilon = \frac{M_E}{M_E + 1000} \approx 1.$$

Thus, the geocentric constant G_{Ei} of gravity for satellites of the Earth which have a small mass is equal to the constant μ.

Table 1 lists the values of Kepler's constant K and geocentric gravity constant $G_{Ei} = \mu_i$, calculated for the satellites Kosmos N (see pp. 270–274 of [13]).

The mean numerical value of the geocentric constant for the presented satellites is $G_E = 4.000\,579 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$.

The larger satellite of the Earth is the Moon, the mass of which is $m_M = 73.5 \times 10^{21}$ kg $= 0.0735 \times 10^{24}$ kg and the average distance is $a = 3.84 \times 10^8$ m; the period of rotation is $T = 23.6 \times 10^5$ s. It follows that the $K = a^3/T^2 = 0.101\,664\,579 \times 10^4\ \mathrm{m^3\,s^{-2}}$, $\mu = 4\pi^2 K = 4.013\,556\,711 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$, $\varepsilon = 0.987\,838\,173$ and

$$G_{EM} = 3.964\,744\,529 \times 10^{14}\ \mathrm{m^3\,s^{-2}}.$$

So, the Earth of mass M_E is acting on the Moon of mass m, with a force

$$F_E = -G_{EM}\frac{m}{\rho_{EM}^2} \tag{41}$$

where G_{EM} geocentric constant of gravity for the Moon. The reverse action of the Moon also exists:

$$F_M = -G_{ME}\frac{M_E}{\rho_{ME}^2} \tag{42}$$

where G_{ME} is the Moon-centric constant of gravity for the Earth. The forces given by equations (40) and (41) are equal according to Newton's third law:

$$G_{ME} \cdot M_E = G_{EM} m. \tag{43}$$

From this we obtain the Moon-centric gravity constant $G_{ME} = G_{EM} m/M_E = 0.048\,812\,174 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$.

At the same time, it is possible to determine the geocentric constant G_{ES} for the Sun. From

$$F_{SE} = -G_{SE}\frac{m_E}{\rho^2}$$

and

$$F_{ES} = -G_{ES}\frac{M_S}{\rho^2},$$

we can write

$$G_{ES} \cdot M_S = G_{SE} m_E \longrightarrow G_{ES} = GS_{SE}\frac{m_E}{M_S}. \tag{44}$$

Thus,

$$G_{ES} = 3.979\,466\,600\,6 \times 10^{14}\frac{0.0597 \times 10^{26}}{198\,91 \times 10^{26}} = 1\,194\,380\,152\ \mathrm{m^3\,s^{-2}}.$$

Table 2. Comparison of the standard and new values of the characteristic constant of gravity.

	Standard value (10^{14} m^3 s^{-2})	New value (10^{14} m^3 s^{-2})
Heliocentric	1 327 124.38	1 320 983.633
Geocentric for a satellite		4.000 579
Geocentric for the Moon	3.986 005	3.964 744 529
Geocentric average	3.986 005	3.986 838 915
Moon-centric average	0.049 027	0.048 812 174

6. Conclusion

Instead of the standard universal gravitational constant (Cavendish's constant of gravity) $f = 6.672 \times 10^{-11}$ m^3 kg^{-1} s^{-2} the following formula is presented:

$$f = \frac{\rho(\dot{\rho}^2 + \rho\ddot{\rho} - v_{\mathrm{or}}^2)}{m_1 + m_2} \tag{45}$$

based on which the Keplerian motion is deduced to be:

$$f_i = \frac{4\pi^2 a_i^3}{(M + m_i)T_i^2} = \frac{\mu_i}{M + m_i}. \tag{46}$$

Instead of the defined characteristic constant of gravity given by

$$\lambda_i = f m_i,$$

the formulae are here defined on the basis of Newton's laws for a central body B of mass M_{B} and its satellite of mass m_i with the primary body-centric constant of gravity

$$G_{\mathrm{B}i} = \frac{4\pi^2 a_i^3}{T_i^2}\frac{M_{\mathrm{B}}}{M_{\mathrm{B}} + m_i} = \mu_i \varepsilon_i, \tag{47}$$

where

$$\varepsilon_i = \frac{M_{\mathrm{B}}}{M_{\mathrm{B}} + m_i}$$

is the corrective factor.

The standard known numerical values of the characteristic constant of gravity are compared in table 2 with the new values.

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