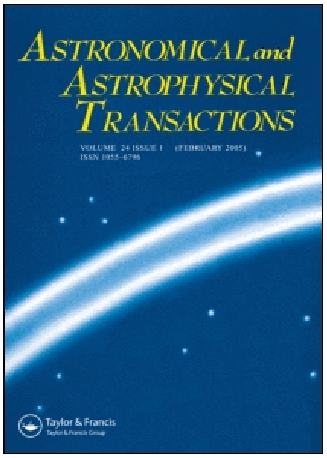

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On the fundamental paradox of stellar dynamics

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Various ways of solving the relaxation paradox for stellar systems are discussed. They include scattering on massive wave or material objects (giant molecular clouds, transient spiral arms and halo black holes), a revision of the Jeans–Chandrasekhar relaxation theory (resonant relaxation, relaxation in the smoothed field and collective relaxation in a steady non-integrable potential and in a non-steady (oscillating) mean field (violent relaxation)).

Keywords: Stellar dynamics; Celestial mechanics; Galaxies; Relaxation

1. Introduction

We shall consider stellar systems as statistical ensembles of N gravitating point masses. Real stellar systems are known to be generally in equilibrium. Their present state could be reached under the action of smoothed or bulk gravitational forces (regular forces according to the terminology employed by Schwarzschild [1]) and random forces due to close stellar encounters (irregular forces according to [1]) [2, 3]. The timescale of regular forces is the so-called crossing time [3, 4] equal to $t_c = (R^3/GM)^{1/2}$. Here, G is the gravitational constant, M is the mass of a system and R is its typical size. According to the classical theory of stellar encounters by Jeans and Chandrasekhar (constructed by analogies with the gas theory developed by Maxwell, Boltzmann and Jeans himself) the relaxation time, i.e. the timescale of irregular forces [3], is $t_r = \sigma^3/(8\pi G^2 m^2 n \ln \Lambda)$, where m is the mass of a star, n is the number density, σ is the typical relative stellar velocity and $\ln \Lambda$ is the so-called Coulomb logarithm (and $\Lambda \approx N$). It follows from the virial theorem that $\sigma^2 \approx (1 + \gamma^2)^{-1} GM/R$, with $\gamma = V/\sigma$, V being a mean streaming velocity. For flat subsystems of our Galaxy, $\Lambda \approx 10 \gg 1$. Then $t_r/t_c \approx [N/(\ln N)]/6\pi\gamma^3$. For galaxies the relaxation time t_r is of the order of 10^{14} years and is much larger than the Hubble time. So, if the Jeans–Chandrasekhar theory is correct, the evolution of galaxies is governed by regular forces only. It was believed for decades that such evolution must be deterministic and the observed structure of galaxies reflects their initial

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state. Ogorodnikov [3] called it a fundamental paradox of classical stellar dynamics. Since the 1950s there have been many attempts to solve that paradox, and we shall try to discuss very briefly their main ideas.

However, first, we must be sure that the paradox exists in reality. It was discussed by the present author in another paper [5]. The existence of moving clusters is the main objection against a short timescale of dynamic evolution (Eddington's [6] consideration, but see the discussion in [7]). The main arguments in favour of irreversible evolution are the following: the universality of the galactic structure (which can be approximated by de Vaucouleurs' profile for elliptical galaxies, or by Sersic's more general formula) and the velocity dispersion–age relation for stars of the Galactic disc (reviewed recently in [8]). Some observational evidence of a certain kind of relaxation in galaxies, groups and clusters of galaxies was also summarized in [9]. The results of numerous N-body simulations also show the exponential divergence of close trajectories, i.e. stochasticity. However, conclusions on mixing and relaxation can be drawn from such experiments only when using the 'Maupertuis parameterization' [10].

2. Scattering on massive objects and arms

The first idea was proposed by Spitzer and Schwarzschild [11]. It was natural to propose that relaxation in stellar systems results from star scattering on massive objects. Then the relaxation time [3] $t_\mathrm{r} = A/m_\mathrm{c}^2 n_\mathrm{c}$, where m_c is the mass of such objects, n_c is their number density and A is almost the same as for stellar encounters. We can choose m_c and n_c and obtain the desirable order for t_r [11]. Usually hypothetical objects of typical mass $10^6 M_\odot$ are considered as perturbers. Neglecting the anisotropy of velocities and solving the Fokker–Planck diffusion equation, Spitzer and Schwarzschild [11] also studied the time dependence of the velocity distribution and were the first to establish a velocity dispersion–age relation. The same idea was proposed also by Lebedinsky [12] but he stressed that, if perturbers lie in the Galactic plane, then the resulting velocity distribution will be triaxial. A more detailed theory of disc heating and thickening was developed by Gurevich [13]. He studied a flattened system with epicyclic unperturbed orbits and found that $\sigma_z/\sigma_R \approx 0.6$ (which is close to observations). At that time the main objection to such theories was the absence of such massive perturbers in the Galaxy.

After the discovery of giant molecular clouds (GMCs) the situation changed. First, Fujimoto [14] developed an analytical theory of scattering on short-lived clouds. It was argued that this assumption is not applicable to GMCs. However, relaxation of unstable fluctuations of star density (as proposed by Lebedinsky [12], Gurevich [13] and Marochnik [15]) or of fluctuations of interstellar gas [16] can be studied by a similar theory. The most detailed theory of star scattering on GMCs was developed by Lacey [17]. He investigated the increase in the velocity dispersions of stars on epicyclic orbits and found that initially the velocity ellipsoid relaxes to a steady shape and this is then followed by steady heating. However, Lacey found that $\sigma_z/\sigma_r \approx 0.8$, i.e. too large a σ_z/σ_r value. Also, velocity dispersions increase too slowly for the observed estimates of GMC masses. The same difficulties were noted also by Kamahori and Fujimoto [18] and Yasutomi and Fujimoto [19]. The main features of the theory were confirmed by numerical investigation [20].

So, some other massive perturbers were discussed. Lacey and Ostriker [21], Kamahori and Fujimoto [18] and Ipser and Semenzato [22] considered relaxation of the black holes of the Galactic halo. The idea was supported by Fuchs *et al.* [23]. In that case some problems arise, both dynamic (the friction of perturbers on stars causes them to spiral to the Galactic centre) and astrophysical (no gas accretion on the black hole is observed) [21]. Carr and Lacey [24] studied relaxation of dark halo clusters that can be composed of 'Jupiters', but at present such objects are unknown.

Most researchers considered scattering on GMCs as only one of the relaxation mechanisms that must be enhanced by others (see, for example, [25]). Ida *et al.* [26] revisited a theory suggested by Lacey and found some drawbacks. According to [26], σ_z was overestimated, and then the equilibrium ratio σ_z/σ_r is about 0.5 in the solar neighbourhood. Let us recall an elegant analytical theory described by Kuzmin [27] and in other publications (see, for example, [28]). The viewpoints of Kuzmin and of Lacey coincide, but Kuzmin found that, if a steady shape of the velocity ellipsoid exists (this was proved later by Lacey [17]), then $(\sigma_R/\sigma_z)^2 = 1 + (\sigma_R/\sigma_\varphi)^2$, which is confirmed by observational data. Kuzmin's theory is abstract. It seems very desirable to revisit it, taking into account the observational characteristics of GMCs and to find a timescale of establishing a steady σ_z/σ_r ratio.

Many workers have considered spiral arms as alternative perturbers. Barbanis and Woltjer [29] were the first to consider the increase in the velocity dispersion in a spiral potential. They generalized the epicyclic theory and averaged squares of the equations of motion. The equations in [29] are time irreversible (owing to the Coriolis forces), but not statistically irreversible [30]. However, it would be interesting to check whether very slow collisional relaxation in a spiral potential provides fast stochastization. It seems very plausible and was confirmed by simulations in [19].

Relaxation of transient spiral density waves was studied by Sellwood and co-workers in a number of papers (see, for example, [31–34]) and also by Marochnik [30], Jenkins and Binney [25], Yasutomi and Fujimoto [19, 35], Fuchs [36], Griv *et al.* [37] and others. Usually spiral arms are considered as effective perturbers but they cannot provide increasing σ_z [4].

Note that scattering on spiral arms cannot solve the relaxation paradox for stars of the Galactic halo, and also for E and SO galaxies. Star relaxation of both spiral arms and massive GMCs can be considered as a process intermediate between 'collisional' star–star relaxation and collisionless relaxation. Antonov *et al.* [38] called it 'quasidiffusion', a special kind of phase–space mixing.

We mention here the ideas of relaxation of 'tidal streamers' [39] and moving clusters [40] and scattering by transient large-scale noise due to discreteness [41, 42]. They are of interest in principle but cannot solve the problem.

3. Classical theory revisited

The only conclusion that undoubtedly can be drawn from the Jeans–Chandrasekhar theory is that the theory is not correct. It is based on the following related approximations [43].

(i) The system is assumed to be infinite and homogeneous, and the regular force is neglected. Kuzmin [44] was the first to try to take regular forces into account.
(ii) With respect to the Markov approximation, a theory given by Prigogine and Severne [45] is free of this assumption.
(iii) Considering the diffusion approximation and using the Fokker–Planck equation, only Agekian [46] and Petrovskaya [47] and their few followers considered the action of irregular forces as a purely discontinuous random process.
(iv) The self-gravity of response is neglected although sometimes it can be significant [41].

The correct theory of relaxation in a gravitational field has not been constructed although some steps towards this have been made. Stars move along complicated orbits governed by regular forces, and the main problem is to take it into account. First we mention resonant relaxation [48], which has been studied by Genkin [49] for a flattened system. The idea is that the interaction of stars with close periods of regular motion is the most effective. Using some formulae from plasma physics, Genkin found that the time of effective relaxation of directions of motion (Chandrasekhar's [2] T_D) is of the order of $(t_c t_r)^{1/2}$ (for simplicity we assumed that the period of epicyclic oscillations is of the order of t_c). A more detailed theory was developed by Rauch and Tremaine [50]. It seems that their value of relaxation time is of the same order. Resonant interactions were noticed in some simulations, but they are too slow to be very effective. Note that they do not influence the relaxation of energies (denoted T_E by Chandrasekhar).

Genkin tried to develope a general theory of relaxation in steady and non-steady regular fields [51–53]. The idea is that regular forces act as an effective accelerator of slow diffusion in velocity space. The most convincing example considered by him is relaxation in the field of a homogeneous system [53]. On the basis of results obtained by Chandrasekhar, Genkin found an effective relaxation time $t_e \approx (t_r t_c^2)^{1/3}$. The same expression had been suggested earlier by Kurth [54] from dimensional analysis. Sementsov [55] stated that, for spherical systems, $t_e \approx (t_r t_c^4)^{1/5}$.

Gurzadyan and Savvidi [56] studied the relaxation problem by trying to apply the ergodic theory and following the method emloyed by Krylov. Using the 'Maupertuis parameterization' they found an expression for Riemann curvature but it was too complicated for analysis. So, they utilized statistical averaging, using the virial theorem (as the system was supposed to be steady), and applied a truncated Holtzmark distribution for an irregular force (which was deduced for a homogeneous gravitating system). Finally the models studied in [56] and in [53] were very similar, and it is not surprising that there the effective relaxation times coincide. The present author hopes to publish some critical comments on [56] in a special paper.

4. Encounterless relaxation

Now it is clear that regular forces can produce chaotic orbits, and oscillations of a smoothed gravitational field can provide stochasticity (an analogy with a pendulum under a periodic external force is useful) [57]. Both phenomena will cause irreversible evolution of star systems. The first was called 'divergent mixing' in [38], and the second 'compulsive mixing'. Divergent mixing has been studied numerically [58, 59]. It is a very fast process but it seems that its real dynamic significance is doubtful for we do not know the portion of chaotic orbits for realistic potentials. If it was a main relaxation process, we can expect a great difference between the structures of galaxies with non-integrable (triaxial?) potentials and the structures of spherical galaxies.

As for compulsive mixing (including the famous violent relaxation as its special case) there have been some attempts to develop its formal theory [60], but it is not clear what physical mechanism will provide stochastization. According to [9], random actions of external galaxies will trigger initially a large-scale relaxation (Poincaré chaos) and then a transition to Maxwell–Boltzmann chaos.

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