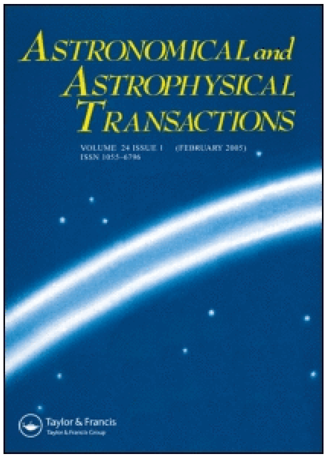

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 August 2006
To cite this Article: Kulikova, N. V. and Chepurova, V. M. (2006) 'Consideration of gravitational perturbations in the evolution of meteor complexes', Astronomical & Astrophysical Transactions, 25:4, 291 - 297
To link to this article: DOI: 10.1080/10556790601000089
URL: http://dx.doi.org/10.1080/10556790601000089

Astronomical and Astrophysical Transactions
Vol. 25, No. 4, August 2006, 291–297

Consideration of gravitational perturbations in the evolution of meteor complexes

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(Received 15 August 2006)

1. Introduction

A deterministic prognostic model of the meteor component of natural space debris allows the expected danger to be assessed not only qualitatively but also quantitatively in the artificial body passage through a certain space region [1]. With this in view, the primary method is a series of experiments based on a universal computer model for the formation and evolution of a meteor complex in the case of the specific parent object. Great interest is at present being expressed in the population of the Solar System and the dynamics of celestial bodies in connection with widening the scope of space research and a continuous increase in space routes. Planned space missions to such celestial objects as planets, planetary satellites, asteroids and comets require the maximum precise information on space contamination by small bodies of natural and artificial origin as well as on the evolution of the trajectories of their motion. This information is necessary to establish the safest space regions and to determine the best intervals between spacecraft launchings and missions. The problems of celestial body entry into near space, including all catastrophic consequences appearing as a result of the passage of these objects through the Earth's atmosphere and the impact of the objects themselves or their disintegration fragments on its surface, may be considered as geospace problems.

As small bodies of the Solar System in the process of evolution display an intricate behaviour which is difficult to describe and to explain within a classical celestial–mechanical theory of motion, it is necessary to develop methods for formalizing the probability process of substance ejection from a parent body at any space point and to study the models of newly appearing classes of small bodies. Also, chance is an essential component of most natural phenomena and a stochastic approach to solving the problems stated is considered to be

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Astronomical and Astrophysical Transactions
ISSN 1055-6796 print/ISSN 1476-3540 online © 2006 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/10556790601000089

appropriate [2]. Substance ejection processes are stochastic when the initial conditions of ejection are unknown *a priori*. Substance ejection can result in the formation of a meteor stream or associated phenomena. One of the criteria for establishing consanguinity between a meteor complex and a supposed parent body is the similarity of orbits. Theoretical studies together with computer simulation and a considerable body of observational data allow the quantitative as well as the qualitative characteristics to be obtained when considering concrete celestial objects.

2. Simulation

Recently, a fair amount of information has been published on some small bodies observed over long time intervals and this permits a probabilistic simulation to be carried out using input data which are considered to be reasonably reliable in terms of human knowledge. Now a computer technology has been developed [3], which is based on a stochastic model of formation and evolution of the orbits of meteor complex fragments formed in the nuclei disintegration of specific comets. The technology has been developed according to a module principle. It can allow one to expand its functions and to change the algorithms applied when necessary. A set of programs consists of a server and five software applications and has been constructed using the technology of dynamic data exchange. A series of computer experiments has been performed on the disintegration of comets most known for their numerous appearances: Comet Halley (26 appearances), Comet Tempel–Tuttle (ten appearances), Comet Giacobini–Zinner (11 appearances), Comet Pons–Winnecke (19 appearances), Comet Grigg–Skjellerup (18 appearances) and others. However, our studies of the formation and the dynamics of meteor complexes have been performed so far within the unperturbed Keplerian motion as the calculations made showed insignificant changes in the fragment orbital elements under the action of non-gravitational effects (the Pointing–Robertson effect, the Yarkovsky–Radzievsky effect, etc.) and secular planetary perturbations [1, 4].

To take into account gravitational perturbations, a separate built-in module is being developed now. The software part of the module is assumed to include the following components.

(i) *Gravitational disturbances in an N-body problem determined by numerical integration.* The RADAU program–Everhart algorithm [5] is used for $N = 2 - 8$.
(ii) *Perturbations from a gravity potential of a major planet in the case of small bodies approaching it.* Analytical formulae of an intermediate non-Keplerian hyperbolic orbit, which were developed previously on the basis of the symmetric version of a generalized two-fixed-centre problem and which consider the oblateness of a central body [6] taken as the planet of approach, are used.
(iii) *Perturbations of the Earth's gravity potential when a small body approaches it.* Calculations are performed according to the formulae for the intermediate hyperbolic orbit which are based on a non-symmetric version of a generalized two-fixed-centre problem.

3. Trajectory of a small body

When moving within the Solar System, the trajectory of a small body may be divided into three parts as follows.

3.1 *First part of the trajectory of a small body*

The largest part is when a body moves in a heliocentric orbit and experiences the greatest gravitational perturbations from major planets. This part of an orbit is considered in component (i) of our module. The Everhart algorithm is a special implicit one-step algorithm constructed from Runge–Kutta methods. It is based on the power expansion of an independent variable on the right-hand side of the equation of motion. A special method for optimal subdivision into substeps is applied (Gauss–Radau division). When all the calculations have been completed, the power expansion coefficients are specified in successive approximations. The integration step is controlled from the last term value in the power series expansion of an independent variable. To this end, two input parameters are introduced into the algorithm: one setting the precision of integration (LL) and another setting the order of power series expansion of a step (NOR). A step is chosen automatically, provided that the last term of power expansion is less than 10^{LL}. The parameters are chosen according to the ratio $0.75 < \mathrm{NOR} < 2\mathrm{LL}$, NOR being chosen from a certain sequence of given numbers. The basic version of the RADA 27 program (RADAU) uses computer representation of numbers with double accuracy and expansion of the right-hand sides of equations to the twenty-seventh order. Figure 1 presents the results obtained with the above algorithm (RADA-17 and RADA-27) for evolution of a semimajor axis and longitude of the node of Comet Temple–Tuttle in its appearances from 1533 to 1899. A diagram of these element variations constructed from observations is also given. It is clear that observations fall within the calculation range from two algorithm modifications; calculations from RADA-27 are closer to the observations. Disintegration fragments of the comet in its appearance in 1533 with the maximum and minimum deviations of Keplerian orbital elements from the parent body orbit seem to bound the space area where the comet evolves.

3.2 *Second part of the trajectory of a small body*

If a body finds itself in the sphere of action of a giant planet, the orbit of a small body may change abruptly when affected by the gravity of the approached planet; this planet may, in some measure, even destroy the body (or may destroy it completely as in case of Jupiter and Comet Levi–Schumacker). Therefore, it is essential to study the motion of a small body in the time interval (even it is small) when it approaches a major planet. The trajectory of a small body relative to the planet approached is often hyperbolic. As is well known, the spherical harmonics expansion of the gravitational potential of the giant planets almost does not involve odd harmonics. Therefore in calculations it is expedient to use an intermediate hyperbolic orbit of the symmetric version of a generalized two-fixed-centre problem [7–11], including perturbations from oblateness (the second harmonics in the potential expansion) of the approached planet relative to which an intermediate orbit is constructed. The algorithms are developed and a computer realization of four problems is performed.

Problem 1 is the transition from the elements of a heliocentric orbit of a small body to its planetocentric coordinates (x, y, z).

Problem 2 is the calculation of the intermediate orbit elements relative to the planet approached using planetocentric coordinates (x, y, z).

Problem 3 is the calculation of the planetocentric rectangular and cylindrical coordinates at any time moment using the intermediate orbit elements obtained in the preceding problem.

Problem 4 is the inverse transition from a planetocentric orbit to a heliocentric orbit.

We now consider these algorithms in more detail.

3.2.1 **Problem 1.** This can be divided into two simpler problems.

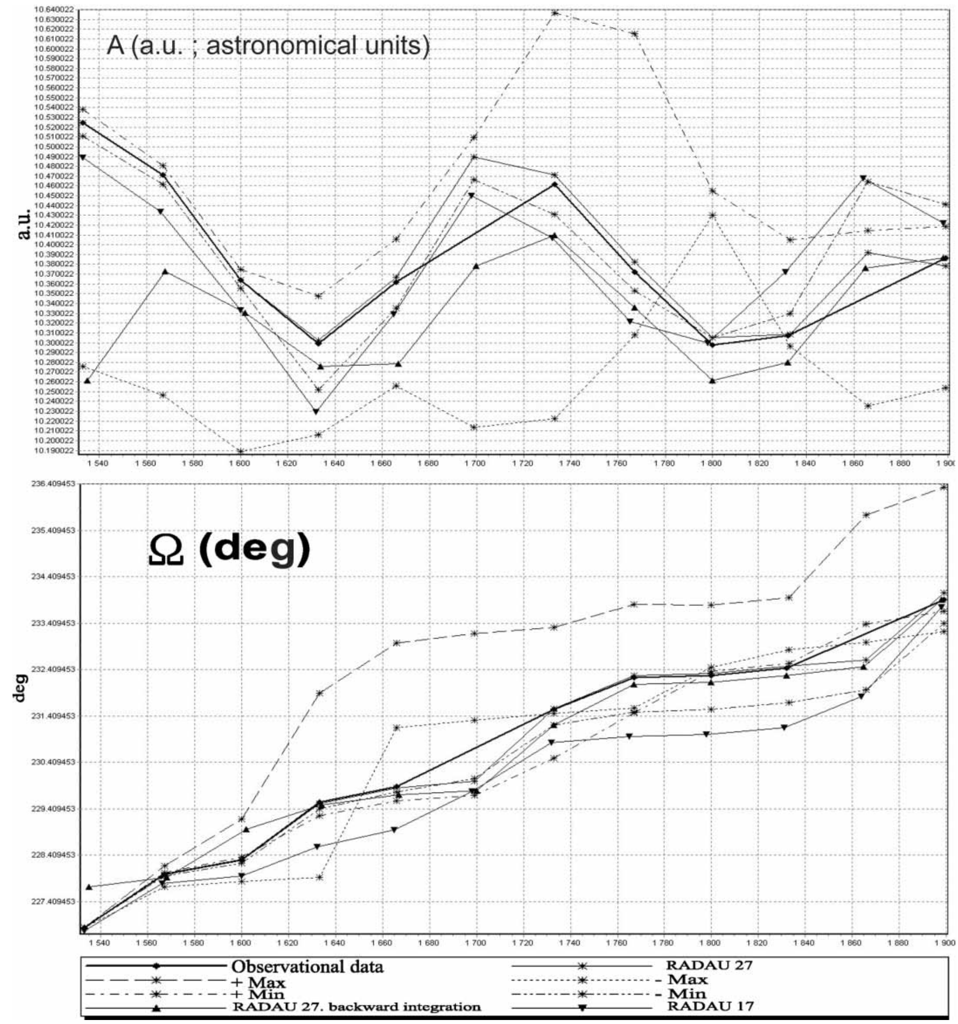

Figure 1. Variations in the Keplerian orbital elements of the disintegration fragments of the comet ((a) the semimajor axis; (b) the longitude of an ascending node) for different observational dates. (a.u.; astronomical units)

Problem 1(a) is as follows: when the elements of a heliocentric orbit of a small body are given, its rectangular coordinates (x', y', z') are calculated from the elements in the solar coordinate system. The equations used are well known (see, for example, [12]) and not given here.

Problem 1(b) concerns the transition from the solar coordinate system to that associated with the planet approached; this procedure is also well known from the literature [9, 12, 13]. As a result, we have a rectangular planetocentric coordinates (x, y, z) of a small body.

3.2.2 Problem 2.

Let us change the coordinates from (x, y, z) to the ellipsoidal coordinates (ξ, η, ζ):

$$2\xi^2 = r^2 - c^2 + [(r^2 - c^2)^2 + 4c^2z^2)]^{1/2}, \quad \xi' = \frac{\xi^2 r' + c^2 z z'}{\xi \boldsymbol{J}},$$

$$\eta^2 = \frac{z^2}{\xi^2}, \quad \eta' = \frac{\xi z' - \xi' z}{\xi^2},$$
$$w = \arctan\left(\frac{y}{x}\right), \left(\cos w = \cos\left[\arccos\left(\frac{x}{\rho}\right)\right],\right.$$
$$\left.\text{i.e. } \operatorname{sgn}(\cos w) = \operatorname{sgn} x\right), w' = \frac{xy' - x'y}{\rho^2}.$$

Here, c is the constant of a generalized two-fixed-centre problem; $c = R_0(-J_2)^{1/2}$, where R_0 is the equatorial radius of the central body, *e.g.* Jupiter; $r^2 = x^2 + y^2 + z^2$ (r is the magnitude of the radius vector of a point); $V^2 = x'^2 + y'^2 + z'^2$ (V is its velocity, and x', y', z' and ξ', η', ζ' are the velocity components); $\rho^2 = x^2 + y^2$; $r' = xx' + yy' + zz'$; $J = \xi^2 + c^2\eta^2$; $J' = \xi'^2 + c^2\eta'^2$.

The arbitrary constants of integration of the differential equations of motion including the energy constant h (for the hyperbola $h > 0$) are calculated, and new arbitrary constants $a < 0$, $e > 1$, $|s| \leqslant 1$ are introduced instead of them:

$$2h = V^2 - 2U, \quad c_1 = xy' - x'y, \quad 2c_2 = JJ' - V^2r^2 - 2Uc^2\eta^2,$$
$$U = \frac{fm_0\xi}{J}$$

(m_0 is the planet mass and f is the gravitational constant),

$$a_0 = -\frac{fm_0}{2h}, \quad e_0^2 - 1 = \frac{4hc_2}{f^2 m_0^2}, \quad 1 - s_0^2 = -\frac{c_1^2}{2c_2}, \quad p = a(1 - e^2) < R_0$$

and a small dimensionless parameter $\varepsilon = c/p$; all necessary values will be presented as a power series ε:

$$a_{i+1} = a_0[1 + \varepsilon_i^2(e_i^2 - 1)(1 - s_i^2) - \varepsilon_i^4 s_i^2(1 - s_i^2)(e_i^2 + 3)],$$
$$e_{i+1}^2 = 1 + (e_0^2 - 1)\{1 - \varepsilon_i^2(1 - s_i^2)(3e_i^2 + 1) + 2\varepsilon_i^4[3e_i^4 + 4e_i^2$$
$$+ 1 - s_i^2(5e_i^4 + 2e_i^2 + 1) + 2s_i^4(e_i^2 - 1)e_i^2]\},$$
$$s_{i+1}^2 = 1 - (1 - s_0^2)[1 - \varepsilon_i^2 s_i^2(e_i^2 - 1) + \varepsilon_i^4 s_i^2(e_i^2 - 1)(e_i^2 + 3 - 4s_i^2)].$$

The last three equations are solved by an iteration method. Then, the constants are determined [7, 9] for an intermediate orbit: a, e, s, ω, Ω, M (and ψ). When $c = 0$, the orbit becomes Keplerian and its constants are denoted as follows: a and e are the semimajor axis and eccentricity respectively, p is the orbital parameter, ψ is the true anomaly, s is the sine of the angle of inclination, ω is the angle which is $\pi/2$ different from the angular pericentre distance of an ascending node and Ω is the longitude of the Keplerian orbit ascending node. By analogy with the Keplerian orbit this allows the constants of an intermediate orbit to be called its elements.

3.2.3 Problem 3. An algorithm for problem 3 (see the formulae in [7–9]) is developed similarly.

3.2.4 Problem 4. This problem is inverse to problem 1 and realizes the change from a planetocentric orbit to a heliocentric orbit. As in the case of problem 1, the equations applied were taken from the same literature sources. The algorithms developed were used to study the motion of comets in their approach to Jupiter [10]. Two passages of Comet Brooks 2 through Jupiter's sphere of action were taken as the reference passages. To assess the relation between the orbital changes and the accuracy of input data (i.e. the coordinates of the comet's entry into Jupiter's sphere of action), the coordinates (x, y, z) and the radius vectors of the orbits for the reference examples have been varied ten times with a 0.002 step for all the coordinates. The calculation results are presented in figures 2 and 3.

According to Dubyago [13], after the close approach of Comet Brooks 2 in 1886 to Jupiter, satellites appeared. This provided evidence of the partial disintegration of a comet or ejections from it. Our calculations confirm notable variations in the coordinates and the radius vector of an orbit relative to Jupiter when leaving its sphere of action (figure 2). In the passage of 1922, the approach was much less. Changes in the coordinates x and y were greater but of opposite sign, which resulted in a parabolic deviation of the radius vector (figure 3).

3.3 *Third part of the trajectory of a small body*

The third part of the trajectory of a small body seems to be essential when a body approaches the Earth and there is a danger for our planet. In this case, it is necessary to determine the real-time trajectory of a hazardous object. In calculations the intermediate hyperbolic orbit from a non-symmetric version of a generalized two-fixed-centre problem including perturbations from the second and the third harmonics in the gravitational potential expansion of the

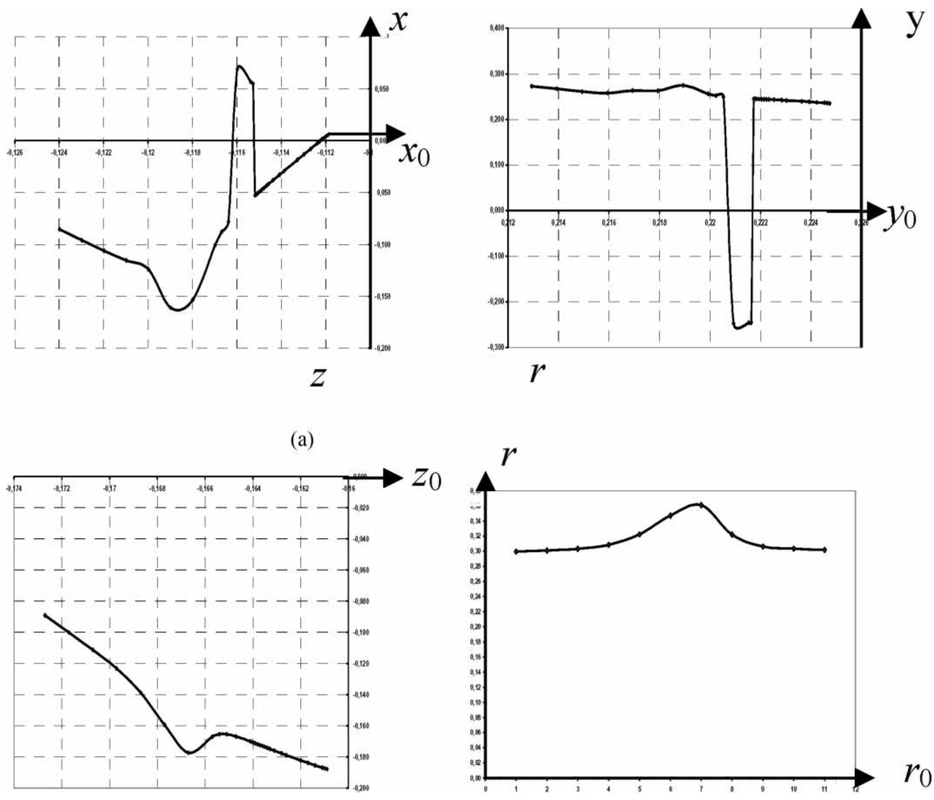

Figure 2. Passage of Comet Brooks in 1886.

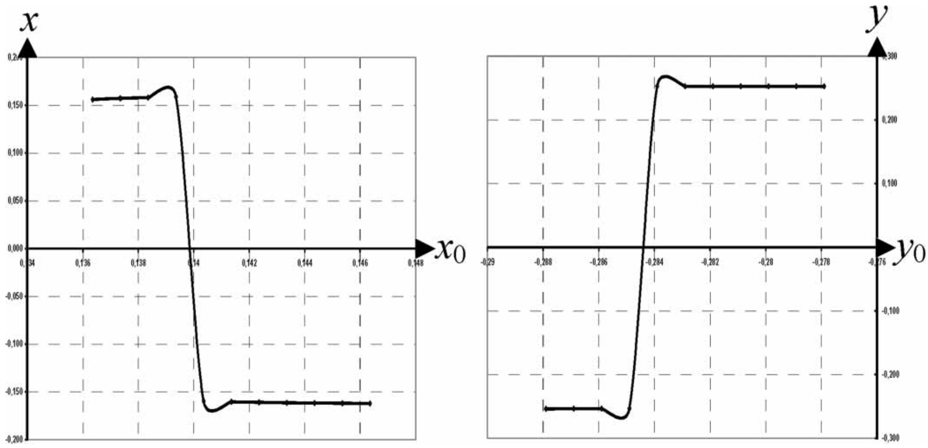

Figure 3. Passage of Comet Brooks 2 in 1922.

central body [14] may be useful. Algorithms describing the motion of a small body have been developed and are now being realized.

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