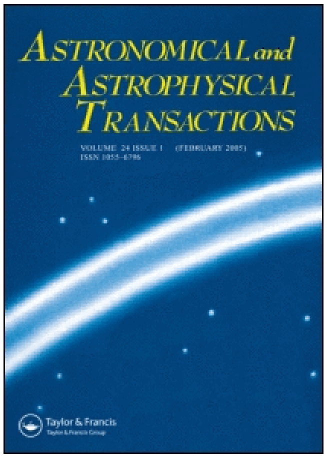

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On the dynamics of non-stationary binary stellar systems with non-isotropic mass flow

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The motion of a test body in the external gravitational field of binary stellar systems with some slowly varying physical parameters of the radiating components is considered on the basis of the restricted non-stationary photogravitational three- and two-body problems with non-isotropic mass flow. The family of polar and coplanar solutions are obtained. These solutions make it possible to give a dynamic and structural interpretation of binary young evolving stars and galaxies.

Keywords: Dynamics; Non-stationary systems; Particular solutions

1. Introduction

At the present time there are intensive investigations of non-stationary dynamic problems in astronomy [1–3]. The variations in some physical parameters of massive celestial bodies allows experimental definition. According to this, the formulation and investigation of celestial mechanical problems incuding the variation in some of those physical parameters with time appear urgent. Consequently, we can take into account the variations in the gravitational effect in the motion of a test body. The motion of a test body in the external gravitational field of a binary star or galaxy system with slowly varying physical parameters (mass, size and form) is considered as the dynamic model. In addition, we take into account the variation in the reduction parameters for radiating and gravitating bodies in the photogravitational formulation of the problem. The motion of particles is investigated in the framework of the restricted non-stationary photogravitational three- and two-body problems with non-isotropic mass flow. The results obtained make it possibile to carry out quantitative and qualitative investigations using an analysis of the effects of variable gravitation in the motion of celestial bodies.

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2. The motion in the neighbourhood of binary stellar system

Let us consider the motion of a test body in the gravitational field of a binary star or galaxy on the basis of the restricted non-stationary photogravitational three-body problem. Particular solutions of the stationary case of the problem have been considered in [4]. For the non-stationary case, analogous particular solutions of the problem were given in [5]. The particular solutions of the restricted three-body problem for the isotropic case of mass variation have been considered by Gelfgat [6], Bekov [7, 8] and Luk'yanov [9]; the non-isotropic case has been discussed by Bekov [10]. In the scheme of the restricted non-stationary photogravitational three-body problem under consideration here, we assume that the masses and reduction coefficients of all three bodies vary with time at the same rate in the presence of reactive forces and are proportional to the rate of mass variation and to the velocity of body motion (the non-isotropic case of variation in the mass of the bodies when the absolute velocities of the separated and added particles are equal to zero). The equations of motion for passive gravitating material in the rotating barycentric coordinate system Oxyz, the x–y plane of which coincides with the plane of motion of the main bodies, and the x axis of which always passes through these points, as in the work by Bekov [10], are of the following form:

$$\begin{aligned} \ddot{x} - 2\omega\dot{y} &= \omega^2 x + \dot{\omega} y - \mu_1 \frac{x - x_1}{r_1^3} - \mu_2 \frac{x - x_2}{r_2^3} - \frac{\dot{m}_3}{m_3}(\dot{x} - \omega y), \\ \ddot{y} + 2\omega\dot{x} &= \omega^2 y - \dot{\omega} x - \mu_1 \frac{y}{r_1^3} - \mu_2 \frac{y}{r_2^3} - \frac{\dot{m}_3}{m_3}(\dot{y} + \omega x), \\ \ddot{z} &= -\mu_1 \frac{z}{r_1^3} - \mu_2 \frac{z}{r_2^3} - \frac{\dot{m}_3}{m_3}\dot{z}, \end{aligned} \tag{1}$$

where r_1 and r_2 are the distances of the test body from main bodies, ω is their angular velocity of motion, and

$$\mu_i = G q_i M_i, \quad \frac{\dot{\mu}_i}{\mu_i} = \frac{\dot{m}_3}{m_3} = \frac{\dot{m}}{m} \quad (i = 1, 2), \tag{2}$$

where the gravitational constant G, the masses M_i of the main bodies and the reduction parameters q_i are functions of time, and also the mass of the test body is a function of time, which means that the masses of the bodieas vary at the same rate.

We now consider the case when the parameters q_i change in the interval of the real scale for the planet systems:

$$0 < q_i \leq 1. \tag{3}$$

Then, as in the case of the restricted three-body problem with non-isotropic mass flow [10], we can give particular solutions for the problem with variable parameters μ_i and m_3 in the form of the Eddington–Jeans law with indices $n = 3$ and $n = 6$:

$$\dot{\mu} = \alpha_i \mu_i^n, \quad \dot{m}_3 = \alpha m_3^n \quad (i = 1, 2) \quad (n = 3, 6). \tag{4}$$

Using the transformations

$$r(x, y, z) = \left(\frac{\mu_0}{\mu}\right)^3 \rho(\xi, \eta, \zeta), \quad \mathrm{d}\tau = \left(\frac{\mu}{\mu_0}\right)^5 \mathrm{d}t, \quad \omega = \left(\frac{\mu}{\mu_0}\right)^5 \omega_0, \tag{5}$$

equations (1) are taken in the autonomous forms

$$\begin{aligned} \xi'' - 2\omega_0\eta' &= \frac{\partial U}{\partial \xi}, \\ \eta'' + 2\omega_0\xi' &= \frac{\partial U}{\partial \eta}, \\ \zeta'' &= \frac{\partial U}{\partial \zeta}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} U &= \frac{\chi\omega_0^2}{2}(\xi^2+\eta^2+\zeta^2) - \frac{\omega_0^2\zeta^2}{2} + \frac{\mu_{01}}{\rho_1} + \frac{\mu_{02}}{\rho_2}, \\ \rho_i^2 &= (\xi-\xi_i)^2 + \eta^2 + \zeta^2 \quad (i = 1, 2), \\ \xi_1 &= -\frac{\mu_{02}}{\mu_0}\rho_{12}, \\ \xi_2 &= \frac{\mu_{01}}{\mu_0}\rho_{12}, \end{aligned} \tag{7}$$

and ρ_{12} and χ are constants given by

$$r_{12}\mu m^2 = \chi C^2, \quad \rho_{12}\mu_0 = \chi C^2(\chi > 0). \tag{8}$$

Here r_{12} is the distance between main bodies and C is the constant of the area integral. Equations (7) coincide with the corresponding equations for the isotropic case of mass flow [3, 8]; consequently, in our non-isotropic case, there exist analogous particular solutions of the problem. The particular solutions of equations (6) are defined by the system of equations

$$\frac{\partial U}{\partial \xi} = 0, \quad \frac{\partial U}{\partial \eta} = 0, \quad \frac{\partial U}{\partial \chi} = 0. \tag{9}$$

There are rectilinear solutions L_i $(I = 1, 2, 3)$ given by

$$\xi_L = \alpha_i, \quad \eta = 0, \quad \zeta = 0 \quad (i = 1, 2, 3), \tag{10}$$

triangular solutions L_4 and L_5 that are defined by the condition

$$\rho_1^3 = \rho_2^3 = \rho_{12}^3 = \frac{\chi^{36}}{\mu_0^3}. \tag{11}$$

If we suppose that $\zeta \neq 0$ in equation (9), then there are the coplanar solutions L_6 and L_7 $(\xi, 0, \xi)$ that may be defined, as is also in the case of the restricted three-body problem with variable masses [8, 9], by the equation

$$2(\xi + \mu_{20}) - 1 - \left(\frac{\mu_{10}\chi}{\xi + \mu_{10}(\chi - 1)}\right)^{2/3} + \left(\frac{\mu_{20}}{-\xi + \mu_{20}(\chi - 1)}\right)^{2/3} = 0. \tag{12}$$

Thus, the considered photogravitational problem has seven particular solutions in the region of varying parameters given by equation (3), analogous to solutions of the restricted variable-mass three-body problem. The additional particular solutions for different laws of the mass and luminosity variation may be determined analogously to the work by Bekov [8].

3. The motion in the neighbourhood of the massive radiating component of the star and the galaxy

Let us now investigate the motion near one of the components of the binary stellar system (star or galaxy); on the assumptions that the gravitational influence from the secondary component is small and that this influence can be considered as a perturbation, we can neglect this in comparison with the influence of the main component. Alternatively, we consider the case when the mass of the secondary component is infinitesimal in comparison with the mass of the main component of the stellar system. Then, as the dynamic model, we consider the motion on the basis of the restricted non-stationary photogravitational two-body problem. Particular solutions of the stationary problem have been considered by Batrakov [11] and Zhuravlev [12]. In our case we additionally take into account the variations in the mass, size and form of the main component, which is taken as a triaxial radiating and gravitating ellipsoid [13]. Let us consider the motion of a passive gravitating point in the external gravitational field rotating with angular velocity Ω and radiating a triaxial ellipsoid with mass $M(t)$, reduction parameter q $(0 < q \leq 1)$, size and form that slowly vary with time. We assume the non-isotropic case of mass variation for the test body (the absolute velocity of separated and added particles is equal to zero); *i.e.* we take into account the reactive force, which is proportional to the rate of mass variation and the velocity of motion for the material point. We suppose also that the slow variations in the ellipsoid's physical parameters do not lead to displacement of its centre of mass. The semiaxes a, b and c of the ellipsoid are all functions of time and, as in the stationary case, the ellipsoid differs little from a homogeneous sphere of radius R and has a volume equal to the volume of this sphere. Then

$$a^2 = R^2 + \alpha', \quad b^2 = R^2 + \beta', c^2 = R^2 + \sigma', \tag{13}$$

where α', β' and σ' are small quantities in comparison with R^2 and which, as a consequence of the equality of the volumes of ellipsoid and sphere, satisfy with sufficient accuracy up to a higher order the correlation $\alpha' + \beta' + \sigma' = 0$.

The equations of motion for a material point in a rotating Cartesian coordinate system $\hat{\mathrm{I}}xyz$ with the origin at the centre $\hat{\mathrm{I}}$ of mass of the ellipsoid, with the axes $\hat{\mathrm{I}}x$, $\hat{\mathrm{I}}y$ and $\hat{\mathrm{I}}z$ coinciding with the main central inertia axes of the ellipsoid, and with the direction of the angular velocity Ω of the ellipsoid rotation coinciding with the $\hat{\mathrm{I}}z$ axis direction have the following forms:

$$\begin{aligned} \ddot{x} - 2\Omega\dot{y} - \dot{\Omega}y - \Omega^2 x &= \frac{\partial \widetilde{V}}{\partial x} - \frac{\dot{m}_3}{m_3}(\dot{x} - \Omega y), \\ \ddot{y} + 2\Omega\dot{x} + \dot{\Omega}x - \Omega^2 y &= \frac{\partial \widetilde{V}}{\partial y} - \frac{\dot{m}_3}{m_3}(\dot{y} + \Omega x), \\ \ddot{z} &= \frac{\partial \widetilde{V}}{\partial z} - \frac{\dot{m}_3}{m_3}\dot{z}, \end{aligned} \tag{14}$$

where

$$\widetilde{V} = \frac{GqM}{r} + \frac{3}{10}GqM\frac{\alpha' x^2 + \beta' y^2 + \sigma' z^2}{r^5} + \cdots \tag{15}$$

represents the external potential of the ellipsoid at small α', β' and σ' and where m_3 is the mass of the material point. Using the transformations

$$\boldsymbol{r}(x, y, z) = l(t)\boldsymbol{\rho}(\xi, \eta, \xi), \quad \mathrm{d}\tau = \Omega\, \mathrm{d}t, \tag{16}$$

where $l^3\Omega^2\kappa = \mu(t)$, $\kappa =$ constant, $\mu(t) = GqM(t)$ and G is the gravitational constant, equations (14) result in the autonomous forms

$$\xi'' - 2\eta' = \frac{\partial V}{\partial \xi}, \quad \eta'' + 2\xi' = \frac{\partial V}{\partial \eta}, \quad \zeta'' = \frac{\partial V}{\partial \zeta}, \tag{17}$$

where

$$V = \kappa U,$$
$$U = \frac{\rho^2}{2} - \frac{1}{\kappa}\frac{\zeta^2}{2} + \frac{1}{\rho} + \varepsilon\frac{\alpha\xi^2 + \beta\eta^2 + \alpha\zeta^2}{\rho^5} + \cdots, \tag{18}$$
$$\rho^2 = \xi^2 + \eta^2 + \zeta^2.$$

Here ε is the parameter ($0 < \varepsilon \ll 1$), α, β and σ are constants that are defined by the correlations

$$\frac{3}{10}\frac{\alpha'(t)}{l^2(t)} = \varepsilon\alpha, \quad \frac{3}{10}\frac{\beta'(t)}{l^2(t)} = \varepsilon\beta, \quad \frac{3}{10}\frac{\sigma'(t)}{l^2(t)} = \varepsilon\sigma. \tag{19}$$

We find the functions $l(t)$ and $\Omega(t)$ that determine the transformation (16) from the correlations

$$l^2\Omega m_3 = l_0^2\Omega_0 m_3 = C_0, \quad \ddot{l} + \frac{\dot{m}_3}{m_3}\dot{l} + (\kappa - 1)\Omega^2 l = 0. \tag{20}$$

Because of the adiabatic invariant

$$l\mu m_3^2 = \kappa C_0^2 = \text{constant} \tag{21}$$

and taking into account that the masses vary at the same rate, i.e.

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{m}_3}{m_3} = \frac{\dot{m}}{m}, \tag{22}$$

where m is a function of the time, meaning that it has the same rate of mass variation, we found the particular solutions for mass variation with the time for the parameters μ and m_3 in the form of the Eddington–Jeans law with the indices $n = 3$ and $n = 6$:

$$\dot{\mu} = \alpha_0\mu^n, \quad \dot{m}_3 = \beta_0 m_3^n \quad (n = 3, 6). \tag{23}$$

The autonomous equations (17) have the same meanings as in the case of the isotropic mass flow [3, 13]; consequently, in our non-isotropic case there exist analogous particular solutions to the problem. The system (17) has particular solutions in the form

$$\xi = \text{constant}, \quad \eta = \text{constant}, \quad \zeta = \text{constant}, \tag{24}$$

analogous to the equatorial and polar solutions of the stationary problem. Equatorial solutions P_i are determined from the expressions

$$\begin{aligned} &P_1(P_3): \xi = \pm 1 \pm \varepsilon\alpha + \cdots, \quad \eta = 0, \quad \zeta = 0, \\ &P_2(P_4): \xi = 0, \quad \eta = \pm 1 \pm \varepsilon\beta + \cdots, \quad \zeta = 0. \end{aligned} \tag{25}$$

The polar solutions are determined in the form

$$P_5(P_6): \xi = 0, \quad \eta = 0, \quad \zeta = \pm\left(\frac{\kappa}{\kappa - 1}\right)^{1/3} \pm \varepsilon\left(\frac{\kappa - 1}{\kappa}\right)^{1/3} + \cdots. \tag{26}$$

Besides these solutions there are other classes of polar solutions, namely z solutions in the neighbourhood of the gravitating and radiating ellipsoid, which lie along the rotation axis of the ellipsoid [13].

4. Conclusion

The results of investigation of the dynamics of binary stellar systems on the basis of the photogravitational three- and two-body problems with variable mass and radiation pressure of the system's components with non-isotropic mass flow are presented as important, because we can investigate the new properties of conforming homographic solutions and build surfaces analogous to the Hill surfaces in order to obtain quantitative and qualitative analyses of the dynamic problem [14]. The particular solutions obtained may be used in difference problems in stellar dynamics, e.g. in the investigation of the motion of gas and dust particles in the neighbourhood of a binary or single forming variable star or the motion of stars and of gas and dust particles in the external gravitational field of a binary galaxy with slowly changing physical parameters of the galaxy's nucleus, and also to obtain supplementary astrophysical data for possible interpretation of transient structural peculiarities originating in the neighbourhood of such evolving stars and galaxies.

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