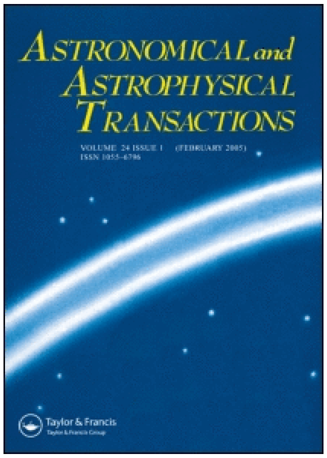

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Jacobi dynamics of variable-mass gravitational systems

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Some consequences of the generalized forms of the Lagrange–Jacobi equation for model gravitational systems with variable masses are given.

Keywords: Stellar dynamics; Variable mass; Gravitational systems

A famous Russian science historian G.K. Mikhailov [1], who was working at the Fesenkov Astrophysical Institute of Non-stationary Dynamic Problems of Astronomy, considered Fesenkov's concept of corpuscular radiation to be a factor in the evolution of the formation and development of the Sun and stars [2].

One can become acquainted with the relevant achievements through the monographs in [3, 4] and the special reviews in [5–7]. In particular, Omarov [8] obtained the following dynamic equation for a gravitational system, which changes its composition along an arbitrary exterior bound S:

$$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} + \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\oint_S r^2\left[\sigma_a(\boldsymbol{r},t) - \sigma_b(\boldsymbol{r},t)\right]\mathrm{d}s\right) + \oint_S \boldsymbol{r}\left[\boldsymbol{u}_a(\boldsymbol{r},t)\,\sigma_a(\boldsymbol{r},t) - \boldsymbol{u}_b(\boldsymbol{r},t)\,\sigma_b(\boldsymbol{r},t)\right]\mathrm{d}s = 2T + W, \tag{1}$$

where I is the barycentric moment of inertia, T is the kinetic energy, W is the potential energy of the system, σ_a and σ_b are the masses along unit area of the surface S at unit time, and the functions $\boldsymbol{u}_a$ and $\boldsymbol{u}_b$ describe the S speed distribution of the particles taken from the system and added to it.

In the case of a gravitational system, the mass of which changed in time only as a consequence of the variation in the mass m of its members, the analogue of the well-known

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Lagrange–Jacobi equation in a classic problem of n bodies [9] has the form

$$\frac{m}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{I}{m}\right) = 2T + W, \quad I = \sum_i m(t)\boldsymbol{r}_i^2, \tag{2}$$

where $\boldsymbol{r}_i$ are the barycentric radius vectors of the bodies ($i = 1, \ldots, n$). Taking into account that the change in the energy $H = T + W$ of such a system obeys the law [3]

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{1}{m}\frac{\mathrm{d}m}{\mathrm{d}t}(T + 2W), \tag{3}$$

equation (2) may be given in the following form [10]:

$$\frac{1}{2}\frac{1}{m^3}\frac{\mathrm{d}m}{\mathrm{d}t}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{I}{m}\right) = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{H}{m^3}\right). \tag{4}$$

Equations (1) and (4) are the basis for the development of Jacobi dynamics variable-mass gravitational systems.

Consider from this viewpoint the following problems of stellar dynamics.

The evaporation of gravitation is one possible mechanism for the non-stationarity of the composition of stellar systems [11]. Adiabatic evolution of such a system is usually considered in the assumption that its total energy is constant. In this approach the dependence $\rho \propto n^{-5}$ between the average density ρ of a stellar system and the variable number n of gravitating members is defined. Let us determine the correct law here with the actual dependence of the quantity H on time. In the case when the system's mass is decreasing at the expense of dissipation of its members with constant masses, i.e.

$$m_i = \text{constant}, \quad i = 1, 2, \ldots, n, \quad n = n(t), \tag{5}$$

equation (1) has the form [3]

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = R\frac{\mathrm{d}^2 M}{\mathrm{d}t^2} + R\left(2\frac{\mathrm{d}R}{\mathrm{d}t} + 2V_R\right)\frac{\mathrm{d}M}{\mathrm{d}t} + 2(2T + W), \tag{6}$$

where $M = mn(t)$ is the system's mass, R is the system's radius and V_R is the average value of the radial velocities that are out of touch with the system's members. The system energy changed according to the law [12]

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \left(\frac{V_R^2}{2} - \frac{1}{2}\frac{GM}{R}\right)\frac{\mathrm{d}M}{\mathrm{d}t}, \tag{7}$$

where G is the gravitation constant and

$$V_R^2 \geqslant 2\frac{GM}{R}. \tag{8}$$

On the basis of equation (7) let us evaluate the change in the system density $\rho = 3M/4\pi R^3$ as a function of n. Let

$$V_R^2 \approx 2\frac{GM}{R}. \tag{9}$$

Then equation (7) takes the form

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{1}{2}\frac{GM}{R}\frac{\mathrm{d}M}{\mathrm{d}t}. \tag{10}$$

In the case in equation (9) the following equality occurs:

$$\sum_{i=1}^{n(t)} H_i = \sum_{i=1}^{n(t)} (T_i + W_i) = T + 2W = H + W \approx \text{constant}. \tag{11}$$

Moreover, one can put

$$W \approx -\frac{GM^2}{R}. \tag{12}$$

From equations (10)–(12) it follows that

$$\frac{3}{2M}\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t} \tag{13}$$

or

$$R = \frac{R_0}{M_0^{3/2}} M^{3/2} = \frac{R_0}{n_0^{3/2}} n^{3/2}, \tag{14}$$

where $M_0 = M(t_0)$, $n_0 = n(t_0)$ and $R_0 = R(t_0)$.

For the average density of a system we have

$$\rho = \frac{3mn_0^{9/2}}{4\pi R_0^3} n^{-7/2}. \tag{15}$$

The dynamic effects of mass loss by the members of a gravitational system have been studied in stellar dynamics by applying it to clusters of galaxies [13, 14]. Let us complete this investigation with an analytical estimation of the time of transition of the system's n bodies of variable masses from the initial stage with a negative energy to the disintegration condition with $H > 0$. In the case of a gravitational system, in which mass is changing with time only on account of the isotropic variability of the individual masses of its members, i.e.

$$m_i = m_i(t), \quad i = 1, 2, \ldots, n, \quad n = \text{constant}, \tag{16}$$

from equation (4) we obtain

$$2\frac{H}{m^3} + \frac{1}{m^3}\frac{\mathrm{d}m}{\mathrm{d}t}\frac{\mathrm{d}J}{\mathrm{d}t} - J\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{m^3}\frac{\mathrm{d}m}{\mathrm{d}t}\right) = \text{constant} - \int_{t_0}^{t} J\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{1}{m^3}\frac{\mathrm{d}m}{\mathrm{d}t}\right)\mathrm{d}t, \tag{17}$$

where t_0 is the starting moment of time and

$$J = \frac{I}{m}. \tag{18}$$

If the mass-changing law $m(t)$ takes place according to the Mestschersky law

$$m(t) = \frac{m(t_0)}{\left(At^2 + 2Bt + C\right)^{1/2}}, \tag{19}$$

where A, B and C are constants, then equation (17) will turn into an exact integral.

When the masses of the bodies are changing in accordance with the Eddington–Jeans law

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha m^k, \tag{20}$$

where α and k are constants, from equation (17) we obtain

$$2\frac{H}{m^3} + \alpha m^{k-3}\frac{\mathrm{d}J}{\mathrm{d}t} - \alpha^2(k-3)m^{2k-4}J = \text{constant} + \Psi, \tag{21}$$

where

$$\Psi = -2\alpha^3(k-2)(k-3)\int_{t_0}^{t} Jm^{3k-5}\mathrm{d}t. \tag{22}$$

The quantity $J = J(t)$ in equation (18) can be expressed by a Taylor series and is limited by the squared approach:

$$J(t) \approx J(t_0) + (t-t_0)\left(\frac{\mathrm{d}J}{\mathrm{d}t}\right)_0 + \frac{(t-t_0)^2}{2}\left(\frac{\mathrm{d}^2J}{\mathrm{d}t^2}\right)_0. \tag{23}$$

In this case from equation (2) it follows that

$$2(2T+W) = m\left(\frac{\mathrm{d}^2J}{\mathrm{d}t^2}\right)_0. \tag{24}$$

Accordingly, equation (4) gives

$$\frac{H}{m^3} = \frac{H_0}{m_0^3} + \frac{1}{4}\left(\frac{1}{m^2} - \frac{1}{m_0^2}\right)\left(\frac{\mathrm{d}^2J}{\mathrm{d}t^2}\right)_0, \tag{25}$$

where $H_0 = H(t_0)$ and $m_0 = m(t_0)$.

Considering that from equation (24) it follows that

$$\left(\frac{\mathrm{d}^2J}{\mathrm{d}t^2}\right)_0 = \frac{2(2T_0+W_0)}{m_0},$$

where $T_0 = T(t_0)$ and $W_0 = W(t_0)$, one can rewrite equation (25) in the form

$$\frac{H}{m^3} = \frac{H_0}{m_0^3} + \left(\frac{1}{m^2} - \frac{1}{m_0^2}\right)\frac{2T_0+W_0}{2m_0}. \tag{26}$$

On the basis of equation (26) we can estimate the time of the system's disintegration.

Let $t = t^*$ be the time in which the system transits into the disintegration condition in view of the masses lost by individual members (equation 16). Then

$$H(t^*) = 0. \tag{27}$$

From equations (26) and (27),

$$\left(\frac{m(t_0)}{m(t^*)}\right)^2 = 1 - \frac{2H_0}{2T_0 + W_0}. \tag{28}$$

The mass-changing law (20) gives

$$\left(\frac{m(t_0)}{m(t^*)}\right)^2 = m_0^2[\alpha(1-k)\,\Delta t^* + m_0^{1-k}]^{2/(k-1)}, \quad k \neq 1, \tag{29}$$

where the system's disintegration time is denoted by

$$\Delta t^* = t^* - t_0. \tag{30}$$

From equations (28) and (29) we obtain the formula defining the time interval in which system transits into the disintegration condition:

$$\Delta t^* = \frac{1}{\alpha(1-k)}\left[\frac{1}{m_0^2}\left(1 - \frac{2H_0}{2T_0 + W_0}\right)^{(k-1)/2} - m_0^{1-k}\right]. \tag{31}$$

In the case when $k = 1$, which was not included in equation (31), from equations (20), (27) and (28) it follows that

$$\Delta t^* = -\frac{1}{2\alpha}\ln\left(1 - \frac{2H_0}{2T_0 + W_0}\right). \tag{32}$$

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