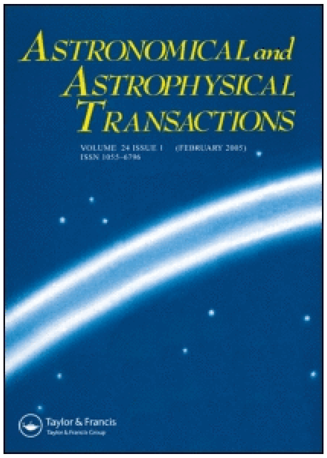

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Comparison and historical evolution of ancient Greek cosmological ideas and mathematical models

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We present a comparative study of the cosmological ideas and mathematical models in ancient Greece. We show that the heliocentric system introduced by Aristarchus of Samos was the outcome of much intellectual activity. Many Greek philosophers, mathematicians and astronomers such as Anaximander, Philolaus, Hicetas, Ecphantus and Heraclides of Pontus contributed to this. Also, Ptolemy was influenced by the cosmological model of Heraclides of Pontus for the explanation of the apparent motions of Mercury and Venus. Apollonius, who wrote the definitive work on conic sections, introduced the theory of eccentric circles and implemented them together with epicycles instead of considering that the celestial bodies travel in elliptic orbits. This is due to the deeply rooted belief that the orbits of the celestial bodies were normal circular motions around the Earth, which was still. There was also a variety of important ideas which are relevant to modern science. We present the ideas of Plato that are consistent with modern relativity theories, as well as Aristarchus' estimations of the size of the Universe in comparison with the size of the planetary system. As a first approximation, Hipparchus' theory of eccentric circles was equivalent to the first two laws of Kepler. The significance of the principle of independence and superposition of motions in the formulation of ancient cosmological models is also clarified.

Keywords: Ancient Greek cosmological ideas; Ancient Greek mathematical models; Aristarchus of Samos

1. Introduction

Since the prehistoric period the Greeks had developed theoretical views of the (up to that moment) known world. As early as the fifteenth century BC, Orpheus was teaching such interesting views to his students. In *The Orphics* [1] (see also [2, 3]), which are attributed to Orpheus and his students), we find the following theories (see paragraphs 45–48 and pp. 86–88 of [1]):

(i) The Earth is spherical.
(ii) It is located at the centre of the celestial sphere.
(iii) It rotates around its axis.

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(iv) The celestial sphere rotates around the axis of the world, which overlaps the rotational axis of the Earth.
(v) The rotation of the Earth and that of the celestial sphere are synchronized.
(vi) The Sun moves in a daily and yearly manner along the ecliptic (see p. 68 of [1]).

The development of cosmological ideas during the ancient Greek civilization began about 600 BC, with Thales of Miletus being the leader of the school of Miletus, and lasted until 400 AD. During the same period the science was systematically developed and reached its peak. Science was not independent of philosophy but was considered part of it. Ancient Greeks did not study the laws of nature in order to take advantage of them; their most important motive was to advance their knowledge and to become wiser (see p. 276 of [4]). The use of mathematical methods was necessary in order to explain and understand the celestial phenomena. Ancient Greek astronomers, mathematicians and philosophers tried to interpret the celestial phenomena that were being observed by formulating several cosmological ideas, theories and mathematical models.

The aim of the present paper, which is based on previous work [5, 6], is to perform a comparative study of the evolution of cosmological ideas and mathematical models in ancient Greece between the sixth century BC and the second century AD. We can conclude that the novel theory of heliocentric system, which was introduced by Aristarchus of Samos, did not arise as a peculiar mathematical model but was the final outcome of long intellectual activity. Anaximander, the Pythagoreans Philolaus, Hicetas and Ecphantus as well as Heraclides of Pontus were the predecessors of Aristarchus of Samos. All of these together created a fervent atmosphere of abundant ideas which influenced Aristarhus' thinking.

We show that the ideas of Plato about space–time in relation to the existence of matter are very close to the relativistic theories, such as the cosmological concept of Big Bang. We also show that Ptolemy was affected by the cosmological model of Heraclides for the explanation of the apparent motions of Mercury and Venus in his formulation of an equivalent system explaining the motions of these two planets. It is also interesting that Aristarchus estimated the size of Universe in comparison with the size of our planetary system.

Finally, we show that ancient Greeks, since the time of Plato, had understood the significance of the principle of independence and superposition of motions which was used in the formulation of their cosmological models.

2. The evolution of the ancient Greek cosmological theories

2.1 *Anaximander of Miletus*

Anaximander (611–546 BC) introduce, according to certain authors, novel ideas about the Earth and other celestial bodies. He can, therefore, be considered as the forerunner of the theories that the Pythagoreans an Aristarchus of Samos would develop later. Theon of Smyrna (p. 198 of [7]) in his book *About Astrology* wrote the following: 'According to what Eudemus (350–290 BC) narrates in his book *The History of Astronomy*, Oenopides discovered first the zodiac circle and the duration of the big year (ἐνιαυτοῦ) . . .. Anaximander had firstly considered Earth to be suspended and moving around the centre of the world'. This means that he was the first who accepted that the Earth moves. Heath (p. 24 of [8]) claimed that there must be some mistake in the text of Theon. Also, Diogenes Laertius (2, 1 of [9]) (see also [10]) wrote: 'Anaximander, who accepts that Infinity is a primary element, considered Earth to be a spheroid and placed it in the centre of the world and the Moon to be heterophotous and lightened by the Sun'. He also mentioned that Anaximander was the first cartographer of the world. According to Plutarch (Γ, 11 of [11]: 'Anaximander mentions that the Earth is suspended, is not held down from somewhere, is located at equal distance from all areas and its shape is circular like the one of a stone column' (this means that it has the shape of a cylinder). According to Theon of Smyrna (p. 198 of [7]), Anaximenes considered that the Moon 'borrows' its light from the Sun.

The above information must be adopted with certain circumspection, since it is from secondary sources. We must point out that Eudemus of Rhodes was one of the two candidates for the direction of the Lyceum after the death of Aristotle (p. 241 of [12]). Furthermore, he lived much closer to Anaximander's time than Diogenes Laertius had (fourth century AD) and wrote among many other things the *History of Astronomy* and the *History of Mathematics*. In these writings the complete (up to that time) knowledge of astronomy and mathematics was included. However, Diogenes Laertius did not write a specific book on mathematics or astronomy. He chose to write one of general interest, the biographies of Philosophers, as Plutarch also did.

2.2 *Pythagoras of Samos*

Pythagoras of Samos (580–490 BC) did not leave behind any written scripts (p. 40 of [13]; p. 86 of [14]) and it is therefore difficult to distinguish with certainty his own astronomical views from those of his students, because information was from secondary sources. He believed that numbers are the essence and the beginning of all life and phenomena. He used to teach that the Earth, the Moon, the Sun, the five (up to that moment) known planets Mercury, Venus, Mars, Jupiter and Saturn and the fixed stars are spherical and that the Earth remains still in the centre of the world. According to certain sources (p. 51 of [8]), with the benefit of doubt, he first discovered that the apparent motions of the Sun and the planets can be analysed into two normal circular motions. The celestial sphere together with the Sun, the Moon and the planets rotate daily westwards (retrograde motion or clockwise) around the axis that goes through the Earth's centre and, at the same time, rotate from day to day in an easterly direction (direct motion or counterclockwise) independently around the same axis. Even though the astronomical ideas of Pythagoras have not been completely verified (p. 86 of [14]), we can certainly regard him as the philosopher who introduced the geocentric theory. His ideas also influenced Plato, Aristotle, Eudoxus of Cnidus, Apollonius of Perga, Hipparchus of Rhodes, Posidonius of Rhodes and Claudius Ptolemy, who contributed towards the improvement and development of that theory.

2.3 *Philolaus the Pythagorian*

However, at the end of the fifth century BC, Philolaus, a contemporary of Socrates and one of the greatest Pythagorian philosophers, formulated the opinion that the Earth, being one of the stars, the Sun, the Moon and the five planets move eastwards (have a direct motion) around the *Central Fire* (Ἑστία τοῦ Παντός) which he called the *Focus*. In the centre of the world, therefore, instead of the Earth, Philolaus places the *Central Fire*. Plutarch (Γ, 11, 13 of [11]) wrote: 'Philolaus the pythagorian claims that first comes the *Fire* which is situated at the centre of the world, because it is the Focus of the Universe, second comes Antichthon (ἀυτίχθωυ), counter Earth, third comes the ecumenical Earth which is placed opposite Antichthon and rotates along with Antichthon around the centre of the world. For that reason those who live in Antichthon are not viewed from the inhabitants of the Earth. ... Other philosophers believe that the Earth is standing still [in the centre of the world]. On the other hand, Philolaus the Pythagorian believes that the Earth moves within a circular orbit around the *Central Fire*, which is slanting, exactly like in the cases of the Sun and the Moon. Heraclides of Pontus'...

(see section 2.9). Later we mention that an analogous idea about Philolaus was pointed out by Aristotle in his book *On Heaven* (XIII, 293α, 18 of [15]). Also Plutarch (H, 1 of [16], see also [17]); (XI of [18]), (see also [19]) (see section 2.5), Aetius (II, 7.1–7.7, pp. 336, 337 and III, 11, 13, pp. 377, 378 of [20]) and Stobaeus (I, 22.1–3, pp. 336, 337 and I, 33–35, pp. 377, 378 of [21]) mentioned the same notions. We should mention that Simplicius (II, 13, 229 of [22], see also [23]) considered Philolaus to be the first to introduce the heliocentric hypothesis.

We make the conjecture that Philolaus meant the Sun. This conclusion can also be deduced from the following.

(i) From the work of Simplicius (as a whole) and especially in the last paragraph, he wrote: 'Those who adopt the most genuine among these theories call central fire the creative power which, coming out from the middle, invigorates the Earth as a whole and heats up again the part of the Earth that has been chilled [during the night]'. The celestial body that has the above properties is the Sun. It is known that, since ancient times people believed that the Sun heats, lightens and invigorates the Earth. They also wrote some hymns for the Sun. One of those who believed that was the great stoical philosopher and astronomer, Posidonius of Rhodes (135–51 BC). He believed that the Sun was the centre of the planets and that its thermogenetic impetus, *the spirit* ($\tau\grave{o}\ \pi\nu\varepsilon\tilde{\upsilon}\mu\alpha$), surges into the world and fills it (pp. 391, of [24]).

Also, since the prehistoric times, in *The Orphics* (pp. 68 and 71 of [1]), we find the ideas of *heliocentrism*, because in the hymns of Orpheus the key words 'the Sun is the Master of the World' (Κοσμοκρὰτωρ ῞Ηλιος), 'the Ruler of the World' (Δεσπότης τοῦ Κόσμου) and its orbit as 'a burning orbit' (Πυρίδρομος) are mentioned. Also, the leading position of the Sun is clarified and the fact that it contains the *seal* by which 'the world was printed'.

(ii) Philolaus was afraid of being condemned for 'not honourable behaviour' (ἐπί ἀσεβείᾳ) because of his cosmological ideas and for that reason he did not argue directly that 'the Earth was moving around the Sun' (see also p. 56 of [25] and p. 4 of [26]). His fear may be justified by the following:
   (a) the annulment of the school of Pythagoras and the slaughter of the students of Pythagoras for which the nobleman Cylon was responsible;
   (b) the persecution of Anaxagoras and of Diogenes Appoloniatis because of the revolutionary theories that they formulated (Anaxagoras escaped to the city of Lampsacos with the help of his student Pericles).

   Also the astronomer E. Antoniadis shared the opinion that the *Central Fire* mentioned by Philolaus was actually the Sun (p. 99 of [31]).

This original idea of Philolaus created therefore a rupture within the scientific thinking of that period.

2.4 *Hicetas and Ecphantus*

During the fifth and fourth centuries BC, two more Pythagoreans from Syracuse, namely Hicetas and his student Ecphantus, modified the system of Philolaus. They believed that the Earth rotates eastwards around its axis. In that way they could explain the daily rotation of the celestial sphere and therefore the succession of day and night.

Aristotle in his book *On Heaven* (XIII, 293α, 18 of [15]) wrote: 'As far as the position [of the Earth] is concerned no one shares [not every body has] the same view and, although most of them say that it is situated in the middle [of the Universe], the Italian Philosophers, the so-called Pythagoreans, say exactly the opposite. They say that in the centre there is the *Fire* and the Earth since it is one of the stars and rotates around the centre creates the day and the night. They also assume another Earth, opposite to ours, which they call Antichthon. . . . Furthermore, many others share the belief of the italian Philosophers, that we should not regard the Earth as being in the centre of the world. . . . therefore in that way of thinking they did not think that the Earth is found in the centre of the celestial sphere. What they do think is that there lies the *Fire*. . . . Some even say that the Earth by occupying the centre oscillates and moves around the axis of the Universe as it is mentioned in Timaeus . . . As we have already said, some characterize the Earth as one of the stars and others regard it as being in the centre of the Universe, eddying (whirling) and moving around its central axis.' Also, Cicero (II, 39, 123 of [27]) mentioned the same for the ideas of Hicetas.

There is an abridgement of Diogenes Laertius (8, 85 of [9]) which said the following: '[Philolaus] believed that everything happens harmoniously and because of a need. He also believed that he was the first to find that the Earth moves in a circular manner. Others, however, attribute it to Hicetas the Syracusian.'

With regard to the ideas of Pythagoreans, Lloyd (pp. 267 of [4]) noted: '. . . it is likely that Leucippus and Democritus accepted the geocentric system.' However, some Pythagoreans did not consider the Earth as the centre of the Solar System. Such statements were first expressed in the late fifth century BC. It is very likely that the theory of Philolaus assumed that both the Earth and the Sun are moving in the Solar System.

2.5 *Plato*

Plato (428–347 BC), however, who repeatedly visited the Pythagorian School during his travels to southern Italy, returned to the views of Pythagoras. He develops his cosmological–cosmogonical ideas in the dialogue of Timaeus. According to his cosmology, the Sun, the Moon, the (up to that moment known) five planets and the stars rotate on circular orbits around the Earth, which, in turn, is placed in the centre of the world. We only mention certain fragments from *Timaeus* (38b, c of [28])†: 'Time was therefore born along with (simultaneously) the sky in order to disappear together [simultaneously] with it—when [if] they disappear in the future—since they were created simultaneously and according to the model of the eternal essence, in order to be as much similar as possible to it concerning the ability. The reason for this is that the model is eternally a being [exists eternally] and, on the other hand, time existed, exists and will continue to do so from the beginning until its end. Through such arguments [reasoning] therefore and such plans of God concerning the creation [birth] of time, the Sun, the Moon and the other five stars, which are called planets, were created in order to determine and conserve the numbers [measurement] of time.'

It can be deduced from this fragment that Plato believed that time was born simultaneously with the creation of the sky, which was the Universe of that time, and will vanish at the same time with the end of the Universe. He talked about the lifespan of the Universe which is finite and the fact that time existed, exists and will exist during this period in contrast with the creator of the Universe, God, the eternal being. Time was therefore born together with matter

†Χρόνος δ' οὖν μετ' οὐρανοῦ γέγονεν, ἵνα ἅμα γεννηθέντες ἅμα καί λυθῶσιν, ἄν ποτε λύσις τις αὐτῶν γίγνηται, καί κατά τό παράδειγμα τῆς διαιωνίας φύσεως, ἵν' ὡς ὁμοιότατος αὐτῷ κατά δύναμιν ᾖ. Τό μέν γάρ δή παράδειγμα πάντα αἰῶνά ἐστιν ὄν, ὁ δ'αὖ διά τέλους τον ἅπαντα χρόνον γεγονώς τε καί ὢν καί ἐσόμενος. Ἐξ οὖν λόγου καί διανοίας θεοῦ τοιαύτης πρός χρόνου γένεσιν, ἵνα γεννηθῇ χρόνος, ἥλιος καί σελήνη καί πέντε ἄλλα ἄστρα, ἐπίκλην ἔχοντα πλανητά, εἰς διορισμόν καί φυλακήν ἀριθμῶν χρόνου γέγονεν.

and space and they will vanish together. These ideas are close to modern cosmological theories and, in particular, to the notion of 'space–time', introduced by Einstein in the formulation of the general theory of relativity at the beginning of the twentieth century. Also, according to the relevant theory introduced by Friedmann, Lemaitre and Gamow, space–time was created following the Big Bang. The cosmological views of Plato are consistent with the cosmology of the Big Bang, but opposite to the steady-state cosmology theory of Bondi, Gold and Hoyle and the cosmological concept of Dirac.

We should note that it would be inappropriate to try to compare modern ideas and theories those of with Plato or ancient Greek scientists, since they belong to different eras. However, modern scientists could use the ideas of Plato or other scientists and philosophers of ancient times as a philosophical basis for their theories.

According to Simplicius (II, 12, pp. 488 and 493 of [22]), Plato believed that the apparent motions of the planets could be described via the composition of normal (uniform) circular motions and that it is apt for geometricians to discover them.

Plato mentioned in *Timaeus* (40a, b of [28]): 'God gave two opposite motions to the divine beings [planets]. One motion is uniform and always takes place in the same area [space], because God has got the same view [thought] about the same objects, and the other occurs towards the front and obeys to the rotation of the immutable and invariable essence.' He then went on to say (40c of [28]): ... 'and the Earth, which nourishes us, which rotates around the axis of the world and which is the guard and creator of night and day, was created by God first and superior to all gods created in heaven'. It is obvious that Plato accepted the daily motion of the Earth around its axis. As we have already mentioned in the previous section, Aristotle pointed out something relevant in his book *On Heaven* (293b of [15]). By studying other parts of the dialogue in *Timaeus*, however, it seems that Plato considered the Earth as being still in the centre of the world.

Until the time of Plato an enormous volume of observational data had been accumulated. It was time for all these data to be used in order to formulate a mathematical model that would offer a theoretical explanation. Plato seemed to have understood that and, as Simplicius (II, 12 of [22]) said, he managed to foresee the need for the use of mathematics by astronomers, in order to explain the celestial phenomena. In this way, he conceived a bright idea: to encourage the astronomers (his students) to focus their efforts on the theoretical aspects of the cosmological problem. They needed to study the mathematical relations that lie behind the observed celestial phenomena in order to suggest mathematical models, which would enact and explain the motions of the celestial bodies and would subsequently explain (save) the phenomena.

Eudoxus of Cnidus was the first who responded to the suggestion of his master and formulated a mathematical model in order to interpret the (up to that moment) known world. That model was the theory of *homocentric spheres*. Plato adopted the system of Eudoxus in order to explain the phenomena. His greatest contribution, however, was the incentive that he created towards the development of certain branches of mathematics and especially geometry.

There are some indications, such as those of Plutarch (H, 1 of [16]; XI, 2, pp. 344 of [18]), that Plato at the end of his life was not satisfied with the central position of the Earth in the celestial sphere. Plutarch wrote: 'Theophrastus additionally points out that in his old age Plato regretted having attributed to Earth the central position of the world without being deserved as such ... but the Universe, the middle of which the Pythagoreans believe consists of *Fire* and they call *Focus* and *Unit*. Concerning the Earth they say that it is not still or found in the middle [centre] of the orbit, but it depicts a circular orbit around Fire. ... Plato, they say, adopted those theories about Earth in his old age, that it is found at another place ...' (and not in the centre of the world).

2.6 *Eudoxus of Cnidus*

The mathematician and astronomer Eudoxus of Cnidus (408–355 BC) asserted, together with his teacher Plato, that the Earth was immobile. Motivated by Plato, he invented the first geometric cosmological model. In order to explain the apparent motions of the celestial bodies and the irregularities of their orbits he formulated the theory of homocentric spheres, which he described in his book *On Velocities* (Περί Ταχῶν). This book has been lost. References to his theory are found in the book by Aristotle (XI, 8, 1073b of [29]; see also [30, 31]) and a more detailed description in the book by Simplicius (II, 12, mainly 221a of [22]), who obtained information from Eudemus of Rhodes through the philosopher Sosigenes. The geometrical details of that theory are not mentioned in these ancient sources and that is the reason why Apelt, Ideler and mostly Schiaparelli tried to explain through their work the basic geometrical principles of Eudoxus' system. Heath [8], Dreyer [13], Dicks [14], Neugebauer (pp. 196, 197 and 228 of [32]; pp. 677 of [33]) were based on these studies (especially on the work of Schiaparelli) in order to describe the theory of Eudoxus in their books. Neugebauer tried to explain the Eudoxus theory and especially the attributes of the figure-of-eight curve, *hippopede*.

In order to explain the apparent motions of the Moon, the Sun and the five planets, Eudoxus replaced the circular orbits that had been formulated before him with a set of 27 revolving concentric spheres, which consisted of eight subsets of spheres: three for the Moon and the Sun, four for each planet and one for the fixed stars.

The basic principles of his system were the following.

(i) The spheres were not perceptible from the human senses and the still spherical Earth was located on their (the sphere's) common centre.
(ii) The spheres would rotate independently one from the other, each with its own constant angular velocity (normal circular motion) around its axis, which would pass through the centre of the Earth.
(iii) The two ends (the poles) of the axis of each sphere in each subset of spheres would be firmly fixed on the internal surface of the immediately preceding sphere. Thus, each sphere takes part in the motion of the preceding spheres, *i.e.* the spheres which were surrounding it. The description of the motion of each celestial body could therefore be achieved through an appropriate combination of motions of the spheres. We observe that Eudoxus had already perceived the principle of *independence and superposition of motions*. However, we have also encountered these ideas in Plato's views in *Timaeus*.

According to that theory, the outermost sphere, upon which the fixed stars were lying, was rotating westwards around its axis, which in turn would go through the world poles. The axis was therefore perpendicular to the plane of the celestial equator. The rotational period of the sphere was 24 h, depicting the daily celestial motion. Exactly the same motion was performed by the first (outer) sphere of each subset of spheres, which would describe the daily motion of the Moon, the Sun and each planet.

In the subset of spheres corresponding to the Moon, the second sphere was rotating eastwards in an opposite way to the first. The axis of rotation was perpendicular to the middle plane of the zodiac circle. In that way, the equator of the second sphere was virtually the ecliptic. The significance of this sphere is not explained clearly in ancient texts. One may conclude that it describes the motion of the Moon along the zodiac circle, namely its monthly motion. The same applies to the second sphere of each subset of spheres corresponding to the Sun and the five planets but with a different period in each case.

The third sphere, on the equator of which the Moon was found, would rotate westwards with a small angular velocity. Its axis was inclined towards the zodiac axis, which is the axis of the second sphere. That sphere aimed to explain the reciprocating motion breadthwise of the Moon (on both sides of the ecliptic). It is quite possible that it would depict the changes in its declination. According to what we have already mentioned, that sphere would participate in the motions of the two spheres that surrounded it. It is not known with certainty which value of angular velocity Eudoxus adopted for the second and the third sphere or which was the inclination of the rotational axis of the third sphere. Probably the motion of the third sphere described one of the Moon's abnormalities, the *regression of the nodes* which has a period of 18.6 years. Opposing views concerning these uncertainties have been formulated by modern authors.

In order to explain the apparent motion of the Sun, Eudoxus considered a mechanism analogous to that of the Moon. The first two spheres would therefore perform motions corresponding to the two first spheres of the Moon, where in this case the rotation of the second sphere would be fairly slow, with a period of 1 year. The internal (third) sphere would rotate, like the second sphere, counterclockwise around its axis, which had a small inclination with respect to the axis of the second sphere. That sphere would rotate more slowly than the second sphere and its purpose was to interpret the breadthwise motion (declination from the ecliptic) which Eudoxus believed that the Sun possessed. That theory was inadequate, since it could not interpret the already observed motions; for example it could not explain the solstices.

Besides their breadthwise declination the planets exhibit, during their apparent direct motions, stationary points (stagnations) and retrograde motion. In order to overcome that difficulty, Eudoxus adopted one extra sphere for each planet. The second sphere, which was rotating towards the direct direction with a period equal to the stellar or zodiac period of the planet, moved on the ecliptic plane. The third sphere would rotate around its axis, which would, have its poles constantly fixed on the ecliptic, meaning the equator of the second sphere. The points on which the two poles were fixed were different for each planet. On the other hand, they were the same for Mercury and Venus. Rotation was taking place towards the direct direction in a period equal to the synodic rotation of each planet. Finally, the fourth sphere, on the equator of which the planet was found, would rotate at a period equal to that of the third sphere, but towards the opposite direction. Its rotational axis would exhibit an inclination with respect to the axis of the third sphere, which would, however, be different for each planet. The superposition of motions of the last two spheres of each planet could interpret the stationary points and the retrograde motions as well as the breadthwise declinations of the planet. The *resultant* motion of the spheres corresponds to an oscillation of the planet, tracing out a figure-of-eight curve, which was along the ecliptic and which was bisected by it. Namely, here we have the superposition of two harmonic oscillations with the same period but towards opposite direction. That curve was named after Eudoxus hippopede (Ἱπποπέδη) and is also known as the *lemniscus* (small lake) (figure 1). Also Neugebauer (pp. 677 of [33]) mentioned that all sources agree that the hippopede is generated by the motion of two concentric spheres which rotate with constant but opposite angular velocity about two inclined axes. However, after some calculations, Neugebauer assumed that the actual retrogradations of each planet are not very large. This means that the latitudes produced by the motion on the hippopede are almost negligible and the Eudoxus did not associate numerical details with his model.

According to the theory of Eudoxus, the distance of the Moon and of each planet, the apparent diameter and the brightness should stay constant, as long as the celestial body moves on a circular orbit around the Earth. The apparent diameter of the Moon together with the apparent brightness of Mars, Mercury and Venus were, however, changing and this indicates that their distance from the Earth was changing too. The concentric spheres of Eudoxus could not therefore explain those observed phenomena.

This problem appeared early. The first who tried to overcome these difficulties was Autolycus of Pitane (about 300 BC), but not successfully (pp. 306 of [14]).

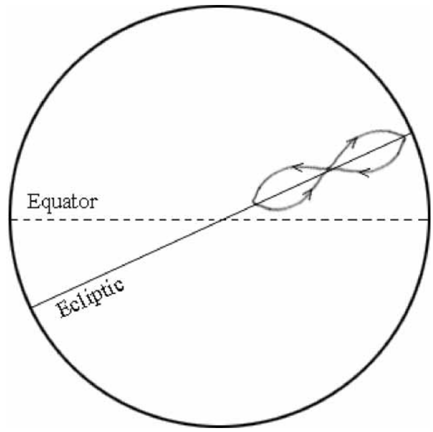

Figure 1. The resultant motion of the third and fourth spheres, the hippopede of Eudoxus of Cnidus.

2.7 *Callippus*

The model of Eudoxus was adopted and extended by his student the astronomer Callippus (370–300 BC). Aristotle in his book *Metaphysica* (XI, 8, 1073b of [29]) (see also pp. 683 of [33]) mentioned concisely that Callippus, in order to approach better the apparent positions of the planets, the Moon and the Sun, added two unstellar spheres for both the Moon and the Sun and one each for Mercury, Venus and Mars. For Jupiter and Saturn he retained the same spheres as Eudoxus. Callippus therefore increased the spheres of Eudoxus from 26 (27) to 33 (34), making the model of concentric spheres more complicated.

2.8 *Aristotle*

Similar theories were supported also by Aristotle (384–322 BC) in his book *On Heaven* (mainly in I, 5, 6, 7, and II, 8, 9, 11, 13, 14 of [15]). He accepted, among other facts, that the Universe is spherical, that the Earth is spherical and that the celestial bodies move around the Earth on normal circular orbits. After developing several arguments, he concluded that there is a need for the Earth to remain still at the centre of the world (XIV, 296b, 22, of [15]). Aristotle adopted the cosmological model of Eudoxus which he modified. He adopted 56 concentric spheres instead of the 26 that Eudoxus had suggested. In those numbers the sphere of the fixed stars has not been added (XI, 8, 1073b–1074a of [29]).

According to certain sources, *e.g.* Simplicius (II, 12, 226a, pp. 504 and 505 of [22]) Aristotle in his late years doubted the theory of Eudoxus, because he was not totally satisfied by it regarding its sufficiency to explain the phenomena. According to Theon of Smyrna (pp. 189 of [7]), Aristotle had already mentioned the idea of eccentricity about the orbits of the planets. The testimony of Theon coincides with that of Simplicius, because the eccentric circles explain the changes in distances and brightness of the celestial bodies. On the other hand, the idea of epicycles and eccentric circles was not probably known up to that day, since otherwise it should have been discussed at Plato's Academy or at least it could have been mentioned by Plato, Aristotle or some other student or partner of Plato. There is a fragment in the book by Geminus of Rhodes (book I, 19, 21 of [34]; see also [35]), according to which it is possible that, if one does not pay sufficient attention, to think that Pythagoreans were the first who invented eccentric circles and epicycles, but that does not seem to be true (pp. 269 of [8]; pp. 143 of [13]).

2.9 *Heraclides of Pontus*

A contemporary of Aristotle and partner of the Academy was a student of Plato and philosopher called Heraclides of Pontus (388–315 BC), who was distinguished for the breadth of his spirit. He was called a paradoxologist (8, 72 of [9]) because of his strange and innovative ideas and therefore his presence was enough to develop a debate in the Academy of Plato. His paradoxology was due, to a great extent, to the two great spiritual influences that he had been subjected to, the Pythagoreans on the one hand and Plato on the other. Even though he had written many books on different subjects, nothing has survived today apart from extract of some of his work.

Disproving the theory of Eudoxus, together with the perceptions of Plato and Aristotle, he partially formulated a heliocentric theory. Testimonies of his system are found in the work of the neoplatonic philosopher Chalcidius who translated *Timaeus* into Latin, of Vitruvius, of Martianus Capella and of Cicero. According to his bright and original theory (pp. 255–260 of [8]; i, pp. 316–317 of [36]), Mercury and Venus (the internal planets), which sometimes seemed to be in front of and at other times behind the Sun and so seem to oscillate from one side of the Sun to the other, rotate on circular orbits around the Sun, which is found at their centre, with constant but different angular velocities (*epicycles*). Simultaneously, the Sun depicts the circumference counterclockwise of a circle having the Earth as its centre in a period of 1 year (*deferent circle* or *deferent* (see figure 3 later)). He consequently explained first the apparent motions of the two planets. We can therefore essentially say that Heraclides was the inventor of the system of epicycles. Later, in section 3.3 we clarify that Ptolemy was influenced by the theory of Heraclides.

He also adopted the view that the daily retrograde rotation of the celestial sphere around the Earth was due to the direct rotation of the Earth around its axis, which is the axis of the world, meaning something similar to what the Pythagorians Ecphantus and Hicetas had formulated (i, pp. 316–317 of [36]). Plutarch (Γ, 13 of [11]), as we have already mentioned (section 2.3), mentioned accordingly: 'Others [philosophers] on one hand believe that the Earth is still [in the centre of the world]. Philolaus the Pythagorian on the other hand . . . Heraclides of Pontus and Ecphantus the Pythagorian believe that the Earth moves, not transitionally within an orbit, but by rotating around its axis towards direct direction, like a wheel around its own axis.' Relevant testimonies can be found in the work of Proclus and some extracts of Simplicius (II, 7, 2000b, p. 444, of [22]): ' . . . for there are some, among whom Heraclides of Pontus and Aristarchus, who believe that they explain the phenomena if they consider the celestial sphere and the stars to be still and that the Earth rotates towards direct direction around the poles of the equator once a day'.

2.10 *Aristarchus of Samos*

Aristarchus of Samos (310–230 BC), who was an astronomer and mathematician, formulated a different cosmological model, namely the heliocentric system. In that system the idea of the Earth's immobility was rejected, since the Sun still remained at the centre of the world. The apparent motion of each planet was a combination of the motion of the Earth and its own around the Sun. The fixed stars were still.

Based on the curious book of Archimedes (287–212 BC) *Psammites* (*Sand-Reckoner*), we find the following (I, 4–6 of [37]; see also [38]): 'Aristarchus of Samos has published some theories [scriptures], from which he considers that the world is much greater than it is said to be. He also assumes that the fixed stars and the Sun remain still and that the Earth moves on a circle around the sun, which is located in the centre of the Earth's orbit. The sphere of the fixed stars, which possesses the same centre with the Sun, is so big, that the ratio of the radius of the circle which the Earth depicts during its revolution to the distance of the fixed stars is equal to the ratio of the centre of the sphere to its surface.'

From the previous analogy, we conclude that Aristarchus had grasped the concept of size of the spherical Universe. He considered the space, which the orbit of the Earth and the planetary system occupied as a whole, to be a point in comparison with the size of the Universe. This is what scientists also adopt today.

Plutarch (B, KΔ of [11]) said: 'Aristarchus formulated the theory that the Sun and the fixed stars remain still, the Earth moves around the Sun and when the Moon is near a node [near the plane of the Earth's orbit] we observe the eclipse of the solar disc.' Aetius (II, 24.8 of [20]) and, as we shall see in the following, Plutarch in another book (H, 1 of [16]) mentioned the ideas of Aristarchus. References to the heliocentric system of Aristarchus can also be found in the books by Vitruvius, Sextus Empiricus and Cicero.

Based on the arguments presented in the previous sections we conclude that the novel heliocentric system invented by Aristarchus did not appear suddenly, as a peculiar mathematical model, but it was the result of a gradual development of astronomical and philosophical ideas as well as of mathematical knowledge until the time of Aristarchus. Anaximander, Philolaus, Hicetas, Ecphantus and Heraclides of Pontus were the predecessors of Aristarchus, who affected his thinking and contributed to Aristarchus' discovery.

Heath (ii, pp. 2 of [36]) commented: 'To Aristarchus belongs the high honour of having been the first to formulate the Copernican hypothesis, which was then abandoned again until it was revived by Copernicus himself. His claim to the title of the ancient Copernicus is still, in my opinion, quite unshaken, ...'

The historian of mathematics, Loria (Vol. I, paragraph 65, pp. 114 of [39]) wrote: 'After him [he means Philolaus the Pythagorian], the movement towards that direction [heliocentric system] was fortified quite impressively. During Plato's time the Greeks had so enormously advanced, that there wasn't much for them to do in order to become masters of the general concept of the heliocentric motion of the planets. It is worthy of all praise that they managed, after a short period of time, to cover even that final step of the way. The great honor which corresponds to that memorable achievement was reserved for a mathematician, contemporary with Archimedes, named Aristarchus of Samos, the Copernicus of antiquity.'

In addition, Loria (Vol. I, paragraph 66, pp. 115 of [39]) said at another point: 'The system [of Aristarchus] that was later called Copernican, was abandoned very quickly in order to be approved the system of eccentric circles and epicyles, something that seems to have been a figment of Apollonius of Pergi's imagination. The latter seemed to be more appropriate for the depiction of the celestial phenomena and the making of the corresponding calculations. The system of Apollonius fully respected the fundamental principle of all astronomical systems that had been proposed from Pythagoras to Copernicus: not to offend through these [systems] all beauty and simplicity of the creation.'

Also, Lloyd (pp. 268 of [4]) in his book wrote: 'Aristarchus' theory of the heliocentric system was complete and managed to include both the daily rotation of the Earth around its axis, as well as the rotation of the Earth around the Sun. However, the theory of the heliocentric system was unpopular among Aristarchus' contemporaries. In particular, among the astronomers who were contemporary to Aristarchus, only the babylonian astronomer Seleucus shared Aristarchus views. The reasons that could account for the unpopularity of this theory are rather complicated and are properly irrelevant to religious issues.'

The ideas of Aristarchus were altering the entirely of human thinking during that time and were opening up new horizons for science. Unfortunately, did not prevail instantly. The revolutionary ideas of Philolaus, Hicetas, Ecphantus, Heraclides and mainly Aristarchus remained in obscurity until the time of Copernicus. The events that led to the prevalence of the heliocentric theory were as follows: the invention of the telescope by Galileo in 1610, the explanation of the aberration of starlight and the stellar parallax, and the discovery of the law of gravity by Newton in 1686, who proved the three empirical laws of Kepler.

2.11 *Seleucus*

After Aristarchus of Samos, Seleucus (second century BC), a contemporary of Hipparchus, an astronomer, a mathematician and a geographer, strongly supported the ideas of Aristarchus. Plutarch commented (H, 1 of [16]): 'He imagines [the Earth] rotating and moving forward, exactly as Aristarchus and Seleucus had showed [mediated] later on.' The same is mentioned in the scripts of Sextus Empiricus. In another book, Plutarch also mentioned that Seleucus followed the theories of Aristarchus (Γ, IZ of [11]): 'Seleucus the mathematician, by moving the Earth [saying that the Earth is moving] as well, claims that due to its rotation and motion it impedes the rotation of the Moon.'

Hence, until Hipparchus' time (190–120 BC), two systems had been developed and formulated: the geocentric and the heliocentric. These two theories were totally conflicting, because they arose from two different and adverse perceptions about the (until then) known world.

3. The theory of epicycles and eccentric circles

3.1 *Apollonius of Perga; Heraclldes of Pontus*

It appears that the ideas of epicycles and of eccentric circles were first introduced at the time of Eudoxus of Cnidus after the formulation of his theory on the concentric spheres and were spread during the third and second centuries BC. The models of epicycles and of eccentric cycles were purely geometrical models and were developed in order to support the geocentric hypothesis.

(i) According to the theory of the eccentric circles, a celestial body H, on the one hand, moves (towards the direct direction) with a constant angular velocity (normal circular motion) on the circumference of a circle with a centre K and a radius $R = \mathrm{KA}$. On the other hand, the Earth, which is considered to the observer Γ, is placed at a distance $\mathrm{K}\Gamma = eR$ from the centre of the circle (figure 2). This circle with eccentricity e is the apparent orbit of the body. In this way the celestial body is moving: firstly, at a different distance from the observer, which explains the changes in its brightness and, secondly, with a different angular velocity at each point of the eccentric circle. In particular, the body has a lower angular velocity around the apogee A, and a higher angular velocity in the region of the perigee Π (figure 2). This motion is essentially equivalent, as a first approximation, to the motion of a celestial body on an ellipse, one focus of which is occupied by the Earth. The eccentricity of the motion is, according to the theory of ellipse, $e = \mathrm{K}\Gamma/\mathrm{KA}$.

(ii) The motion within the system of epicycles is also similar to the previous motion and constitutes a better approximation. Usually, in this case, a celestial body H moves with a constant angular velocity ω_2 on a small circle of radius r, the *epicycle*, either clockwise or counterclockwise. The centre B of the epicycle moves by convention counterclockwise, with a constant angular velocity ω_1 (usually different from ω_2) on the circumference of a

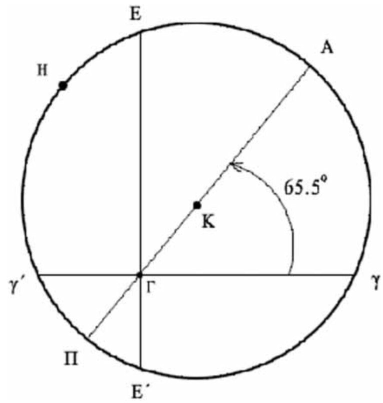

Figure 2. The eccentric motion, which is performed by a celestial body H. The Earth is placed at point Γ and its distance from the centre K of the cirle, divided by the radius KA, is equal to the eccentricity of the eccentric motion: $e = \mathrm{K}\Gamma/\mathrm{KA}$. If H is the Sun, points E and E′ are summer and winter solstice; and points γ and γ' the spring and autumn equatorial points respectively. A is the apogee and Π is the perigee of the eccentric motion.

great circle with a radius R, the deferent, which has the Earth as its centre Γ. The plane of the epicycle may be at an angle with the plane of the deferent. This complex motion, which is a superposition of two simple harmonic oscillations, is called epicycle motion (section 7.2 of [40]). It is possible that this is equivalent to the motion which takes place on an eccentric circle. The eccentricity therefore of the apparent orbit of the body is found by calculating the ratio of the radii of the two circles: $e = r/R$ (figure 3) (p. 264 of [33]). In general, provided that those two angular frequencies are combined in a suitable way (their ratio must be a rational number), the resulting motion will be an ellipse (or a helical curve, etc.).

In order to make these theories clearer, we shall use modern notation. A mechanical analogue of the epicycles motion and its equivalence, under certain initial conditions, to a eccentric circle

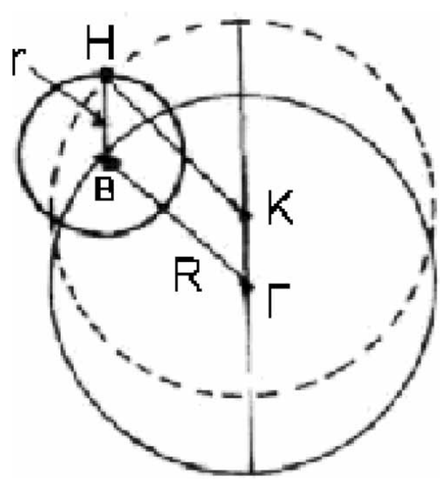

Figure 3. The epicyclinal motion, which is performed by a celestial body H. H moves on the epicycle with B as its centre, which in its own turn moves on the *deferent* with radius $R = \Gamma\mathrm{B}$. If H is the Sun then the epicyclic motion is equivalent to the eccentric circle (dashed circle) with K as its centre and a radius KH. The distance between the Earth Γ and the centre K is equal to the radius $r(= \mathrm{BH})$ of the epicycle. Therefore, the eccentricity of the apparent orbit of H is $e = \mathrm{K}\Gamma/R = r/R$.

arises in the motion of a satellite which goes round the Earth in a stable circular orbit of radius r_o, angular velocity ω_o and period T_o. Let us perturb the satellite's energy E by a tiny amount ΔE, which results in a change in its radius by $\Delta r(\Delta r/r_o \ll 1)$, while maintaining constant angular momentum. The satellite will then execute as a first approximation a simple harmonic oscillation around the position $\bar{r}_o$ of equilibrium and between the limits $r_o - D$ and $r_o + D$, where D is the width of oscillation with angular frequency ω_1 and period T_1. The satellite will therefore execute simultaneously two harmonic oscillations. As a result of the superposition of these two oscillations, the satellite changes slightly its initial orbit. In the case of the Earth's gravitational field, the ratio of the angular velocities (or the ratio of the corresponding periods) for the two harmonic motions is a rational number and, in particular, $\omega_o/\omega_1 = T_1/T_o = 1$ (we omit the mathematical calculations) (see also p. 284 of [40]). It can be deduced from this relation that the resultant motion is a new circular orbit shifted from the initial orbit and of the same period. The centre O′ of the new orbit does not coincide with the centre O of the initial circular orbit (i.e. the centre of the Earth) but is shifted to the direction of greater radial deviation. Hence, the resultant motion is an eccentric circle (figure 4).

As far as the inventor of the system of epicycles and the eccentric circles is concerned, it seems that the first who implemented the epicycles was Heraclides, as we mentioned in section 2.9, while the great geometrician Apollonius of Perga (265–190 BC) first applied eccentric circles.

Apollonius studied the theory of eccentric circles and proved that it applies only to the three exterior planets Mars, Jupiter and Saturn, since it interprets to a certain extent their apparent motions, while it could not be implemented for the interior planets. He also studied the cases of equivalence between the epicycles and the eccentric circles. It is obvious therefore that Apollonius was the first who introduced eccentric circles. This assumption also coincides with the fact that Apollonius introduced conic sections (he is the author of a relevant book) since, as we have already mentioned, the motion on an eccentric circle is, as a first approximation, a motion on an ellipse, where the Earth possesses one of its foci with eccentricity $e = \mathrm{K}\Gamma/\mathrm{KA}$ (figure 2).

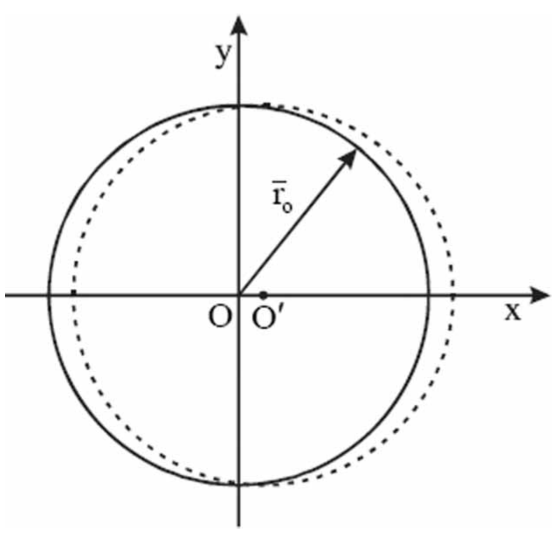

Figure 4. The orbit shown as a dashed circle is the epicyclic orbit (perturbed orbit) which is almost the circumference of circle; however, the centre O′ is shifted from the centre O of the Earth in the direction of the maximum radial deviation that is outside the circumference of circle of radius r_o.

We do not know, however, for sure whether a complex theory of the planets was developed in ancient Greece and who its inventor was. Some historians such as the astronomers Schiaparelli, Heath and Tannery tried to do that and studied the corresponding fragments of ancient authors. Thus, some final arguments have been given by modern authors, which coincide to a certain degree. After a thorough study of certain ancient writers and modern historians, one can conclude that the theory of Heraclides was also implemented for the three exterior planets. Hence, a system similar to the system that became known in the fifteenth century, namely the system of Tycho Brache, was developed in ancient times. According to that system the five planets move on epicycles or eccentric circles around the Sun, which simultaneously moves around the Earth. The inventor of this system is not known with certainty. It is possible, however, that the authorship of this system belongs to Apollonius, without of course excluding the possibility that it belongs to some other contemporary or previous astronomers, who lived between Heraclides and Apollonius (pp. 113 of [5]). This problem, however, is still being discussed. The development of that system, although it retained the immobility of the Earth at the centre of the world together with normal circular motions, consisted of one important step towards the evolution of cosmological ideas and geometrical models.

In the context of the geocentric system, the geometrical models of the epicycles and the eccentric circles were novel and created new horizons towards its mathematical interpretation. These geometrical models were invented and dominated thereafter because of the deeply rooted belief that the Earth was still at the centre of the world and that the orbits of the celestial bodies were normal circular motions. They consequently provided a first satisfactory explanation of the apparent orbits of the planets. This must be the reason why Apollonius introduced eccentric circles and implemented them together with the epicycles instead of considering that the celestial bodies depict elliptic orbits.

3.2 *The adoption of the geocentric system by Hipparchus of Rhodes*

During the following century, based on arguments analysed in previous work (pp. 115 of [5]), Hipparchus adopted the geocentric theory and rejected the heliocentric theory. In order to explain the geocentric system by using a mathematical model, he first used for the planets and especially for the Sun and the Moon the model of eccentric circles and later the model of epicycles, which, he claimed, belonged to him (pp. 188 of [7]). About the theory of epicycles, Hipparchus said that it was more preferable than the theory of the eccentric circles because the celestial bodies move symmetrically with respect to the centre of the world, the Earth. Hipparchus also used the observations that were available at that time in order to interpret the apparent orbit of the Sun. He knew that the four seasons of the year and especially the arcs $\mathrm{E}\gamma$, $\gamma\mathrm{E}'$, $\mathrm{E}'\gamma'$ and $\gamma'\mathrm{E}$ (figure 2) were not equal. This inequality led him to the formulation of two models about the motion of the Sun.

(i) During one tropical year, the Sun H moves, within a constant angular velocity, along one circle around the Earth with radius $R = \mathrm{KA}$ a centre K, which is placed at a distance $\mathrm{K}\Gamma = eR$ from the observer Γ (from the Earth), where e is the eccentricity of the orbit (figure 2).
(ii) During the tropical year, the Sun H moves on an epicycle with a radius $r = eR$ westwards. The centre B of the epicycle, depicts a direct direction (eastwards) and during the same period the deferent circle with radius R and centre Γ, the Earth (figure 3).

He proved that these two theories are equivalent, meaning that the motion on an epicycle is identical with the motion along an eccentric circle. Each of these two theories could interpret (determine) fairly well the apparent motion of the Sun with an error of 1 minute of the arc (pp. 163 of [13]) That inaccuracy was so small that it remained acceptable for 1700 years. As we have already mentioned, Apollonius seems to be the first who tried to prove the equivalence of the two motions for certain planets. According to Ptolemy (Γ, 4 of [41]; see also [42]). Hipparchus correctly made the assumption that the orbit of the Sun was an eccentric circle. In order to interpret the differences between the lengths of the seasons, he placed the centre of the eccentric circle towards the direction of the arc which is found meaning between the spring equinox γ and the summer solstice E, meaning the direction of the apogee A, and chose the right values for the eccentricity e and the length of the apogee (p. 41 of [43]). Hipparchus first talked about the apogee and perigee Π of the Sun's orbit and found that its eccentricity was $e = 1/24 = 0.0416$, which was very satisfying, since its current value is approximately $e = 0.0167$. He also found that the length of the apogee, which is the arc between the spring equinox and A, is the angle $(\gamma \Gamma \mathrm{A}) = 65°30'$, also a very good value. In that way he explained the inequality of the four seasons of year, since the Sun was moving with different angular velocities along the eccentric circle (slower towards the apogee and faster in the region of the perigee). Additionally, he proved that the orbit of the Sun coincided with the ecliptic, while previous astronomers considered that the orbit was inclined towards the ecliptic.

Consequently, as a first approximation, the theory of Hipparchus was equivalent to the first two laws of Kepler.

As far as the orbit of the Moon is concerned, Hipparchus knew that its motion is more complex than the Sun's motion and that it includes many abnormalities. He also formulated two models about the motion of the Moon.

Initially, he considered that an eccentric circle could depict the apparent orbit of the Moon. He then calculated first abnormality of the Moon's orbit (which in first approximation is an ellipse), namely its eccentricity and found it equal to $e = 0.0875$. This is a very good value, since its current value is $e = 0.055$.

Hipparchus (E 7 of [41]; p. 265 of [33]) observed that the orbit of the Moon appears to be a great circle on the celestial sphere and is totally found within zodiac zone. He additionally calculated that the inclination of the Moon's orbit in relation to the ecliptic is 5° and therefore its greatest declination is $23°\ 51' + 5° = 28°\ 51'$ approximately. At the time of Hipparchus, the inclination, the obliquity of the ecliptic, was not $23°\ 27'$, like today, but $23°\ 43'$ and according to Eratosthenes' calculations $23°\ 51'$ approximately (i, 22, 67 of [41]; p. 40 of [44]; pp. 3–14 of [45]; pp. 134–136 of [46]). He also knew that the advance of the perigee, *i.e.* the line of apsides, was being shifted from west to east and performed a complete rotation in a period of approximately 9 years. We now know that the rotation of the line of apsides on the plane of the Moon's orbit is $40°\ 40'\ 35''$ per year and so it has a period of 8.85 years. This result together with other abnormalities of the Moon's motion is due to the tidal forces (the attractions by the Sun and the planets) acting upon the Moon. He therefore understood that the theory of the eccentric circle was inadequate because it could not possibly explain the observations, meaning the other abnormalities of the Moon's orbit. This is exactly the reason why he was led to the conclusion that he should implement the theory of epicycles. He was then able to determine the second abnormality of the Moon's orbit, which is due to the regressive motion of the line of apsides around its average position. Hence the Moon is shifted on both sides of its average position up to a width of $1°.25$ and with a period equal to the time between two transitions of the Sun through the line of apsides of its orbit. The latter abnormality is due to the change in the curvature of the orbit and therefore in the eccentricity.

Hipparchus thus accepted that the Moon rotates clockwise on an epicycle, the centre of which moves on the *deferent* counterclockwise around the Earth. The plane of the *deferent* coincides with the ecliptic. The plane of the epicycle forms an angle of 5° with the plane of the deferent circle and therefore intersects the ecliptic at an angle of 5°. Hence, the *nodes* move from east to west and perform a complete rotation around the axis of the ecliptic in a period of $18(2/3) = 18.6$ years. This abnormality is called '*the regression of the nodes*'. Hipparchus calculated the length of the radius r of the Moon's epicycle and the position of the apogee. For the radii of the deferent circle and the epicycle he used the following values: $R = 60$ and $r = 5(1/4)$, respectively; therefore $e = 5(1/4)$: $60 = 0.0875$. The angular velocity of the deferent circle was $\omega_1 = 13^\circ\ 10'\ 35''$ per day and that of the epicycle $\omega_2 = 13^\circ\ 3'\ 54''$ per day (pp. 240 and 243 of [32]) did not completely coincide because of the direct motion of the line of apsides in approximately 9 years. As a consequence, the difference in these angular velocities justifies the draconitic or nodical month.

Because of the few and simple astronomical instruments that Hipparchus had at his disposal he could not possibly determine or interpret all the abnormalities of the motion of our satellite. These abnormalities were explained only after the invention of the telescope and of the law of gravity by Newton. Despite all this, however, the calculations and results that he provided about the motion of the Moon and the theory that he formulated in order to interpret some of the abnormalities of its orbit are remarkable.

As far as the study of the motion of the five known planets is concerned, Hipparchus assumed that each planet would move on the epicycle, the centre of which would turn around the Earth on the *deferent*. In this way, Hipparchus laid the foundations of planetary theory. He did not, however, have at his disposal sufficiently accurate observations that would help him to form a complete theory. The great astronomer Claudius Ptolemy undertook later the shaping and perfecting of Hipparchus's ideas.

3.3 *Claudius Ptolemy*

Claudius Ptolemy (about 100–170 AD) accepted the geocentric system. Based on the ideas and theories of Heraclides, Apollonius and mostly Hipparchus, meaning the theories of epicycles and eccentric circles, he formulated his own cosmological model. In order to overcome some imperfections of the geocentric theory, Ptolemy relied upon the mathematical model of Hipparchus, which he modified and enlarged upon the motions of the planets. This system was called the Ptolemaic system. All the details of his system are mentioned in his well-known treatise *Syntaxis* or *Almagest* (*Megale Syntaxis tes Astronomias* see [41], [42]), which consists of 13 volumes. This treatise contributed to the spreading of knowledge of the ancient Greek astronomers to the Arabs and the western world.

Together with the previous mathematical-cosmological models, the Ptolemaic system relied upon the superposition of normal circular motions, *i.e.* oscillations, which arose from the combination of epicycles and eccentric circles. This system interpreted and predicted the apparent positions of Moon, the Sun and planets.

According to that system, each planet was rotating around the Earth by performing simultaneously two normal circular motions. It was therefore moving on an epicycle, the centre of which was moving eastwards on the deferent circle. Each planet had its own deferent circle with centre K, which would not coincide with the Earth Γ, since it would be placed outside the Earth. Each deferent circle was therefore an eccentric circle. The eccentric circles would increase their radius as one goes from the Moon to Mercury, Venus, Sun, Mars, Jupiter and Saturn (figure 5). Also, Ptolemy considered that the constant angular velocity (uniform angular motion) by which the centre of each epicycle moves on the deferent circle should be measured not from the Earth, but rather from the third point E, the *equant point*. This point should be placed *at the other end of the diameter* joining the Earth to the centre of the *deferent circles*, which is placed in the middle of the line ΓE. Therefore, the retrograde motions of Mars, Jupiter and Saturn could be quite easily and accurately explained.

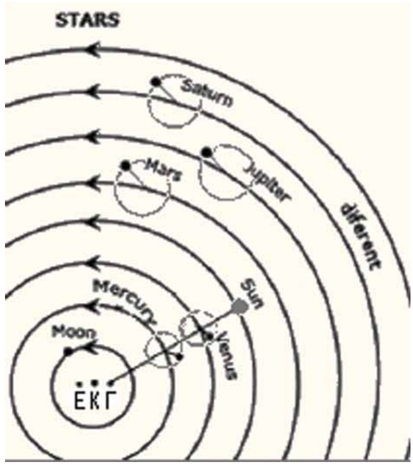

Figure 5. The cosmological model of Ptolemy. K is the centre of the *deferent circles*, E is the *equal point* and Γ is the centre of the Earth.

As far as the interior planets are concerned, they never stay far from the Sun as seen from the Earth, because these planets oscillate from one side of the Sun, to the other. The greatest elongation of Mercury never exceeds 28° and that of Venus never exceeds 48° approximately. For this reason, Ptolemy considered that the centres of the epicycles of different radii corresponding to the two planets are always found along the same straight line which connects the Earth to the Sun, as the centres of the epicycles move around the corresponding deferent circle.

The resultant motion that the two planets perform is then equivalent to that contained in the theory of Heraclides about Mercury and Venus. This conclusion can be obtained as follows: firstly, the centres of the epicycles of the two planets are projected always on the centre of the Sun and, secondly, as the two planets rotate on their epicycles, with angular velocities arranged in such a way that their apparent positions could be described in an optimal way, their orbits are projected on the celestial sphere. As a result, each planet appears on either one or the other side of the solar disc, *i.e.* they seem to oscillate from one side of the Sun to the other. We may therefore make the conjecture that Ptolemy invented this system based on the existence of the theory of Heraclides of Pontus.

The Moon and the Sun were moving around the Earth on their own eccentric circle, eastwards within a period of a month for the Moon and a year for the Sun. Ptolemy considered, in contrast with Hipparchus' ideas, that epicycles were not needed because the motions of the Moon and of the Sun are always direct. At the same time they participated in their daily motion.

Regarding the motions of the fixed stars, Ptolemy supposed that the stars are found to be riveted in a sphere, which surround the deferent circles of all planets. The rotation of that sphere around the Earth in 24 h was initiating the rises and sets of the stars.

Using the observations that had been collected until his time, Ptolemy could always improve his system so that his statements coincided with the future apparent positions of the planets. The radii of the *deferent* and the *epicycle*, the orientation of the plane of the epicycle relative to the deferent circle, the angular velocities of the planet on the epicycle and of the centre of epicycle on the deferent circle could be modified and combined in such a way that the resultant motion of the planet could fit fairly well to its apparent position. The Ptolemaic system was therefore a very flexible system and that is the reason why it prevailed and remained the only acceptable planetary system for 1400 years after Ptolemy, until Copernicus laid the foundations of the heliocentric system. Galileo achieved the first proof of the heliocentric system in 1610 after the invention of the telescope. Later, having many observations at his disposal, Kepler discovered and formulated the three laws of motion of the planets around the Sun. Finally, Newton, after the discovery of the law of gravity in 1686, proved theoretically the three laws of Kepler.

4. Conclusions

The use of mathematics and, in particular, geometry led to significant contributions towards the development of cosmological ideas and theories. The models of epicycles and of eccentric circles were complex geometrical models and created new horizons in the mathematical interpretation of the geocentric system. These geometrical models were invented and dominated thereafter because of the deeply rooted belief that the Earth was still at the centre of the world and that the orbits of the celestial bodies were normal circular motions. This could be the reason why Apollonius, who wrote the definitive work on conic sections, introduced eccentric circles and implemented them together with epicycles instead of considering that celestial bodies travel in elliptic orbits.

A thorough study of ancient sources and modern authors implies that the heliocentric hypothesis of Aristarchus of Samos was an innovative idea that created new horizons not only in astronomy but also in human thought in general. However, the heliocentric theory did not arise as a peculiar mathematical model but was the product of a long and fervent intellectual activity created by the precursors of Aristarchus, namely Anaximander, Philolaus, Hicetas, Ecphantus and Heraclides of Pontus, who influenced Aristarhus' thinking. On the other hand, Simplicius considered that it was Philolaus who introduced the heliocentric system. In order to explain this consideration of Simplicius, we assume that, when Philolaus mentioned the *Central Fire*, he implied the Sun.

Despite the importance of Aristachus' hypothesis the geocentric system finally became the dominant cosmological model until the time of Copernicus. An explanation of this fact requires a detailed analysis, which will be presented elsewhere. This was because it was supported by the eminent intellects of Pythagoras, Plato, Aristotle, Eudoxus, Apollonius, Hipparchus as well as Ptolemy. Also, the models of epicycles and of eccentric cycles contributed to this. Kuhn (p. 140, 141 and 148 of [47]) noted among others: 'If the solution to a problem appears at a time when there is not much dispute in the relevant science field, then new ideas may be not become widely known and be forgotten. The heliocentric model of Aristarchus was ignored because the geocentric model was the dominant theory at the time. This model [geocentric] seemed not to have any weak point that a new model could overcome.'

We have concluded that Plato believed in a simultaneous beginning of time and space (he named it 'sky') in connection with the existence of matter and that they will vanish at the same time. These views of Plato are close to modern cosmological ideas introduced by Einstein in the formulation of the general theory of relativity and are in accordance with the Big Bang model. On the other hand, they are not consistent with the cosmology theories of Gold, Bondi and Hoyle and that of Dirac. We should note that it would be inappropriate to try to compare modern ideas and theories with those of Plato or ancient Greek scientists, since they belong to different eras. The ideas of Plato or other ancient scientists and philosophers could be used as a philosophical basis by modern scientists.

The principle of independence and superposition of motions was introduced in the formulation of cosmological models as early as Plato's time. According to Simplicius, Plato believed that the apparent motions of planets could be described by the superposition of normal circular motions or, in other words, by the superposition of simple hannonic oscillations.

Based on a fragment of *Timaeus* we conclude that Plato accepted the daily motion of the Earth around its axis. Something similar is also mentioned in Aristotle's book *On Heaven*.

In a different chapter of *Timaeus*, Plato seems to consider the Earth as the fixed centre of the world.

Simplicius noted that Plato could foresee the need to use mathematics in astronomy and especially in the formulation of mathematical models in order to explain celestial phenomena. He also emphasized this need to his students.

After studying the theory of homocentric spheres we conclude that Eudoxus had also realized the principle of superposition of motions. For example, the motion resulting from the motions of the two last spheres of each planet is a superposition of two harmonic oscillations that resembles a figure of eight.

Heraclides of Pontus was the first to explain the apparent motions of Mercury and Venus. Heraclides was therefore the inventor of the theory of epicycles. According to the theory of Ptolemy, the resultant motion executed by each of these two planets is equivalent to the motion resulting from the theory of Heraclides. We may therefore conjecture that Ptolemy's ideas were based on the theory of Heraclides of Pontus in order to formulate his system describing the motions of Mercury and Venus.

A mechanical analogue of the epicyclical motion and its equivalence, under certain initial conditions, to an eccentric circle arises in the motion of a satellite which goes round the Earth in a stable circular orbit, if we perturb the satellite's energy E by a tiny amount ΔE while maintaining the angular momentum constant.

Hipparchus' theory of eccentric circles is, as a first approximation, equivalent to the first two laws of Kepler.

Aristarchus formulated an analogy between the dimensions of the planetary system and those of the celestial sphere. From this analogy we conclude that Aristarchus conceived the actual size of Universe. He considered the space occupied by the orbit of the Earth and the planetary system as a point in comparison with the space occupied by the stars. This is equivalent to an estimation of the size of the Universe. Today, astronomers make similar considerations.

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