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Electromagnetic and gravitational radiation of graviatoms

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The conditions for the existence of a graviatom have been found. The graviatoms (quantum systems around mini-black-holes) satisfying these conditions contain the following charged particles: the electron, muon, tau lepton, wino, pion and kaon. Electric dipole and quadrupole radiation and gravitational radiation are calculated for the graviatoms and compared with the Hawking mini-hole radiation.

Keywords: Graviatoms; Electromagnetic radiation; Gravitational radiation

1. Introduction

The motion of microparticles on scales larger than the Compton length is quantized in curved space–time. Kuchař [1] has shown that the latter reduces to the Schrödinger equation with the Newtonian potential in a non-relativistic case. The behaviour of charged particles in a centrally symmetric gravitational field was considered by DeWitt and DeWitt [2] who obtained a so-called self-force, acting on the charge, apart from the Newtonian gravitational force.

The quantum-mechanical problem of electron motion in the gravitational field of a mini-hole was considered by Gaina [3] who obtained hydrogen-like solutions.

The general case of charged-particle motion in the Schwarzschild field was considered later [4–6], taking account of the self-force obtained by DeWitt and DeWitt. Primordial black holes (or mini-holes) can capture charged particles owing to gravitational interaction. Bound quantum systems maintaining a charged particle in orbit around a mini-hole were called graviatoms [6].

In the present article, we shall derive the conditions for the existence of a graviatom and calculate the electromagnetic radiation and gravitational radiation to be compared with the Hawking mini-hole radiation.

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2. Theoretical solution to the problem

The Schrödinger equation for the graviatom [4] given by

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR_{pl}}{dr} \right) - \frac{l(l+1)}{r^2} R_{pl} + \frac{2m}{\hbar^2} \left(E - \frac{mc^2 r_Q r_g}{4r^2} + \frac{mc^2 r_g}{2r} \right) R_{pl} = 0 \quad (1)$$

describes a radial motion of a particle with the charge Q and mass m in the effective mini-hole potential, taking into account the self-interaction obtained by DeWitt and DeWitt, as follows:

$$U_{\text{eff}} = -\frac{mc^2 r_g}{2r} + \frac{mc^2 r_Q r_g}{4r^2} + \frac{\hbar^2 l(l+1)}{2mr^2}, \quad (2)$$

where $r_g = 2GM/c^2$ and M are the mini-hole gravitational radius and mass, respectively and $r_Q = Q^2/mc^2$ is the classical radius of the charged particle.

The solution to (1) has the form

$$R_{pl} = \text{constant} \times \rho^s e^{-\rho/2} F(-p, 2s+2, \rho), \quad (3)$$

where $F(-p, 2s+2, \rho)$ is the confluent hypergeometric function,

$$\rho = \frac{2(-2mE)^{1/2}}{\hbar} r, \quad s(s+1) = \frac{2mA}{\hbar^2} + l(l+1),$$

$$A = \frac{mc^2 r_Q r_g}{4}, \quad B = \frac{mc^2 r_g}{2}, \quad p = n - s - 1,$$

$p = 0, 1, 2, \dots, l \leq n$, $n = 1, 2, 3, \dots$, and n and l are the principal and orbital quantum numbers, respectively.

The energy spectrum of the charged particle captured by the mini-hole is

$$E = -\frac{2B^2 m}{\hbar^2} \frac{1}{\left\{ 2p+1 + [(2l+1)^2 + 8mA/\hbar^2]^{1/2} \right\}^2}. \quad (4)$$

If we take account of only the electromagnetic and gravitational interactions and neglect the strong interaction, determining nuclei sizes, the solutions (3) and (4) for the nuclei with the mass $m = 2Zm_p$ and charge $Q = Ze$ can be divided into two cases: firstly, the hydrogen-like case for light nuclei with $r_g r_Q / \lambda_c^2 \ll 1$, whose energy spectrum is given by the formula

$$E = -\frac{4Z^3 m_p^3 G^2 M^2}{\hbar^2 n^2}, \quad (5)$$

where $\lambda_c = \hbar/mc$ is the Compton wavelength, m_p is the proton mass and G the gravitational constant; secondly, the case for the Kratzer potential, which is valid for heavy nuclei with $r_g r_Q / \lambda_c^2 \gg 1$, whose energy spectrum takes the form

$$E = -\frac{mc^2 r_g}{4r_Q} + \frac{\hbar c}{r_Q} \left(p + \frac{1}{2} \right) \left(\frac{r_g}{2r_Q} \right)^{1/2} + \frac{\hbar^2}{2mr_Q^2} \left(l + \frac{1}{2} \right)^2$$

$$- \frac{3\hbar^2}{2mr_Q^2} \left(p + \frac{1}{2} \right)^2 - \frac{3\hbar^3}{m^2 c r_Q^2 (2r_g r_Q)^{1/2}} \left(p + \frac{1}{2} \right) \left(l + \frac{1}{2} \right)^2, \quad (6)$$

where the second term describes oscillations, the third term rotations, the fourth term the anharmonicity of oscillations and the fifth term the oscillation-rotation coupling. Below we shall call this case an oscillatory case.

3. The conditions for the existence of a graviatom

A graviatom can exist if the following conditions are fulfilled:

- (i) the geometrical condition $L > r_g + R$, where L is the characteristic size of the graviatom and R is the characteristic size of a charged particle;
- (ii) the stability condition given by
 - (a) $\tau_{\text{gr}} < \tau_{\text{H}}$, where τ_{gr} is the graviatom lifetime and τ_{H} is the mini-hole lifetime, and
 - (b) $\tau_{\text{gr}} < \tau_{\text{p}}$, where τ_{p} is the particle lifetime (for unstable particles);
- (iii) the indestructibility condition (due to tidal forces and the Hawking effect) $E_{\text{d}} < E_{\text{b}}$, where E_{d} is the destructive energy and E_{b} is the binding energy.

If we introduce the dimensionless quantity

$$\alpha = \frac{GMm_{\text{p}}}{e^2}, \quad (7)$$

then the hydrogen-like case will correspond to

$$\alpha \ll \left(\frac{\hbar c}{e^2}\right)^2 \frac{m_{\text{p}}}{mZ^2} \quad (8)$$

and the oscillatory case to

$$\alpha \gg \left(\frac{\hbar c}{e^2}\right)^2 \frac{m_{\text{p}}}{mZ^2}. \quad (9)$$

The characteristic size of a hydrogen-like graviatom $L = a_{\text{B}}^{\text{g}}$, where

$$a_{\text{B}}^{\text{g}} = \frac{\hbar^2}{GMm^2} \quad (10)$$

is the Bohr radius for the graviatom. The characteristic size of an oscillatory graviatom is $L = r_{\text{Q}}$. The characteristic size of a nucleus is $R = 1.25 \times 10^{-13} A^{1/3}$ cm, where A is the atomic weight of the nucleus.

The graviatom lifetime is

$$\tau_{\text{gr}} = \left(\frac{\hbar c}{e^2}\right)^5 \left(\frac{m_{\text{p}}}{\alpha m}\right)^4 \frac{\hbar}{Z^2 m c^2}, \quad (11)$$

and the mini-hole lifetime is

$$\tau_{\text{H}} = \frac{15360\pi G^2 M^3}{\hbar c^4}. \quad (12)$$

The lifetime of an unstable particle is

$$\tau_{\text{p}} = \frac{\hbar}{\Gamma}, \quad (13)$$

where Γ is the natural linewidth.

The destructive energy is $E_{\text{d}} = \{U_{\text{t}}, E_{\text{H}}\}$, with the tidal energy

$$U_{\text{t}} = \frac{GMmR}{r^2} \quad (14)$$

and the Hawking radiation energy

$$E_{\text{H}} = \frac{b}{8\pi} \frac{\hbar c}{e^2} \frac{m_{\text{p}} c^2}{\alpha}, \quad (15)$$

where $\hbar\omega_{\text{m}} = bKT$, according to the Wien displacement law, and $b = 2.822$.

The binding energy is $E_b = \{E_W, I_{\text{ion}}\}$, where E_W is the nuclear binding energy. The graviatom ionization energy is

$$I_{\text{ion}} = \frac{m^3 e^4 \alpha^2}{2n^2 \hbar^2 m_p^2}. \quad (16)$$

In terms of α , the conditions for the existence of a graviatom are

$$\alpha < \frac{1}{2^{1/2}} \frac{\hbar c m_p}{e^2 m} \quad (a_B^g > r_g, R \ll r_g), \quad (17)$$

$$\alpha > \left(\frac{\hbar c}{e^2} \right)^{8/7} \left(\frac{m_p^7}{15360\pi Z^2 m_{\text{pl}}^2 m^5} \right)^{1/7} \quad (\tau_{\text{gr}} < \tau_{\text{H}}), \quad (18)$$

where $m_{\text{pl}} = (\hbar c/G)^{1/2}$ is the Planck mass,

$$\alpha > \left(\frac{\hbar c}{e^2} \right)^{5/4} \frac{m_p}{m} \left(\frac{\Gamma}{mc^2} \right)^{1/4} \frac{1}{Z^{1/2}} \quad (\tau_{\text{gr}} < \tau_p), \quad (19)$$

$$\alpha > \frac{b}{8\pi} \frac{\hbar c m_p c^2}{e^2 E_W} \quad (E_{\text{H}} < E_W), \quad (20)$$

$$\alpha > \frac{\hbar c m_p}{e^2 m} \left(\frac{bn^2}{4\pi} \right)^{1/3} \quad (E_{\text{H}} < I_{\text{ion}}), \quad (21)$$

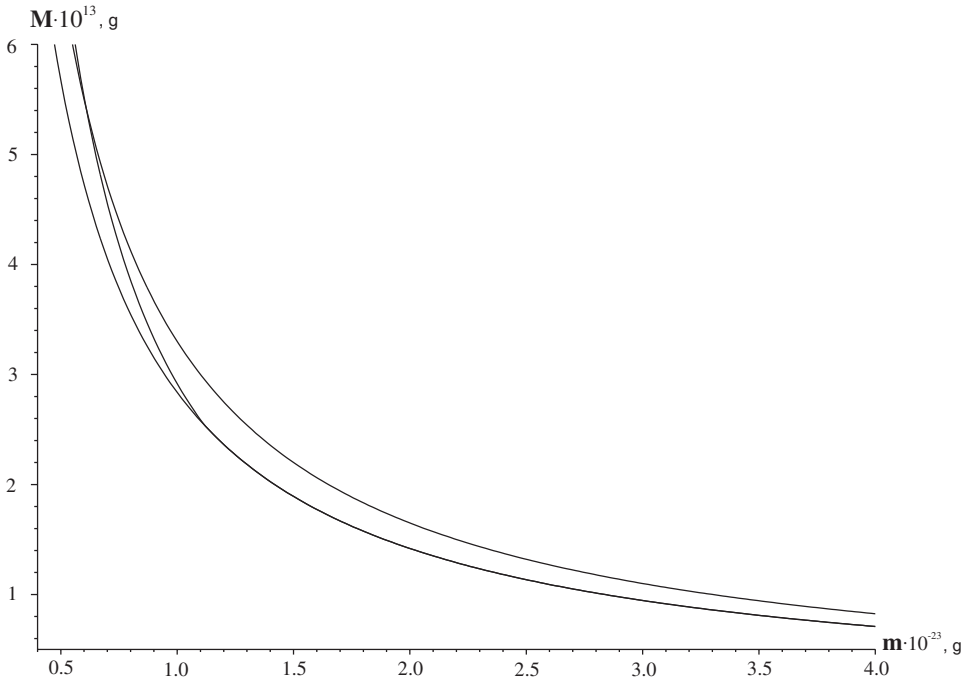


Figure 1. The dependence of mini-hole masses on the charged-particle masses satisfying the conditions for the existence of a graviatom. The light curves indicate the range of values related to the geometrical condition (the upper curve) and to the ionization condition of the Hawking effect (the lower curve). The heavy curve is related to the particle stability condition ($\tau_p = 10^{-22}$ s).

$$\alpha > \left(\frac{\hbar c}{e^2}\right)^{4/3} \frac{m_p}{m} \left(\frac{E_W e^2}{m^2 c^4 R}\right)^{1/3} \quad (U_t < E_W), \quad (22)$$

$$\alpha < \left(\frac{\hbar c}{e^2}\right)^2 \frac{m_p}{m} \frac{e^2}{2n^2 m c^2 R} \quad (U_t < I_{\text{ion}}). \quad (23)$$

The conditions (17)–(23) prove to be fulfilled only for hydrogen-like graviatoms, i.e. if the condition (8) is satisfied. Moreover, it is the graviatoms containing leptons and mesons with $Z = 1$ that satisfy these conditions. On the contrary, neither the graviatoms containing hadrons nor atomic nuclei can satisfy them for both the hydrogen-like case (hadrons and light nuclei) and the oscillatory case (heavy nuclei). The charged particles that are able to be constituents of the graviatom are the electron, muon, tau lepton, wino, pion and kaon.

The conditions for the existence of a graviatom restrict the quantity α (proportional to the mini-hole mass M) to within a narrow range of its values depending on the charged-particle mass m (figure 1).

4. Graviatom radiation

The intensity of the electric dipole radiation of a particle with mass m and charge e in the gravitational field of a mini-hole is

$$I_{fi}^d = \frac{2\hbar e^2 \omega_{if}^3 f_{if}}{m c^3}, \quad (24)$$

where $\omega_{if} = (E_i - E_f)/\hbar$ is the frequency of the transition $i \rightarrow f$ and f_{if} is the oscillator strength [7]. The condition that the radiation is a dipole one is

$$\alpha < \frac{m_p \hbar c}{m e^2}, \quad (25)$$

which almost coincides with the geometrical condition (17) to be satisfied for the graviatoms under consideration.

The electric dipole radiation intensity for a hydrogen-like graviatom performing the transition $2p \rightarrow 1s$ is

$$I_{12}^d = \frac{2\hbar e^2 \omega_{21}^3}{m c^3} f_{2p \rightarrow 1s}, \quad (26)$$

with the oscillator strength

$$f_{2p \rightarrow 1s} = \frac{2^{13}}{3^9} = 0.4162 \quad (27)$$

and the transition energy

$$\hbar \omega_{12} = \frac{3\alpha^2 e^4 m^3}{8 m_p^2 \hbar^2}. \quad (28)$$

Finally, we obtain

$$I_{12}^d = \frac{2^5 \alpha^6 e^{12} m^8}{3^6 c^3 \hbar^8 m_p^6}. \quad (29)$$

The electric dipole radiation intensity for the transition $3p \rightarrow 1s$ is

$$I_{13}^d = \frac{2\hbar e^2 \omega_{31}^3}{m c^3} f_{3p \rightarrow 1s}, \quad (30)$$

with the oscillator strength

$$f_{3p \rightarrow 1s} = \frac{3^4}{2^{10}} = 0.0791 \quad (31)$$

and the transition energy

$$\hbar\omega_{31} = \frac{2^2 \alpha^2 e^4 m^3}{3^2 \hbar^2 m_p^2}. \quad (32)$$

Finally, we obtain

$$I_{13}^d = \frac{\alpha^6 e^{14} m^8}{2^3 3^2 c^3 \hbar^8 m_p^6}. \quad (33)$$

Hence, it follows that

$$\frac{I_{12}^d}{I_{13}^d} = \frac{2^8}{3^4} = 3.161, \quad \frac{\omega_{21}}{\omega_{31}} = \frac{3^3}{2^5} = 0.844.$$

The electric quadrupole radiation intensity for the transition $3d \rightarrow 1s$ is

$$I_{13}^q = \frac{6\hbar e^2 \omega_{31}^3}{mc^3} f_{3d \rightarrow 1s} \quad (34)$$

with the oscillator strength

$$f_{3d \rightarrow 1s} = \frac{3^7}{2^{16}} \left(\frac{Mm}{m_{pl}^2} \right)^2, \quad (35)$$

where the transition energy ω_{31} is given by equation (32). Finally, we obtain

$$I_{13}^q = \frac{\alpha^8 e^{18} m^{10}}{2^3 3^4 c^5 \hbar^{10} m_p^8}. \quad (36)$$

The gravitational radiation intensity for the graviatom performing the transition $3d \rightarrow 1s$ is

$$I_{13}^g = \frac{6\hbar GM \omega_{31}^3}{c^3} f_{3d \rightarrow 1s}, \quad (37)$$

where the oscillator strength and the transition energy are given by equations (32) and (35), respectively. Finally, we obtain

$$I_{13}^g = \frac{\alpha^9 e^{18} m^{11}}{2^3 3^4 c^5 \hbar^{10} m_p^9}. \quad (38)$$

The mini-hole creates particles near its horizon that can ionize graviatoms and split nuclei, which are their constituents. The power due to the Hawking effect is given by the formula

$$P_H = \frac{1}{15360\pi\alpha^2} \left(\frac{\hbar c}{e^2} \right)^2 \frac{(m_p c^2)^2}{\hbar}. \quad (39)$$

The Hawking energy is

$$E_H = \frac{b}{8\pi\alpha} \frac{\hbar c}{e^2} m_p c^2. \quad (40)$$

The mass M of the mini-hole decreases owing to evaporation due to the Hawking effect, i.e.

$$M_f = \left(M_i^3 - \frac{\hbar c^4}{5120\pi H G^2} \right)^{1/3}, \quad (41)$$

where M_f and M_i are the final and initial mini-hole masses, respectively and H is the Hubble parameter. $M_f = 0$ corresponds to the minihole mass M_f that has evaporated during the

Table 1. Graviatom parameters for the electron, muon and tau lepton.

	Value for the following charged particles					
	e		μ		τ	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
mc^2 (MeV)		0.511		105.659		1.777×10^3
τ_p (s)		∞		2.200×10^{-6}		2.900×10^{-13}
M (g)	3.12×10^{17}	3.62×10^{17}	1.51×10^{15}	1.75×10^{15}	9.96×10^{13}	1.04×10^{14}
r_g (cm)	4.69×10^{-11}	5.46×10^{-11}	2.27×10^{-13}	2.64×10^{-13}	1.35×10^{-14}	1.57×10^{-14}
a_B^g (cm)	5.46×10^{-11}	6.35×10^{-11}	2.64×10^{-13}	3.07×10^{-13}	1.57×10^{-14}	1.83×10^{-14}
$\hbar\omega_{12}$ (MeV)	0.071	0.096	14.64	19.81	246.2	333.2
$\hbar\omega_{13}$ (MeV)	0.084	0.114	17.35	23.48	291.8	394.9
$I^d(2p \rightarrow 1s)$ (erg s $^{-1}$)	1.02×10^{10}	2.55×10^{10}	4.39×10^{14}	1.09×10^{15}	1.24×10^{17}	3.08×10^{17}
$I^d(3p \rightarrow 1s)$ (erg s $^{-1}$)	3.25×10^9	8.05×10^9	1.39×10^{14}	3.44×10^{14}	3.93×10^{16}	9.74×10^{16}
$I^d(3d \rightarrow 1s)$ (erg s $^{-1}$)	1.33×10^8	4.47×10^8	5.70×10^{12}	1.91×10^{13}	1.61×10^{15}	5.41×10^{15}
$I^g(3d \rightarrow 1s)$ (erg s $^{-1}$)	1.11×10^{10}	4.36×10^{10}	4.75×10^{14}	1.85×10^{15}	1.34×10^{17}	5.24×10^{17}
E_H (MeV)	0.081	0.094	16.78	19.52	282.2	328.3
P_H (erg s $^{-1}$)	2.63×10^{10}	3.56×10^{10}	1.13×10^{15}	1.52×10^{15}	3.18×10^{17}	4.31×10^{17}

Table 2. Graviatom parameters for the wino, pion and kaon.

	Value for the following charged particles					
	\tilde{W}		π		κ	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
mc^2 (MeV)		8.000×10^5		139.568		493.994
τ_p (s)		5.000×10^{-10}		2.600×10^{-8}		1.200×10^{-8}
M (g)	1.99×10^{11}	2.31×10^{11}	1.14×10^{15}	1.33×10^{15}	3.22×10^{14}	3.75×10^{14}
r_g (cm)	2.99×10^{-17}	3.49×10^{-17}	1.72×10^{-13}	1.99×10^{-13}	4.86×10^{-14}	5.65×10^{-14}
a_B^g (cm)	3.49×10^{-17}	4.06×10^{-17}	1.99×10^{-13}	2.33×10^{-13}	5.65×10^{-14}	6.57×10^{-14}
$\hbar\omega_{12}$ (MeV)	1.11×10^5	1.50×10^5	19.34	26.17	68.44	92.62
$\hbar\omega_{13}$ (MeV)	1.31×10^5	1.78×10^5	22.92	31.02	81.12	109.78
$I^d(2p \rightarrow 1s)$ (erg s $^{-1}$)	2.52×10^{22}	6.24×10^{22}	7.66×10^{14}	1.90×10^{15}	9.60×10^{15}	2.38×10^{16}
$I^d(3p \rightarrow 1s)$ (erg s $^{-1}$)	7.96×10^{21}	1.97×10^{22}	2.42×10^{14}	6.01×10^{14}	3.04×10^{15}	7.53×10^{15}
$I^q(3d \rightarrow 1s)$ (erg s $^{-1}$)	3.27×10^{20}	1.10×10^{21}	9.95×10^{12}	3.34×10^{13}	1.25×10^{14}	4.18×10^{14}
$I^g(3d \rightarrow 1s)$ (erg s $^{-1}$)	2.72×10^{22}	1.06×10^{23}	8.29×10^{14}	3.23×10^{15}	1.04×10^{16}	4.05×10^{16}
E_H (MeV)	1.27×10^5	1.48×10^5	22.16	25.78	78.44	91.25
P_H (erg s $^{-1}$)	6.45×10^{22}	8.73×10^{22}	1.96×10^{15}	2.66×10^{15}	2.46×10^{16}	3.33×10^{16}

Table 3. Relations valid for all graviatoms.

Relation	Value	
	Minimum	Maximum
$I^g(3d \rightarrow 1s)/I^q(3d \rightarrow 1s)$	83.295	96.899
$I^g(3d \rightarrow 1s)/I^d(2p \rightarrow 1s)$	1.082	1.703
$I^g(3d \rightarrow 1s)/I^d(3p \rightarrow 1s)$	3.419	5.383
$I^d(2p \rightarrow 1s)/P_H$	0.390	0.715
$\hbar\omega_{12}/E_H$	0.872	1.015
$(Mm)^{1/2}/m_{pl}$	0.780	0.841

Universe's lifetime T :

$$M_i = \left(\frac{\hbar c^4}{5120\pi H G^2} \right)^{1/3}, \quad (42)$$

which gives $M_i = 4.38 \times 10^{14}$ g for $H = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ($H = 1/T$).

Tables 1 and 2 present the graviatom parameters: the mini-hole and charged-particle masses satisfying the conditions for the existence of a graviatom, the energies and intensities of the electromagnetic radiation, the gravitational radiation and the Hawking radiation. Also, unstable particle lifetimes (for the wino see [8]) and the Bohr graviatom radii are indicated.

As the mini-holes are constituents of the graviatoms, these mini-holes are formed because of the Jeans gravitational instability at times of about $r_g/c = 10^{-27} - 10^{-21}$ s from the initial singularity. The mini-hole masses for the graviatoms involving electrons, muons and pions exceed the value of 4.38×10^{14} g, which means that it is possible for such graviatoms to have existed up to now [9].

Table 3 presents the relations valid for all graviatoms: the gravitational-to-electromagnetic radiation intensity ratios, the dipole-to-Hawking radiation ratio as well as the quantity equal to the square root of $Gm/\hbar c = 0.608 - 0.707$. The latter is the gravitational equivalent of the fine-structure constant. The gravitational radiation intensities exceed the electromagnetic radiation intensities by two orders of magnitude. The graviatom dipole radiation energies and intensities have proved to be comparable with those for the Hawking effect with the mini-holes being constituents of the graviatoms. The gravitational equivalent of the fine-structure constant does not exceed unity; thus the perturbation theory remains valid.

5. Conclusion

We have considered the conditions for the existence of a graviatom, which proves to be satisfied for charged leptons and mesons but not baryons (protons and nuclei). The baryon sizes appear to exceed the mini-hole gravitational radii, which means that neither the hydrogen-like nor the oscillatory case can occur, i.e. stable graviatoms with baryon constituents become impossible. Instead of these, a so-called quantum accretion of baryons on to a mini-hole occurs. The internal structure of the baryons, consisting of quarks and gluons, should be taken into account. The whole problem is solvable within the framework of quantum chromodynamics and quantum electrodynamics. The radiation of baryons and quarks is also worth consideration later. In the future, it is also of interest to consider the graviatoms as sources of the electromagnetic background radiation and their possible contribution to dark matter.

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