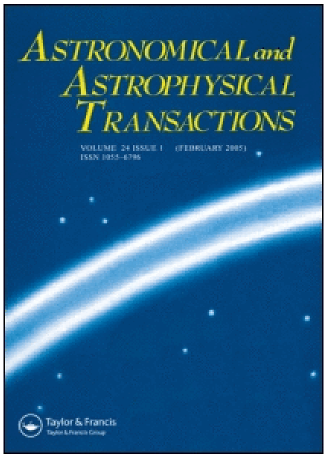

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Synthetic Hertzsprung – Russell diagrams of low-mass stars and the efficiency of colour transformation in determining the open cluster ages

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The scales of the effective temperature T_{eff} and bolometric correction (BC) are necessary for isochrones to be compared with observed colour – magnitude diagrams (CMDs) of star clusters. Reliable scales can be found, providing improved numerical relations between observed and theoretical quantities. A detailed analysis is presented of the effects which the log T_{eff}, $B - V$ and BC scales have on the transformation of evolutionary tracks and time isochrones from the theoretical Hertzsprung – Russell diagram to the CMD. We present a comparison between different scales of published transformations to find the ages of some selected open clusters.

Keywords: Low-mass stars; Hertzsprung – Russell diagram; Colour – magnitude diagrams; Colour transformation; Open cluster ages

1. Introduction

The last step in comparing theoretical isochrones with a star cluster colour – magnitude diagram (CMD) is the transformation of luminosity and effective temperature T_{eff} into brightness and colour. While there are a number of both theoretical and empirical sets of transformations available, this last step is, nonetheless, difficult and critical. In this paper, we demonstrate that existing empirical and theoretical transformations provide very different $M_V - (B - V)$ relationships when stellar evolutionary models of low-mass stars are taken into account. Details of the low-mass stellar models can be found in [1]. For the choice as to which transformation we apply to our isochrones, the following requirements need to be fulfilled.

(i) The transformation which is better to explain the CMD of our theoretical models should be chosen.
(ii) The isochrone should fit a cluster well in the $M_V - (B - V)$ plane for all transformations.

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In section 2 we present a number of recently published transformations and show the differences between the log T_{eff}, $B - V$ and bolometric correction (BC) scales. We show that considerable differences in the resulting colours exist. Section 3 presents comparisons of the transformation scales with our stellar evolutionary models [1]. In section 4, we then present tests to verify its usefulness for the case of our own isochrones applied to open clusters suitable for our pre-main-sequence (PMS) evolutionary models.

2. A comparison of different published colour transformations

2.1 *Transformation sources*

For the purpose of this investigation we have used a number of recently published transformations, which we briefly describe in the following.

2.1.1 The Flower 1977 (F77) [2] model. The T_{eff}, $B - V$ and BC scales are presented for supergiants and giants as well as for main sequence stars hotter than the Sun. $T_{\text{eff}} - (B - V)$ relations are found for each of the luminosity classes V, III and I. In [2], the new temperatures, colours and BC values have been included for stars hotter than the Sun, and for M-type supergiants and giants. Also the temperature – colour relation has been given for G and K supergiants.

2.1.2 The Flower 1996 (F96) [3] model. This paper collected temperature and BC measurements for 335 stars for the purpose of establishing refined numerical relations between T_{eff} and BC and between T_{eff} and $B - V$. The stars span luminosity classes from main sequence stars (V) to supergiants (I) and temperatures from 2900 to 52 500 K. Temperature and BC measurements from the basis of the new T_{eff}, $B - V$ and BC scales have been derived in this work.

2.1.3 The Kenyon–Hartmann 1995 (K95) [4] model. Kenyon and Hartmann tabulated the absolute magnitudes M_{J}, M_{K} and M_{bol} for tracks using T_{eff} values, BC values and broadband colours (see table A5 of [4]).

2.1.4 The Siess *et al.* 1997 (S97) [5] model. Siess *et al.* gave synthetic diagrams which have been modified as functions of various factors. They adopted the T_{eff} scale and BC compilation of [6] for spectral types earlier than K5, and those of [7] for spectral types K7 and later.

2.2 *Comparison of transformations*

2.2.1 The log T_{eff} and $B - V$ scales. From figures 1–3 the results of various colour transformations (indicated in figures) are displayed. In figure 1 we have plotted log T_{eff} and $B - V$ values from [2] as a medium-dashed curve. The solid curve is from [3]. The long-dashed curve is the log $T_{\text{eff}} - (B - V)$ relation of [4]. The data from [5] are shown as a short-dashed curve. The results provide a fit around the magnitudes $0.40 \leq B - V \leq 0.80$. From the figure, it can be seen that deviations between different transformations for late spectral types are

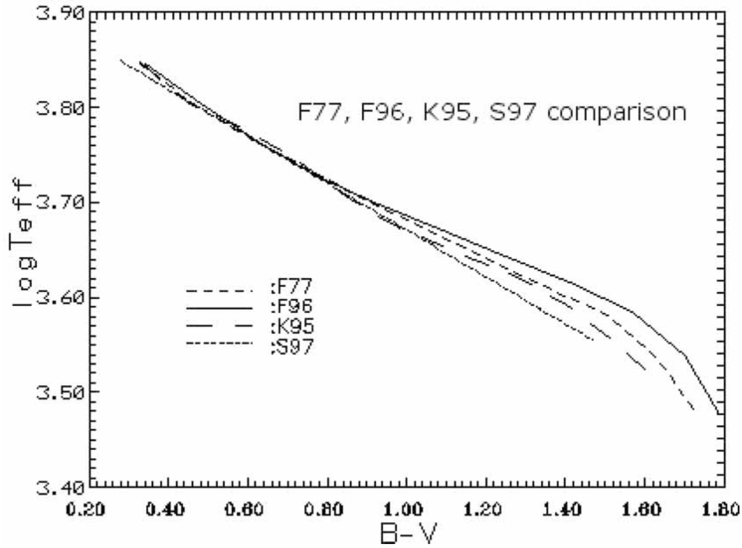

Figure 1. A comparison between the $(B-V)-\log T_{\text{eff}}$ scales for F77, F96, K95 and S97.

evident. For early spectral types the computations match each other. The linear fits for this graph are

$$\text{F77:}\quad \log T_{\text{eff}} = 3.9155 - 0.2343(B-V), \tag{1}$$

$$\text{F96:}\quad \log T_{\text{eff}} = 3.9000 - 0.2095(B-V), \tag{2}$$

$$\text{K95:}\quad \log T_{\text{eff}} = 3.9171 - 0.2394(B-V), \tag{3}$$

$$\text{S97:}\quad \log T_{\text{eff}} = 3.9181 - 0.2470(B-V). \tag{4}$$

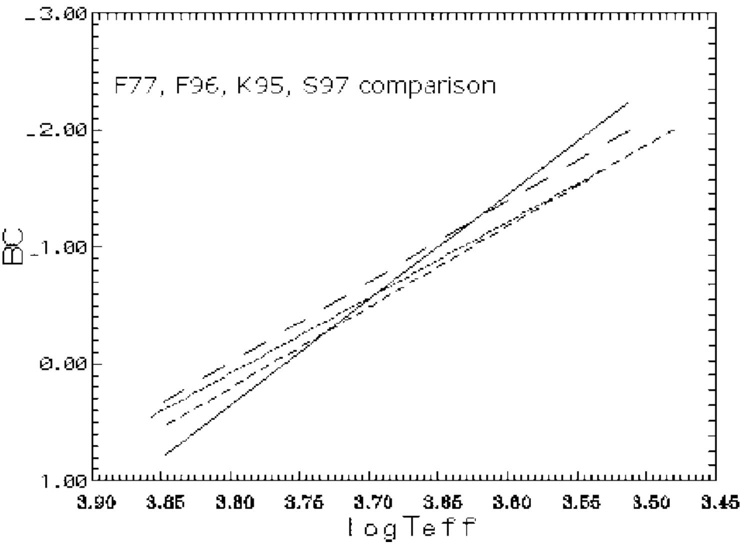

Figure 2. The relation between BC and T_{eff} for all transformations. The different symbols for the lines are the same as in figure 1 (see text).

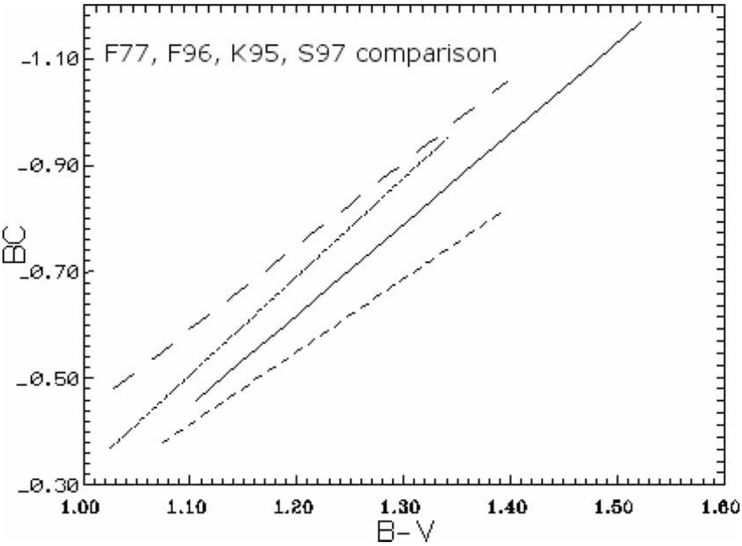

Figure 3. A comparison between the $(B - V) - BC$ scales for F77, F96, K95 and S97.

2.2.2 The log T_{eff} and BC scales. The comparison is displayed in figure 2. The linear fits for this graph are

$$\text{F77:} \quad \text{BC} = 6.9215 \log T_{\text{eff}} - 26.0942, \tag{5}$$

$$\text{F96:} \quad \text{BC} = 9.0493 \log T_{\text{eff}} - 34.0295, \tag{6}$$

$$\text{K95:} \quad \text{BC} = 6.9270 \log T_{\text{eff}} - 26.3326, \tag{7}$$

$$\text{S97:} \quad \text{BC} = 6.4691 \log T_{\text{eff}} - 24.5017. \tag{8}$$

2.2.3 The $B - V$ and BC scales. The difference between transformations is evident in figure 3. For cooler temperatures the scale in [3] is hotter than the scale in [2], and it agrees at around 5000 K with the others but deviates after that temperature. For a given value of BC $= -0.50$ the differences are $\Delta(\text{S97, F77}) \approx 0.20$ and $\Delta(\text{F96, K95}) \approx 0.06$. The linear fits for this graph are

$$\text{F77:} \quad \text{BC} = -1.3643(B - V) + 1.0869, \tag{9}$$

$$\text{F96:} \quad \text{BC} = -1.7063(B - V) + 1.4286, \tag{10}$$

$$\text{K95:} \quad \text{BC} = -1.5662(B - V) + 1.1307, \tag{11}$$

$$\text{S97:} \quad \text{BC} = -1.8468(B - V) + 1.5248. \tag{12}$$

3. Comparison of transformation scales with our theoretical models

Evolution of the gravitational contraction phase of stars having masses $0.3M_{\text{Sun}} \leq M/M_{\text{Sun}} \leq 1.5M_{\text{Sun}}$ were studied together with deuterium burning [1]. The equation of state developed in [8] and OPAL opacity tables were used. Convection was treated by the mixing-length theory. The ratio of the mixing length to the pressure scale height was assumed to be 1.74. For the chemical composition we took the fractional hydrogen abundance $X = 0.699$ and the heavy-element abundance $Z = 0.019$, by mass. Details of the models can be found in [1]. These evolutionary models are used to generate a synthetic Hertzsprung – Russell diagram (SHRD) of low-mass stars. The method is as follows. The difference between the bolometric and absolute visual magnitudes is the BC, given by

$$\mathrm{BC} = M_{\mathrm{bol}} - M_V, \tag{13}$$

$$= +2.5\ \log\left(\frac{L_V}{L}\right) + \text{constant}, \tag{14}$$

with

$$L_V = \frac{4\pi R_* \pi \int_0^\infty cF(\lambda) S_V\ \mathrm{d}\lambda}{4\pi R_* \pi \int_0^\infty F(\lambda)\ \mathrm{d}\lambda}, \tag{15}$$

where R_* is the radius of the star, $F(\lambda)$ is the monochromatic flux, $S_V(\lambda)$ is the response function of the V band, and c is a constant. After combining these relations, we obtain

$$\mathrm{BC} = -2.5\ \log\left(\int_0^\infty F(\lambda)\ \mathrm{d}\lambda\right) + 2.5\ \log\left(\int_0^\infty F(\lambda) S_V(\lambda)\ \mathrm{d}\lambda\right) + \text{constant}, \tag{16}$$

$$\mathrm{BOL} = -2.5\ \log\left(\int_0^\infty F(\lambda)\ \mathrm{d}\lambda\right) = -2.5\ \log\left(\frac{\sigma T_{\mathrm{eff}}^4}{\pi}\right), \tag{17}$$

$$\mathrm{BC} = \mathrm{BOL} - V + \text{constant}. \tag{18}$$

where

$$V = -2.5 \int_0^\infty F(\lambda) S_V(\lambda)\, \mathrm{d}\lambda$$

For the solar composition the theoretical evolutionary tracks of stars of low masses [1] are obtained and displayed in figure 4. This diagram is transformed to the SHRD in the $M_V - (B - V)$ plane by using all the transformation scales that we considered.

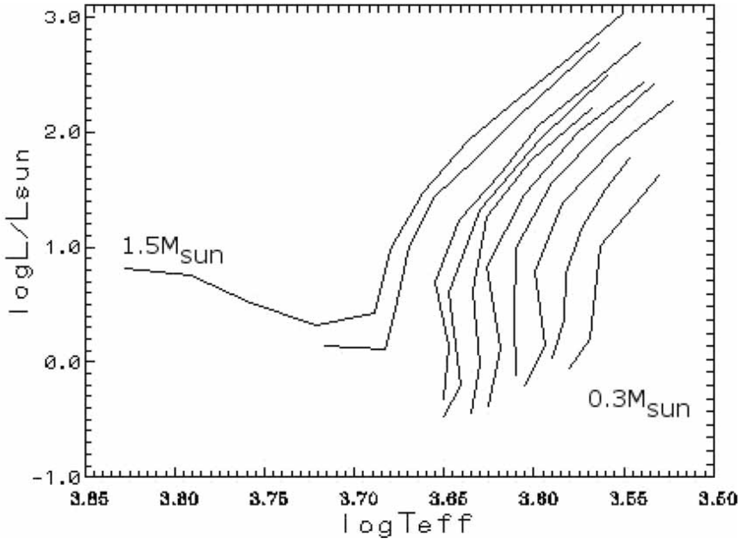

Figure 4. Theoretical evolutionary tracks of low-mass stars of masses $(0.3 - 1.5)M_{\mathrm{Sun}}$.

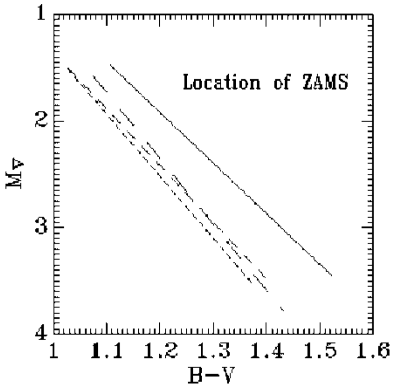

Figure 5. Zero-age main sequence of our models calculated from different transformation scales.

In order to emphasize the key influence of transformations, we compare in figure 5 the location in the $M_V - (B - V)$ diagram with the zero-age main sequence of our models derived from transformation scales. The differences due to the conversion relations are obvious and the difference in M_V has a magnitude of about 0.7 between [3] and [5], which causes our models to become redder if we take into account the scales in [3].

Figure 6 shows the SHRD of our low-mass model tracks if we take into account the transformation scale in [3].

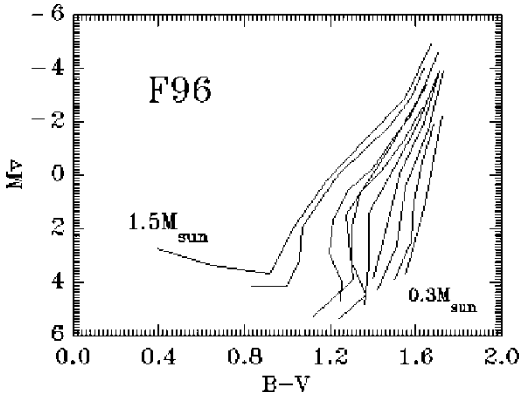

Figure 6. SHRD obtained by F96 transformations.

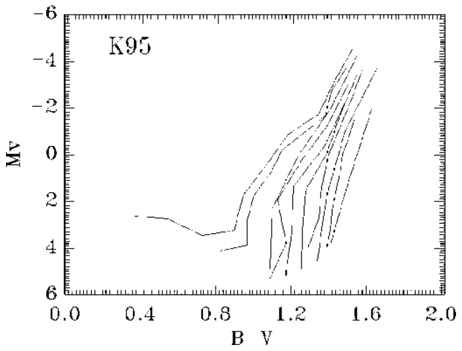

Figure 7. SHRD obtained by K95 transformations.

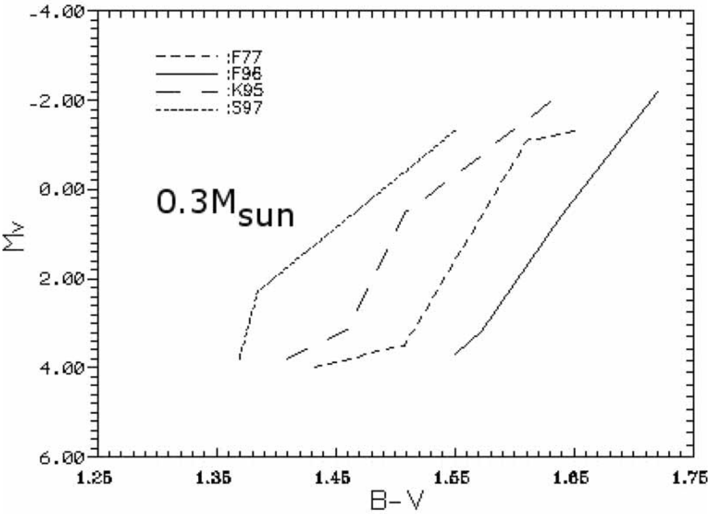

Figure 8. SHRD comparison for $0.3 M_{\rm Sun}$ obtained from all transformations.

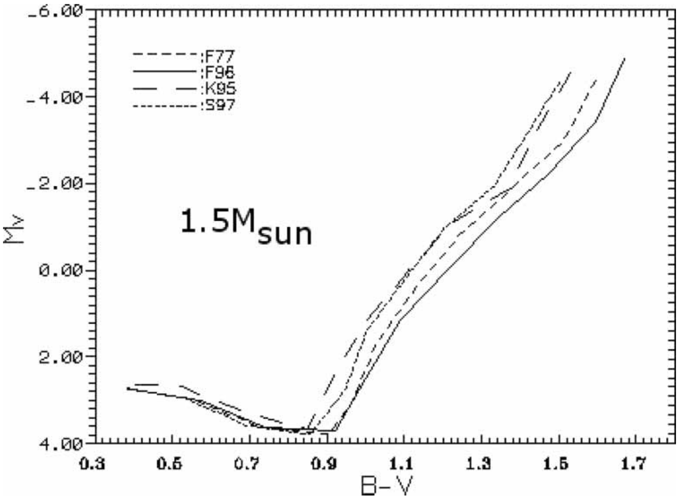

Figure 9. SHRD comparison for $1.5 M_{\rm Sun}$ obtained from all transformations.

The SHRD of our low-mass model tracks is also displayed in figure 7 for the transformations in [4].

Figure 8 displays the differences in the evolutionary track of a $0.3 M_{\rm Sun}$ star. From this figure, one can conclude that the results in [5] are hotter than the others.

We display the same comparison in figure 9 for the $1.5 M_{\rm Sun}$ track. The difference is achieved more at the cool end of the track while there exists a fit towards the main sequence.

4. Isochrones

We present comparisons between the published transformations needed to generate the SHRD of the early PMS phases of low-mass stars with $Z = 0.021$ [1]. We compare our models in

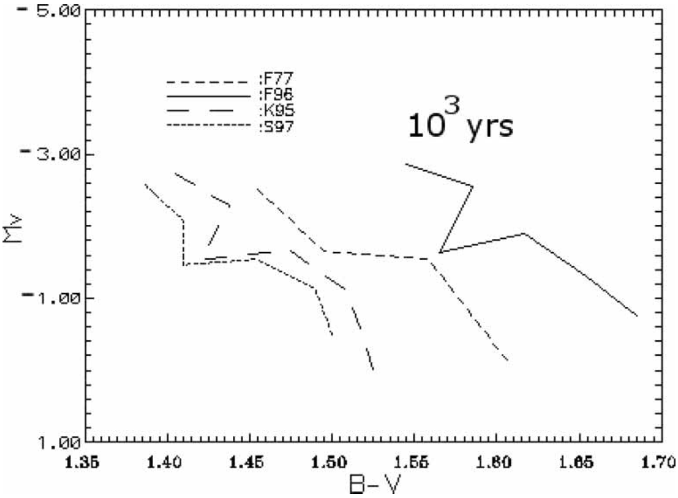

Figure 10. Comparison of the isochrones for 10^3 years for all transformations.

the $M_V - (B - V)$ plane and we show important deviations in the PMS tracks for stellar masses towards $0.3M_{\text{Sun}}$. The disagreement for all scales is obvious down to lower masses. For transforming our isochrones to the $M_V - (B - V)$ plane we have tested all the transformations mentioned above. The presented SHRD obtained with $Z = 0.021$ is suitable for the clusters chosen for determining their ages. The estimated ages of the selected clusters NGC 6318 and Trapezium are shown to be shifted somewhere between the isochrones for 10^3 and 10^4 years and between those for 10^6 and 10^7 years respectively.

Figures 10 and 11 display the isochrone comparisons for 10^3 and 10^7 years respectively.

Figure 10 displays major differences for the 10^3 years line between transformations. The result in [5] is hotter than others and gives longer ages. In figure 12 we display the fit for the isochrones for 10^3 and 10^4 years and the position of the open cluster NGC 6318.

If we take into account the transformations in [3], NGC 6318 is older than 10^3 years but, if we consider [5], 80% of the stars of this cluster are older than 10^3 years. Another comparison

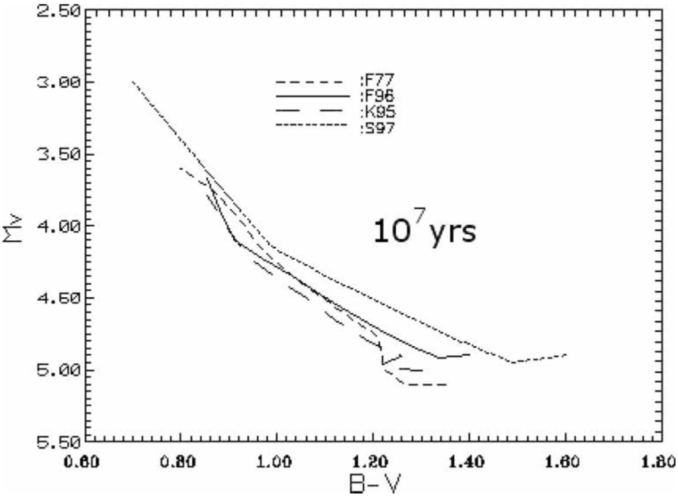

Figure 11. Comparison of the isochrones for 10^7 years for all transformations.

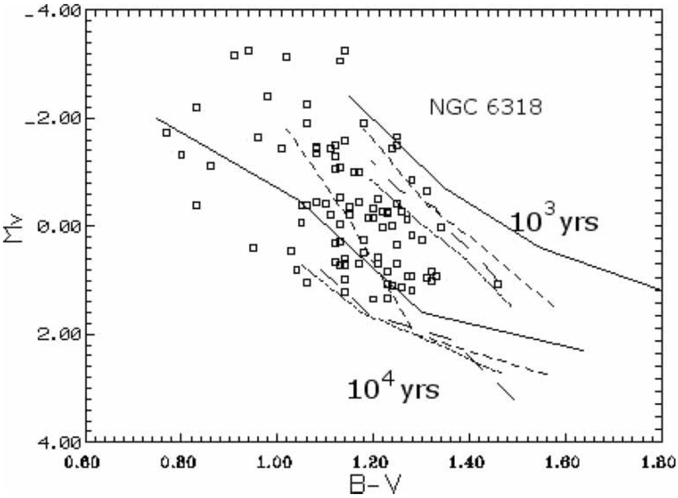

Figure 12. Fit of the isochrones for 10^3 and 10^4 years to the NGC 6318 cluster transformations.

is shown in figure 13 with the isochrones for 10^4 and 10^5 years. According to [3], for NGC 6318, 70% of the stars are found to have ages less than 10^5 years while for [5] all stars have ages around 10^5 years. In this figure the Trapezium open cluster is displayed, too.

A similar effect on age can be seen in figure 14 which is obtained for 10^6 and 10^7 years for the Trapezium cluster. Half of its stars can be considered to have ages less than 10^7 years when the isochrone in [3] is taken into account. According to the isochrone in [5] this cluster has an age of 10^7 years.

It is evident from these illustrations that colour differences up to 0.20 magnitude exist for some typical transformations and there are no two transformations giving the same result for the lower end of our evolutionary stages.

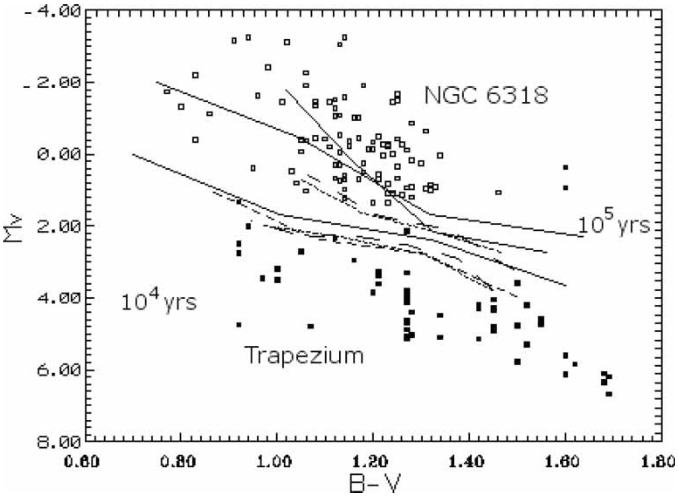

Figure 13. Fit of the isochrones for 10^4 and 10^5 years to the NGC 6318 and Trapezium clusters.

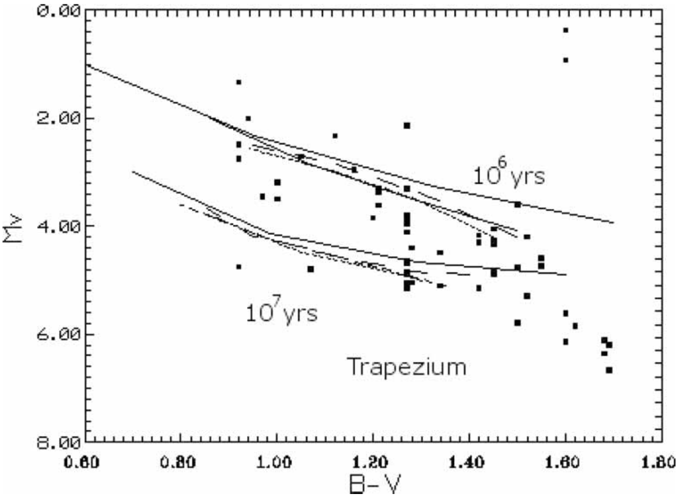

Figure 14. Fit of the isochrones for 10^6 and 10^7 years to the Trapezium cluster.

5. Results and conclusions

We present comparisons between published transformations needed to generate the SHRD of the early PMS phases of low-mass stars with $Z = 0.021$. The main attention has been paid to the transformation scales from our theoretical Hertzsprung – Russell diagram to the observational CMD. By adopting the scales in [2–5] we compared our models in the $M_V - (B - V)$ plane and we showed important deviations in the PMS tracks for stellar masses towards $0.3 M_{\rm Sun}$. Because SHRDs are useful tools to study the ages of young open clusters, we analysed the ages of some selected open clusters (NGC 6318 and Trapezium) by taking into account each of the transformation scales and found different ages for different scales. The colour differences up to 0.20 magnitude cause shifts in the isochrones and they give up to 50% difference in the cluster ages considered. We can say that the problem of determining a good empirical scale for BC against $T_{\rm eff}$ is of primary importance for matching the stellar evolutionary tracks to the observations. The number of sample stars for the analysis of the BC – log $T_{\rm eff}$ relationship must be increased. Besides this, observational errors directly affect the computed value of the integrated flux in the observed regions, since they introduce uncertainties in the parameters derived from the fit and thus in the choice of the model needed to complete the spectral distribution.

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