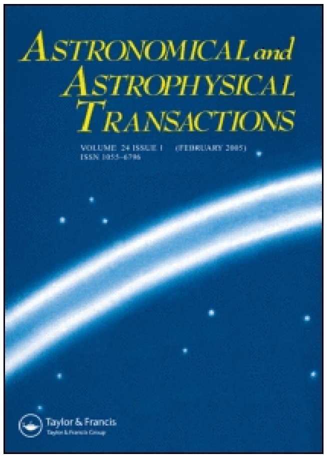


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# On the description of the dark energy by the pressure-dominance condition

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To describe the state of dark energy a new condition, namely the pressure-dominance condition ($p < -\rho$), was first proposed. This allowed us to write the dark-energy equation of state in the form of the equations in nonlinear acoustics. It is shown that a system of two gravitating scalar fields allowed the realization of the pressure-dominance condition.

*Keywords*: Dark energy; Equation of state

One of the basic problems of modern cosmology is the theoretical description of the dark energy associated, for example, with quintessence, the $\Lambda$ term, etc. Its observable properties are the scale homogeneity and absence of clustering [1, 2]. An important theoretical aspect of this problem is the question of the dark-energy equation of state. The simplest equation of state for quintessence was chosen in the linear form $p = w^2\rho$, where the magnitude of the coefficient of proportionality is within the interval $-1 < w^2 < -1/3$ [3]. However, observable data on supernova demonstrate that $w^2$ may be less than $-1$ [4–9]. At the same time these data led to the result about the energy-dominance violation in the dark energy.

This has been discussed in a number of articles where the energy-dominance condition was found by using nonlinear equations of state. In fact, in [10] it was expressed as the equation of state for a Thait liquid, $p = \omega_0\rho^\gamma$, where $\omega < 0$ and $\gamma > 0$. In [11, 12] the Chaplygin gas equation of state, $p = -A/\rho^n$, where $A > 0$ and $n > 1$, was used for its description. Modification of the Chaplygin gas equation of state [13] and other attempts (see, for example, [14]) to describe the dark energy were also used.

In the present article, the new theoretical interpretation of dark-energy non-clustering is proposed. To understand its meaning, consider the general condition of any substance clustering.

It is well known that clustering of a baryonic substance takes place in cases when its density satisfies the energy-dominance condition

$$-\rho \le p \le \rho. \tag{1}$$

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Hence, it is possible to interpret the absence of clustering as a violation of condition (1), or as a violation of the energy-dominance principle in cosmology.

The violation of the energy-dominance principle in classical cosmology has been discussed, for example, in [15]. It was shown there that, if the general relativity is modified by using the conformal gravitational constant $G \to G\psi(\rho)$, then the energy-dominance principle for high densities may be violated by choosing a suitable function $\psi(\rho)$.

However, we shall not consider the situation outside the framework of general relativity but shall give another treatment of dark-energy non-clustering. In fact, we shall interpret the non-clustering of dark energy as the possibility of describing its state by a pressure-dominance condition, *i.e.* by imposing the following conditions on it:

$$p < -\rho \quad \text{and} \quad p > \rho. \tag{2}$$

To find the equation of state for dark energy, note that in the above-cited articles the magnitude of the coefficient has rather wide limits: $-1.0 < w^2 < -1.3$. According to other estimates, $-1.3 < w^2 < -1.6$ and also just $-2.4 < w^2 < -1.0$ [7]. From the first two estimates it is easy to see that it is possible to describe the state of dark energy with the required accuracy by a small deviation from the vacuum state. Thus, to deduce this, we shall use the approximation method.

So, we write the equation of state for an arbitrary barytropic substance in the general form

$$p = p(\rho) \tag{3}$$

and express its pressure and energy density as

$$\begin{aligned} p &= p_0 + \delta p, \quad \delta p > 0, \\ \rho &= \rho_0 + \delta\rho, \quad \delta\rho > 0, \end{aligned} \tag{4}$$

where $\delta p$ and $\delta\rho$ are small incremental terms added to an unperturbed pressure $p_0$ and unperturbed energy density $\rho_0$, which are determined by the propagation of sound in a substance. Equation (3) can be expressed as a Taylor series,

$$p = p_0 + \left(\frac{\partial p}{\partial \rho}\right)_0 \delta\rho + \frac{1}{2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_0 (\delta\rho)^2 + \cdots, \tag{5}$$

according to [16].

Further we utilize the basic values of pressure and energy density that satisfy the vacuum state, namely

$$p_0 = -\rho_0, \tag{6}$$

while we consider the dark-energy state as a sum of the vacuum state and a small incremental term added to it. As

$$\left(\frac{p}{\rho}\right) = \upsilon^2 > 0 \tag{7}$$

is the speed of sound in a substance (we use the atomic unit system, *i.e.* $\hbar = c = 1$), then to describe the dark energy we limit ourselves to only the three terms in equation (5). Bearing in mind equation (6) we express the pressure in the nonlinear form

$$p = -\rho_0 + \upsilon^2\left(1 + \kappa\frac{\delta\rho}{\upsilon}\right)\delta\rho = p_0 + \delta p, \tag{8}$$

where $\kappa = (\partial\upsilon/\partial\rho)_0$ is the dispersion index of the substance. If $\kappa > 0$, $\delta\rho > 0$, then dispersion is normal and equation (8) in total will describe the state of a substance satisfied by the energy-dominance condition (1). However, if $\kappa < 0$, $\delta\rho > 0$, then dispersion will be abnormal and equation (8) may effectively describe the state of dark energy that now satisfies the pressure-dominance condition (2). Note that the choice of the nonlinear equation of state type (8) allows us to describe the dynamics of dark energy effectively in the framework of the standard inflationary model.

In doing this, consider the self-consistent problem for mutual evolution of fields and the Universe. A suitable system of Einstein's equations and equations of two interacting scalar fields is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{4\pi G}{3}\left(\dot{\varphi}^2 + m_\varphi^2\varphi^2 + \dot{\psi}^2 + m_\psi^2\psi^2 + \frac{\lambda_\varphi}{2}\varphi^4 + \frac{\lambda_\psi}{2}\psi^4 + v\varphi^2\psi^2\right), \tag{9}$$

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + m_\varphi^2\varphi + \lambda_\varphi\varphi^3 + v\psi^2\varphi = 0, \tag{10}$$

$$\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} + m_\psi^2\psi + \lambda_\psi\psi^3 + v\varphi^2\psi = 0. \tag{11}$$

To search for the possibility of pressure-dominance realization in this model, let the masses and fields correlate with each other as $m \ll m_\psi$, $\psi \gg \psi$, while the self-action coefficients satisfy the inequality $\lambda_\varphi \ll \lambda_\psi \ll 1$. Hence, the period of oscillations for the field $\varphi$ is larger than the period of oscillations for the field $\psi(T_\varphi \gg T_\psi)$. In other words, in the time when the field $\psi$ changes, the basic field $\varphi$ almost does not change, *i.e.* we can describe this by the conditions

$$\dot{\varphi} \approx 0, \quad \varphi \approx \text{constant}. \tag{12}$$

By neglecting the self-action of the fields, we can simplify our system of equations and reduce it to another system given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{4\pi G}{3}(m_\varphi^2\varphi^2 + \dot{\psi}^2 + \hat{m}_\psi^2\psi^2), \tag{13}$$

$$\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} + \hat{m}_\psi^2\psi = 0, \tag{14}$$

on which we shall perform our analyses. Here $\hat{m}_\psi^2 = m_\psi^2 + v\varphi^2$ is the square of the effective mass for the field $\psi$, determined by the proper field mass and its interaction with the field $\varphi$.

In the following, it is necessary to determine the masses of the scalar fields and their initial amplitudes. According to [17] their typical magnitudes are

$$m_\varphi \ll \lambda_\varphi^{1/2} M_{\mathrm{P}}, \quad m_\psi \ll \lambda_\psi^{1/2} M_{\mathrm{P}}, \quad \varphi_0 \approx \lambda_\varphi^{-1/4} M_{\mathrm{P}}, \quad \psi_0 \approx \lambda_\psi^{-1/4} M_{\mathrm{P}}, \tag{15}$$

where $M_{\mathrm{P}}$ is the Planck mass. Bearing in mind these constraints, consider the case when

$$m_\varphi\varphi \gg \hat{m}_\psi\psi, \quad m_\varphi\varphi \gg \dot{\psi}. \tag{16}$$

For our model the inequalities (16) occur if

$$\lambda_\psi \ll \lambda_\varphi \ll 1. \tag{17}$$

Conditions (16) mean that the energy of the basic field $\phi$ is essentially larger than the energy of the additional field $\psi$. Under this assumption the system (13) and (14) becomes simpler and takes the form

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{4\pi G}{3} m_\varphi^2 \varphi^2, \quad \ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} + \hat{m}_\psi^2 \psi^2 = 0. \tag{18}$$

It is easy to see that equations (18) reduce to one linear differential equation of the second order given by

$$\ddot{\psi} + \mathfrak{J}\dot{\psi} + \mathfrak{R}\psi = 0,$$

with the coefficients $\mathfrak{J} = (12\pi G)^{1/2} m_\varphi \varphi$ and $\mathfrak{R} = \hat{m}_\psi^2$. We looks for its solution in the standard form $\psi = \psi_0 \exp(Mt)$; hence we obtain the following algebraic equation:

$$M^2 + \mathfrak{J}M + \mathfrak{R} = 0$$

which has two roots

$$M_{1,2} = -\frac{\mathfrak{J}}{2} \pm \left(\frac{\mathfrak{J}^2}{4} - \mathfrak{R}^2\right)^{1/2}. \tag{19}$$

From equations (16) and (17) we obtain $m_\varphi \gg \hat{m}_\psi$. Thus it is possible to express $\mathfrak{J}^2/4 - \mathfrak{R}^2$ as a Taylor series with respect to the small value $\hat{m}_\psi / m_\varphi$ and to obtain two solutions

$$M_1 = -\frac{\hat{m}_\psi^2}{2(3\pi G)^{1/2} m_\varphi \varphi}, \quad M_2 = -2(3\pi G)^{1/2} m_\varphi \varphi \tag{20}$$

Note that the second solution is approximate with zero accuracy with respect to $\hat{m}_\psi / m_\varphi$. Thus the required solutions of the field $\psi$ take the forms

$$\psi_1 = \psi_0 \exp(M_1 t) = \psi_0 \exp\left(-\frac{\hat{m}_\psi^2 t}{2(3\pi G)^{1/2} m_\varphi \varphi}\right), \tag{21}$$

$$\psi_2 = \psi_0 \exp(M_2 t) = \psi_0 \exp[-2(3\pi G)^{1/2} m_\varphi \varphi]. \tag{22}$$

From the right-hand side of equation (9) it is easy to find the incremental terms added to the energy density and pressure:

$$\delta\rho = \frac{1}{2}\dot{\psi}^2 + \frac{m_\psi^2}{2}\psi^2 + \frac{\upsilon}{2}\varphi^2\psi^2, \tag{23}$$

$$\delta p = \frac{1}{2}\dot{\psi}^2 - \frac{m_\psi^2}{2}\psi^2 - \frac{\upsilon}{2}\varphi^2\psi^2. \tag{24}$$

Putting these into the nonlinear equation of state (8) and bearing in mind the constraints on the field's character (negligible self-action), we obtain the main dispersion term

$$\kappa \approx \kappa_0 = 2\frac{(\dot{\psi}^2 - \hat{m}_\psi^2 \psi^2) - \upsilon^2(\dot{\psi}^2 + \hat{m}_\psi^2 \psi^2)}{\upsilon(\dot{\psi}^2 + \hat{m}_\psi^2 \psi^2)^2}. \tag{25}$$

(Other terms that are proportional to the coefficient of interaction are omitted here.) Putting equations (21) and (22) into equation (25), we obtain

$$\kappa_{0_{1,2}} = 2\frac{(M_{1,2}^2 - \hat{m}_\psi^2) - \upsilon^2(M_{1,2}^2 + \hat{m}_\psi^2)}{\upsilon(M_{1,2}^2 + \hat{m}_\psi^2)^2 \psi_0^2} \exp(-2M_{1,2}t). \tag{26}$$

To estimate the sign of dispersion, assign specific values to the roots in equation (26). For the first root the dispersion is

$$\kappa_{0_1} = -2\frac{(1 - \hat{m}_\psi^2/12\pi G m_\varphi^2\varphi^2) + \upsilon^2(1 + \hat{m}_\psi^2/12\pi G m_\varphi^2\varphi^2)}{\upsilon\hat{m}_\psi^2(1 + \hat{m}_\psi^2/12\pi G m_\varphi^2\varphi^2)\psi_0^2}\exp\left(\frac{\hat{m}_\psi^2}{\sqrt{3\pi G}m_\varphi\varphi}\cdot t\right). \quad (27)$$

For the second root we have the following expression for dispersion:

$$\kappa_{0_2} = -2\frac{(1 - 12\pi G\varphi^2 m_\varphi^2/\hat{m}_\psi^2) + \upsilon^2(1 + 12\pi G\varphi^2 m_\varphi^2/\hat{m}_\psi^2)}{\upsilon\hat{m}_\psi^2(1 + 12\pi G\varphi^2 m_\varphi^2/\hat{m}_\psi^2)\psi_0^2}\exp[4(3\pi G)^{1/2} m_\varphi\varphi t]. \quad (28)$$

Remembering that because of equations (16) and (17) the masses of the fields correlate as $\hat{m}_\psi \ll m_\varphi$ and the gravitational constant $G = M_{\mathrm{P}}^{-2}$, we obtain the following estimation:

$$\frac{\hat{m}_\psi^2}{G m_\varphi^2\varphi^2} \approx \frac{\lambda_\psi}{\lambda_\varphi^{1/2}} \ll 1.$$

Hence, the expression for abnormal dispersion (27) is simplified radically and takes the form

$$\kappa_{0_1} = -2\frac{1+\upsilon^2}{\upsilon\hat{m}_\psi^2\psi_0^2}\exp\left(\frac{\hat{m}_\psi^2}{\sqrt{3\pi G}m_\varphi\varphi}\cdot t\right) < 0. \quad (29)$$

From this we see that the time given by

$$T_1 \approx \frac{G^{1/2}m_\varphi\varphi}{m_\psi^2} \gg \left(\frac{\lambda_\varphi}{\lambda_\psi^4}\right)^{1/4} M_{\mathrm{P}}^{-1} \gg \left(\frac{\lambda_\varphi}{\lambda_\psi^4}\right)^{1/4} \times 10^{-43}c. \quad (30)$$

is the time scale for the existence of dark energy. Thus, at typical magnitudes of the self-interaction constants $\lambda_\psi \approx 10^{-14}$ and $\lambda_\varphi \approx 10^{-12}$, the exponent index will be of the order of $10^{-5}$ during the prehot stage $t \approx 10^{-37}c$ of evolution of the Universe. This means that the magnitude of abnormal dispersion is

$$\kappa_{0_1} \approx -2\frac{1+\upsilon^2}{\upsilon m_\psi^2\psi^2} \gg -2\frac{1+\upsilon^2}{\upsilon\lambda_\psi^{1/2}}M_{\mathrm{P}}^{-4} \quad (31)$$

or, in the usual units, $\kappa_{0_1} \gg -10^{-87}\mathrm{cm}^3\ \mathrm{g}^{-1}$. This estimation shows that, despite the minuteness of the dispersion (31) at the initial stages of the evolution of the Universe, the state of the substance differs from that of the vacuum-like substance.

Because of the same conditions (15) and (17) we also obtain the following estimation:

$$G\varphi^2\frac{m_\varphi^2}{\hat{m}_\psi^2} \approx \frac{\lambda_\varphi^{1/2}}{\lambda_\psi} \gg 1; \quad (32)$$

hence the dispersion is positive (normal dispersion):

$$\begin{aligned}\kappa_{0_2}(t) &\approx 2\frac{1-\upsilon^2}{\upsilon G m_\varphi^2\varphi_0^2\psi_0^2}\exp[(12\pi G)^{1/2}m_\varphi\varphi t] \\ &\approx 2\frac{1-\upsilon^2}{\upsilon}\left(\frac{\lambda_\psi}{\lambda_\varphi^2}\right)^{1/2} M_{\mathrm{P}}^{-4}\exp[(12\pi G)^{1/2}m_\varphi\varphi t] > 0. \quad (33)\end{aligned}$$

The state of the system with positive dispersion, as follows from equation (32), will exist on the time scale

$$T_2 \approx \frac{1}{G^{1/2} m_\varphi \varphi} \gg \lambda_\varphi^{-1/4} M_{\mathrm{P}}^{-1} \gg \lambda_\varphi^{-1/4} \times 10^{-43} c. \tag{34}$$

Hence, the exponent index becomes very large (much greater than $10^2$) even at the initial lifetime of the Universe $t > T^2$. However, because of the condition $\upsilon = 1$ [18], the dispersion (32) will always tend to zero, *i.e.* $\kappa_{0_2}(t) = 0$.

So, the given analysis provides the result that the system of two gravitating scalar fields, one of which, namely $\varphi$, is in the vacuum-like state (12) and the other, namely $\psi$, is exponentionally decreasing as equation (21), can naturally describe the state of dark energy by the pressure-dominance condition.

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