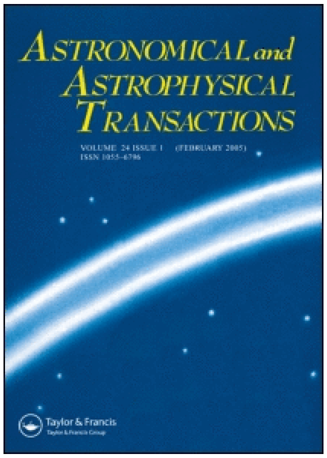

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The accelerated expansion of the Universe and the multidimensional theory of gravitation

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From formulae for multidimensional gravitation representing the generalization of the general theory of relativity for n dimensions, the condition of the accelerated expansion of the Universe is deduced. Initially a model single-component ideal isotropic fluid with power diagonal metrics was used. Restrictions on the equation conditions for our three-dimensional space are considered and additional dimensions are obtained.

Keywords: Cosmology; Multidimensional gravitation; Multidimension; Dark matter; Expansion of the Universe

1. Introduction

In the last few years of the twentieth century, a very important astronomical discovery was made: the accelerated expansion of the Universe [1]. The result of observing flashes from far supernovae and measurements of their characteristics has now been related not only to the value of Hubble's parameter but also to the parameter of acceleration of the Universe. The fact that the Universe is undergoing accelerated expansion has been proved to be true by several independent groups of researchers.

On the other hand, significant progress was achieved in the unification of physical interactions. The grand unification, supersymmetric, string and superstring theories were developed, as also were membrane, p-bran and the so-called μ and F theories. Multivariate gravitational models and scalar–tensor theories of gravitation have become the theoretical basis of the explanation of time and spatial variations in fundamental physical constants.

Multivariate theories of gravitation represent the generalization of the relativity theory which has been checked reliably for weak fields and in part for strong fields (double pulsars) to within 0.001. Since a suitable grand unification theory until now has not been created, it is useful to consider simple but sufficiently general multidimensional models based on the multivariate models of Hilbert and Einstein in vacuum or with various sources (cosmological constant, ideal or viscous liquid, scalar and electromagnetic fields and their interactions etc.) [2, 3].

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2. Theory

In this work, the opportunity to obtain formulae for the accelerated expansion of the Universe is considered. The Universe in the framework of cosmological models describes evolution of n one-dimensional spaces at present as a single-component ideal liquid. The metrics of model is given by

$$\mathrm{d}s^2 = -\mathrm{d}t_\mathrm{s}^2 + \sum_{i=1}^{n} a_i^2(t_\mathrm{s})\varepsilon^i(\mathrm{d}y^i)^2,$$

where t_s is the synchronous time and $\varepsilon^i = \pm 1, i = 1, \ldots, n$. The range of definition of the metric is $M = (t_-, t_+) \times R^n$.

The energy–momentum tensor

$$T_N^M = \mathrm{diag}(-\rho(t), p_1(t), \ldots, p_n(t))$$

describes an anisotropic fluid, where ρ is the density and p_i is the pressure in the ith one-dimensional space.

The pressure in all one-dimensional spaces is assumed to be proportional to the density. The scale factor is

$$a_i = a_i(t_s) = A_i t^{\nu_i},$$

where

$$\nu_i = \frac{2u^i}{\langle u^\Lambda - u, u\rangle}$$

and $u^\Lambda = 2$ corresponds to a component of the Λ term ($p = -\rho$). Also

$$u^i = \sum_{j=1}^{n} G^{ij} u_j,$$

$$\langle u^\Lambda - u, u\rangle = \sum_{i,j=1}^{n} G^{ij}(2 - u_i)u_j,$$

$$G^{ij} = \delta_{ij} + \frac{1}{2 - D},$$

where $D = 1 + n = 1 + 3 + d$ in the total number of dimensions (including time), $d = n - 3$ is the number of additional dimensions and

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

We consider our Universe as an isotropic three-dimensional space with $u_1 = u_2 = u_3 = u$ and $p = (1 - u)\rho$ with additional dimensions $u_4 = \cdots = u_n = v$ and $p_\mathrm{in} = (1 - v)\rho$.

Accelerated expansion occurs if $a_i > 0, i = 1, 2, 3$. In the case of the power metrics, $a_i(t) = t^{\nu_i}$; this condition holds at $\nu_i > 1$. Restrictions on the properties of the additional dimensions impose also the requirement of constancy of the gravitational constant G:

$$G(t) = \left(\prod_{i=4}^{n} a_i(t)\right)^{-1} = \text{constant} \implies \sum_{i=4}^{n} \nu_i(t) = 0$$

at $a_i(t) = t^{\nu_i}$. Invariance of the gravitational constant is a fundamental property of our world as well as in the isotropic Universe.

Let us consider first the case when our three-dimensional space is combined only with one additional dimension, *i.e.* $d = 1$, $D = 1 + 3 + 1 = 5$:

$$G^{ij} = \delta_{ij} - \frac{1}{3}.$$

In the considered case, $u_1 = u_2 = u_3 = u$ and $u_4 = v$. As a result,

$$u^1 = u^2 = u^3 = -\frac{1}{3}v, \quad u^4 = -u + \frac{2}{3}v,$$

$$\langle u^\Lambda - u, u\rangle = 3[G^{11}u(2-u) + G^{12}u(2-u) + G^{13}u(2-u)] + 3G^{14}u(2-v) + 3G^{41}v(2-u) + G^{44}v(2-v).$$

After substitution of G^{ij}, we obtain

$$\langle u^\Lambda - u, u\rangle = 2\left(-u - \frac{1}{3}v + uv - \frac{1}{3}v^2\right),$$

$$\nu_1 = \nu_2 = \nu_3 = \frac{-1/3v}{-u - 1/3v + uv - 1/3v^2}, \quad \nu_4 = \frac{-u + 2/3v}{-u - 1/3v + uv - 1/3v^2}.$$

With the additional requirements of accelerated expansion of the Universe and constancy of the gravitational constant ($\nu_4 = 0$), we obtain $v = 3/2u$. The restrictions on the parameters in the equation of state in our space, u, and in additional dimensions, v, are deduced from

$$\frac{4}{3} < u < 2, \quad 3 < v < 2.$$

Let us generalize our calculation for arbitrary dimensionality d, with $D = d + 4$:

$$G^{ij} = \delta_{ij} - \frac{1}{d+2}$$

$$G^{ij} = \begin{cases} \dfrac{d+1}{d+2}, & i = j, \\ -\dfrac{1}{d+2}, & i \neq j. \end{cases}$$

If $u_1 = u_2 = u_3 = u$ and $u_4 = \cdots = u_n = v$; by analogy to the previous case, we have

$$u^1 = u^2 = u^3 = \frac{d-1}{d+2}u - \frac{d}{d+2}v,$$

$$u^4 = \cdots = u^n = -\frac{3}{d+2}u - \frac{2}{d+2}v,$$

$$\langle 2 - u, u\rangle = \frac{1}{d+2}[-6u - 3u^2(d-1) + 6duv - 2dv - 2dv^2],$$

$$\nu_1 = \nu_2 = \nu_3 = \frac{2[(d-1)u - dv]}{-6u - 3u^2(d-1) + 6duv - 2dv - 2dv^2},$$

$$\nu_4 = \cdots = \nu_4 = \frac{2(-3u + 2v)}{-6u - 3u^2(d-1) + 6duv - 2dv - 2dv^2}.$$

The condition of constancy of the gravitational constant, $\sum_{i=4}^{n} \nu_i(t) = 0$, in the case of isotropic additional dimensions will be written as $\nu_i(t) = 0$, $i = 4, \ldots, n$.

As a result we obtain $v = \frac{3}{2}u$, as well as in the four-dimensional case. Substituting this value in the expression for ν_i, $i = 1, 2, 3$, we obtain

$$\nu_1 = \nu_2 = \nu_3 = -\frac{2}{3(u-2)} \quad \forall d.$$

Apparently, in the case of isotropic internal dimensions, our result will not depend on the number of dimensions.

The final condition of the accelerated expansion of the Universe will be precisely the same as in the four-dimensional case:

$$\frac{4}{3} < u < 2, \quad 3 < v < 2.$$

Finally, it is possible to obtain restrictions on the equations of state in our space with additional dimensions. The external pressure of our three-dimensional space is

$$-\rho < p < -\frac{1}{3}\rho.$$

The internal pressure of the additional dimensions is

$$-2\rho < p_{\text{in}} < -\rho.$$

Our considerations hold only in the case of an isotropic equation of state with additional dimensions, *i.e.* for $u_4 = \ldots = u_n = v$. In the anisotropic case (in relation to additional measurements) the properties of our Universe depend on the anisotropy parameter of internal space, Δ:

$$\Delta = \sum_{i=4}^{n} u_i^2 - \frac{1}{d}\left(\sum_{i=4}^{n} u_i\right)^2 > 0.$$

$\Delta = 0$, if $u_4 = \ldots = u_n = v$. For $\Delta \neq 0$ the cndition of accelerated expansion of the Universe will be

$$\frac{4}{3} + \sigma < u < 2 + \sigma,$$

where $\sigma > 0$ depends on the anisotropy parameter ($\sigma \approx \Delta/u$). For example, in the case of two various additional dimensions $u_4 = v_1$ and $u_5 = v_2$, where $v_1 \neq v_2$, we obtain $v_1 + v_2 = 3u$,

$$\nu^1 = \nu^2 = \nu^3 = \frac{-u}{-3u + 6u^2 - (v_1^2 + v_2^2)} = \frac{-u}{-3u + 3/2u^2 - \Delta} = \frac{1}{3 - 3/2u + \Delta/u},$$

where $\Delta = v_1^2 + v_2^2 - \frac{1}{2}(v_1 + v_2)^2$.

From the condition $\nu_i > 1$, ($i = 1, 2, 3$) it follows that

$$\frac{4}{3} + \frac{2}{3}\frac{\Delta}{u} < u < 2 + \frac{2}{3}\frac{\Delta}{u}, \quad -\left(\frac{1}{3} + \frac{2}{3}\frac{\Delta}{u}\right)\rho < \rho < -\left(1 + \frac{2}{3}\frac{\Delta}{u}\right)\rho.$$

Apparently, these ratios, in the case of anisotropic additional dimensions, can influence the dynamics of our world.

During the twentieth century a choice between the basic cosmological models depended only on the amount (density) of ordinary observable space substance [4]. The discovery of dark matter and then accelerated expansion force us to change our usual representations. Already there is a talk of an unknown cosmic substance which determines the dynamics of our world.

The further development by observational engineering and construction of new theoretical models will allow us to make a definite conclusion about the physical nature of this substance. There are proofs [5] that the invariance of Hubble's constant at the transition from near to distant cosmological scales is connected to a scalar field ($p = -\rho$). The same field, only at a highest density, caused the initial inflation, as a result of which the early stage of evolution of the size of the world instantaneously increased by many many nonillions of times [6].

Observations of SNIa flashes and anisotropy of relic radiation allow us to draw a reliable conclusion on the deviation from the linear law of expansion of the Universe. It is not possible to talk about the exact law of expansion yet; however, already it is possible to tell that acceleration began some billions of years ago. Before then, expansion occurred within the framework of the classical Freedman cosmology [4]. By considering the benefits of each model, this could help us to obtain the density of the ordinary substance in the Universe. Then the situation is such that the dynamics of expansion of the Universe allow us to judge whether a substance of an unknown physical nature is probably included in the Universe.

As the calculations made in this work, are based on an n-dimensional generalization of the general theory of a relativity, the metrics of the model should satisfy the Einstein equations. According to the studies of [7–11], there correspond two choices for this condition Ivashchuk and co-workers: exponential and power. An exponential law results after substitution of the equation conditions of a cosmological vacuum, $p = -\rho$. In this case, however, acceleration will be positive in any case, and accelerated compression is possible. The power law of expansion supposes, in turn, various equation conditions.

In this work, power diagonal metrics were studied. The benefit of choosing power metrics can also be connected with a rather small deviation of observational dynamics from a linear law, although there is a certain amount of uncertainty. The dynamic characteristics of the scale factor have allowed us to draw a conclusion about the physical nature of the initial substances.

A range is obtained for the equation conditions allowing accelerated expansion of the Universe: $-\rho < p < -(1/3)\rho$; this will be coordinated with the estimations obtained earlier [12].

In the range found, the equation conditions include two-dimensional topological defects: domain walls, $p = -(2/3)\rho$. According to modern representations [13], topologically stable point defects were formed on boundary surfaces, where areas with various orientations meet. They could have dimensions from zero to three and consist of a vacuum with unbroken symmetry [13].

The possibility that additional dimensions exist is simultaneously investigated, together with their physical nature and influence on the dynamics of our three-dimensional world. The equation conditions of additional measurements are in the range $-2\rho < p_{\text{in}} < -\rho$, which corresponds to the so-called phantom or illusive matter when the weak condition is broken with energy dynamics $p + \rho < 0$. The same condition is required for the existence of 'wormholes'.

All calculations were carried out for single-component ideal isotropic substances. The considerations of a more complex model is possible, in which, for example, alongside the usual physical substance and the scalar field there exists a third unknown component, serving as the principal cause of the existence of latent dark matter.

3. Conclusions

(1) The formulae for multidimensional gravitation as applied to real cosmological models are tested.
(2) The relation between the dynamics of cosmological expansions and the properties of the initial substance is shown.

(3) Restrictions on the equation conditions of our three-dimensional space and additional dimensions are obtained.
(4) It is shown that, for certain parameters, the dynamics of the Universe do not depend on the number of additional dimensions.
(5) The properties of additional dimensions, which are required to influence the dynamics of our Universe, are specified.

A Magister Dissertation from the Institute of Gravitation and Cosmology, Peoples' Friendship University of Russia, and also a report given to the Fifth Summer Astronomical School in Odessa in August 2005 have served as a basis for this paper.

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