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Constructing self-consistent galactic models by Schwarzschild’s method

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A code for constructing self-gravitating galactic models in a given potential by Schwarzschild’s method was developed. As an example a distribution function for a two-component model of the Galactic potential by Kutuzov and Ossipkov was found.

Keywords: Stellar dynamics; Celestial mechanics; Galaxies; Modelling

1. Introduction

Constructing self-consisting models of gravitating systems is one of the main purposes of galactic dynamics [1]. There have been many attempts to develop both theoretical and semiempirical models of stellar systems but very often they suffer from principle [2] or technical [3] drawbacks. A numerical method for finding steady distribution functions in a given potential was proposed by Schwarzschild [4]. It includes integrating many orbits and in principle does not depend on the number of isolating integrals of motion in the potential under study. The method has been used by many researchers for calculating distribution functions for both integrable and non-integrable potentials. A modification of the method was proposed by Contopoulos and Grøsbol [5].

In this study, we develop a set of computer programs for constructing self-consistent models in a given mass distribution. We assume that the system is steady and axisymmetric, the density is non-negative and the total mass is finite.
2. Schwarzschild’s method

Let us recall briefly the main points of Schwarzschild’s method. The configuration space is considered as a set of \( N \) closed cells. The phase space is stratified into \( K \) orbits calculated in a given mass distribution. The portion of time spent by any orbit in any cell is called the stay time of the orbit in that cell. So, the mass of any cell is proportional to the sum of such times over orbits crossed by the cell. The orbit weights (i.e. their masses) are unknown and are to be found under the restriction that they must be non-negative. This is equivalent to the problem of solving a linear system in that the number of equations is equal to the number of cells, the number of unknown quantities (weights or masses) is equal to the number of orbits, and the right-hand sides are equal to the densities (prescribed or found from a given potential) under the condition that unknown quantities are non-negative. Some variants of the simplex method of linear programming are used as a rule for solving the system.

3. Set of programs

The set of programs developed includes the following modules:

(i) a module of space dividing into a given number of three-dimensional cells;
(ii) a module for the calculation of the starting parameters for computing orbits, namely \((E, J)\) (integrals of energy and momentum) and \((R, z, v_R, v_z)\) (cylindrical coordinates);
(iii) a module for integration of the orbits over a sufficient time for dense filling of cells (under the given accuracy);
(iv) a module for the creation of a library of orbits;
(v) a module for working with a database of data structured over orbits and cells;
(vi) a module for calculation of the weights by the simplex method;
(vii) a module for calculation of the kinematic parameters of the obtained model.

4. An example: a model by Kutuzov and Ossipkov

A two-component model of our Galaxy reported by Kutuzov and Ossipkov [6, 7] was considered as an example for applying the method developed. The starting description function of the model is the gravitational potential. The first component (the ‘disc’) is a combination of the generalized isochrone potential obtained by Kuzmin and Malasidze [8] and the equipotentials given by Miyamoto and Nagai [9]. It depends on three dimensionless structure parameters. The second component (the ‘halo’) is a spherical ‘limiting’ model published by Kuzmin and Veltmann [10] known also as Hernquist’s [11] model. It depends on one structure parameter. The two-component Galaxy model depends also on two scale parameters and on the weights of components. The values of parameters were estimated to fit the observational data of our Galaxy [6, 7].

5. Some results

Orbit calculations were fulfilled on the basis of Everhart’s [12] method with double precision. The intervals employed in the calculation were some hundreds of orbital revolution times (the orbital periods near the Galactic centre were much less than those at 15 or 20 kpc).
Conservation of the integrals of motion for $E$ and $J$ along the orbits was controlled with an accuracy of $10^{-8}–10^{-9}$. Finding the solutions for the orbit weights by the simplex method was a procedure that demanded most of the computation time. Until now we have calculated only orbits with the following initial conditions: $z = 0$, $v_R = 0$. Note that all the calculated orbits were found to be ordered of the box type. Figures 1(a) and (b) show examples of the density profiles along the $R$ and $z$ axes respectively for a two-component model of the Galactic potential given by Kutuzov and Ossipkov. There are discrepancies in the obtained distribution functions of density with respect to the theoretical values only at the extreme bounds of model.

6. Conclusions

(1) Generally, a solution is not unique. A given mass distribution can be obtained from various sets of orbits and various kinematic properties. In particular, to construct models with smaller $\sigma_z$, orbits must be calculated with initial $v_R \neq 0$ (under fixed $E$ and $J$).

(2) When models with a given potential are considered, a domain containing 90–99% of the total mass must be taken into account.

(3) After finding a steady distribution function, it is necessary to check whether it is stable and can survive under real conditions. For this purpose the model can be approximated as an $N$-body system and observed by simulation.
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