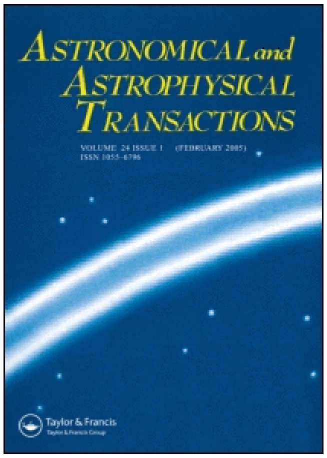

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 June 2005
To cite this Article: Barkin, Yu. V. and Shatina, A. V. (2005) 'Deformations of the Earth's mantle due to core displacements', Astronomical & Astrophysical Transactions, 24:3, 195 - 213
To link to this article: DOI: 10.1080/10556790500496339
URL: http://dx.doi.org/10.1080/10556790500496339

Astronomical and Astrophysical Transactions
Vol. 24, No. 3, June 2005, 195–213

Deformations of the Earth's mantle due to core displacements

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(Received 15 September 2005)

The planet is assumed to consist of an absolutely rigid sphere to which the viscous elastic spherical shell (the mantle) is connected from the external side. In the undeformed state the centres of mass of the mantle and the core coincide and the shells have concentric positions. The centre of mass of the core is considered to be displaced according to a definite law relative to the centre of mass of the mantle in its undeformed state among to the differential action from external celestial bodies. The solution of the problem of elasticity is obtained using a restricted treatment by taking into account only the mutual interaction of the mantle and the moving core and by neglecting the non-sphericities of the core and the mantle. The corresponding effects of the mantle deformations caused by the external bodies are known and can be studied separately on the basis of the principle of superposition. The deformations of the Earth's mantle due to secular drift of the core along the polar axis are described. The phenomenon of the contrasting tendencies in the deformations northern and southern hemispheres of the Earth (expansion and contraction, respectively) is discovered. The evaluation of the velocity of the core drift relative to the mantle's centre of mass has been obtained and was found to be equal to 8.0 cm year^{-1} .

Keywords: Planetary deformations; Core motion; Elastic mantle; Expansion and contraction of the hemispheres

1. Introduction

In the model problem the relative displacements of the mantle and core and deformations of the mantle are studied. We consider the mantle as an elastic (homogeneous and isotropic) layer, and the core as a rigid homogeneous spherical body. In the undeformed state of the mantle and the core, their centres of mass coincide and the shells have concentric positions. The mantle and the core are considered to execute a given relative motion, i.e. the centres of the mantle and the core move by a definite law. Furthermore, in this paper we shall concentrate attention on the secular drift of the core along the polar axis of the Earth. The dynamic reason for and the origin of the similar relative displacements lie in the gravitational differential action on the non-spherical core and mantle from external celestial bodies [1–6].

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Astronomical and Astrophysical Transactions
ISSN 1055-6796 print/ISSN 1476-3540 online
http://www.tandf.co.uk/journals
DOI: 10.1080/10556790500496339

Because of this action the shells undergo forced mutual mechanical interaction. The core pushes on the mantle (and the mantle on the core), attracts its material points and deforms its external surface and all mantle layers. The gravitational attraction of the particles of the mantle is changed also by the gravitational attraction of any external body, and the mantle undergoes by additional deformations. We give an analytical description of the above-mentioned deformations of the mantle due to small displacements of the core. Here, we consider a restricted treatment of the problem and believe that the displacements of the core relative to the centre of mass of the planet (by undeformed mantle) are given. Formally, this means that here we do not consider the deformations of the mantle under the attraction of the external celestial body. In accordance with the linear theory of elasticity this can be studied separately. Furthermore, using the solution of the above-mentioned problem of elasticity we neglect the non-sphericities of the core and the mantle (in its undeformed state) and do not take into account the rotation of the planet.

The solution of the problem of elasticity of the mantle's deformations due to the core displacements was first used for the study of the variations in the geopotential coefficients [7]. These results were obtained for a compressible elastic body with a concentric mass distribution (in the undeformed state of the mantle). However, in this paper we consider and study in more detail a similar solution for the homogeneous elastic mantle described in [8, 9].

We pay special attention to the study of the deformations of the mantle's external surface by the secular trend of the core along the polar axis. Using the given constant velocity of the drift we observe the inversion phenomenon of the expansion of the southern hemisphere and the contraction of the northern hemisphere of the planet. We have applied the solution obtained to the Earth's core–mantle system and obtained the following evaluations. Owing to the secular drift of the core to the North Pole with a velocity of 8.3 cm year^{-1} and owing to the gravitational action of its superfluous mass the mantle's surface is deformed so that the 45° S parallel of the southern hemisphere tests lengthens and the 45° N parallel shortens at a velocity of 1.67 cm year^{-1}. This effect has been predicted theoretically in the Barkin [2, 3, 10] as a 'flux inversion phenomenon' and has been confirmed by space geodesy methods in the last few years.

Chinese workers first detected the planetary 'flux phenomenon' from observational data [11, 12]. In these papers, the global planetary form change deduced from geophysical research was identified by space geodesy data from very-long-baseline interferometry, Global Positioning System and satellite laser ranging measurements. Using the data on the geodesic rates and vertical velocities of stations, three kinds of data and their integration have been obtained with consistent results; within the midlatitude belt 20–50° latitude in the northern hemisphere there may be a contraction of about 8–10 mm year^{-1}; within the midlatitude belt–(20–50°) latitude in the southern hemisphere there may be an expansion of about 12–14 mm year^{-1} [11]. The dependence of the length of the parallels on the latitude was also studied [12] and the velocities of expansion of the middle parallels have been evaluated as 16–20 mm year^{-1}. Using the above-mentioned Chinese results we have evaluated the velocity of the centre of the core of the Earth relative to the mantle centre as 8.0 cm year^{-1}.

2. Treatment of the problem and equations of motion

2.1 *Displacement vector*

The planet is a system of two shells: the mantle and the core. We consider the mantle as an elastic (homogeneous and isotropic) layer, and the core as a rigid spherical body with a radius r_0. In the undeformed state the mantle occupies a domain $\Omega = \{\boldsymbol{r} \in \mathrm{E}^3 \mid r_0 \leq |\boldsymbol{r}| \leq r_1\}$. In the undeformed state the centres of mass of the mantle and the core coincide and the shells have concentric positions. The mantle and the core are subjected to differential action from external celestial bodies. Because of this action the shells undergo a mutual mechanical interaction. The core pushes on the mantle and deforms its inner surface and all mantle layers. The gravitational attraction of the particles of the mantle is also changed and the mantle is exposed to additional deformations. The goal of our work is to give an analytical description of the above-mentioned deformations of the mantle due to small displacements of the core. Here, we consider a restricted treatment of problem and believe that displacements of the core relative to the centre of mass of the planet (by the undeformed mantle) are given. Formally, this means that we do not consider deformations of the mantle under the attraction of an external celestial body. Also we do not take into account the rotational motion of the planet. We shall characterize the elastic properties of the homogeneous mantle using the standard parameters of the theory of elasticity: E is the Young's modulus; ν is the Poisson's ratio; μ, k and λ are the Lamé coefficients.

To describe the planetary motion and deformations let us introduce the following reference systems. C_0xyz and C_cxyz are two Cartesian reference systems with parallel axes and with the origin at the centre C_0 of mass of the undeformed planet and as the centre C_c of mass of the core respectively. The axes C_0z and C_cz coincide and are directed along the polar axis of the planet. From the secular trend of the core along the polar axis with a constant velocity it can be seen that both reference systems are inertial. The base reference system in the considered problem for the construction of the equations of motion will be C_cxyz.

We suggest that the core's centre of mass undergoes a given small relative motion with respect to the reference system C_0xyz. We shall determine its position by the radius vector $\boldsymbol{\rho}$. The radius vector of the mantle point M is

$$\boldsymbol{R}(\boldsymbol{r}, t) = \boldsymbol{r} + \boldsymbol{u}(\boldsymbol{r}, t),$$

where $\boldsymbol{r} \in V$ (V is a domain in occupied by the planet in the undeformed state) is a displacement vector of points of the elastic mantle. The distance between the centre of the core and an arbitrary mantle point is determined from the formula

$$\boldsymbol{R}(\boldsymbol{r}, t) = -\boldsymbol{\rho}(t) + \boldsymbol{r} + \boldsymbol{u}(\boldsymbol{r}, t). \tag{1}$$

The displacement vector $\boldsymbol{\rho}(t)$ is considered to be a known function of time, but the main attention in the paper will be given to the case of secular drift of the core along the polar axis with a constant velocity $\dot{\rho} = |\dot{\boldsymbol{\rho}}|$:

$$\boldsymbol{\rho}(t) = \dot{\boldsymbol{\rho}} t. \tag{2}$$

2.2 *Potential energy*

The potential energy of gravitational interaction of the core with the elastic mantle is determined by the volume integral

$$\Pi = -f\,\Delta m_c \int_{\Omega} \frac{\delta}{[(\boldsymbol{r} + \boldsymbol{u} - \dot{\boldsymbol{\rho}})^2]^{1/2}}\,\mathrm{d}v. \tag{3}$$

Here, Δm_c is the additional mass of the core, δ is the density of the mantle, f is a gravitational constant, $\gamma = f\,\Delta m_c$ and $\mathrm{d}v$ is the elementary volume of the mantle. Taking into account the geometrical relation $\rho \ll r_0$ we develop the potential energy in a series. Employing only the main terms of this series we have

$$\begin{aligned}
\Pi &= -\gamma\delta \int_\Omega [(\boldsymbol{r}+\boldsymbol{u}-\boldsymbol{\rho})^2]^{-1/2}\,\mathrm{d}v \\
&= -\gamma\delta \int_\Omega [(\boldsymbol{r},\boldsymbol{r}) + 2(\boldsymbol{r},\boldsymbol{u}-\boldsymbol{\rho}) + (\boldsymbol{u}-\boldsymbol{\rho},\boldsymbol{u}-\boldsymbol{\rho})]^{-1/2}\,\mathrm{d}v \\
&= -\gamma\delta \int_\Omega \frac{1}{r}\left(1 + \frac{2(\boldsymbol{r},\boldsymbol{u}-\boldsymbol{\rho})}{r^2} + \frac{(\boldsymbol{u}-\boldsymbol{\rho},\boldsymbol{u}-\boldsymbol{\rho})}{r^2}\right)^{-1/2}\mathrm{d}v \\
&= -\gamma\delta \int_\Omega \frac{1}{r}\left[1 - \frac{1}{2}\left(\frac{2(\boldsymbol{r},\boldsymbol{u}-\boldsymbol{\rho})}{r^2} + \frac{(\boldsymbol{u}-\boldsymbol{\rho},\boldsymbol{u}-\boldsymbol{\rho})}{r^2}\right)\right. \\
&\quad \left. + \frac{3}{8}\left(\frac{2(\boldsymbol{r},\boldsymbol{u}-\boldsymbol{\rho})}{r^2} + \frac{(\boldsymbol{u}-\boldsymbol{\rho},\boldsymbol{u}-\boldsymbol{\rho})}{r^2}\right)^2 + \cdots\right]\mathrm{d}v \\
&= -\gamma\delta \int_\Omega \frac{1}{r}\left(1 - \frac{(\boldsymbol{r},\boldsymbol{u}-\boldsymbol{\rho})}{r^2} - \frac{1}{2}\frac{(\boldsymbol{u}-\boldsymbol{\rho},\boldsymbol{u}-\boldsymbol{\rho})}{r^2} + \frac{3}{2}\frac{(\boldsymbol{r},\boldsymbol{u}-\boldsymbol{\rho})^2}{r^4} + \cdots\right)\mathrm{d}v \\
&= -\gamma\delta \int_\Omega \left(\frac{1}{r} - \frac{(\boldsymbol{r},\boldsymbol{u}-\boldsymbol{\rho})}{r^3} - \frac{(\boldsymbol{u}-\boldsymbol{\rho},\boldsymbol{u}-\boldsymbol{\rho})}{2r^3}\right. \\
&\quad \left. + \frac{3}{2}\frac{(\boldsymbol{r},\boldsymbol{u})^2 - 2(\boldsymbol{r},\boldsymbol{u})(\boldsymbol{r},\boldsymbol{\rho}) + (\boldsymbol{r},\boldsymbol{\rho})^2}{r^5} + \cdots\right)\mathrm{d}v.
\end{aligned}$$

2.3 *Equations for the elasticity problem and boundary conditions*

In this section we obtain the equations of deformations for the problem based on the standard method developed in [8, 13] and using the known classical results of the theory of elasticity [14]. We introduce the functional of the potential energy of elastic deformations in accordance with the linear theory of elasticity. The functional of dissipative forces corresponds to the Kelvin-loight model:

$$D[\dot{\boldsymbol{u}}] = \chi E[\dot{\boldsymbol{u}}] \tag{4}$$

The variation principle of d'Alembert and Lagrange will be formulated in the following form:

$$\int_V (\ddot{\boldsymbol{u}}, \delta\boldsymbol{u})\rho\,\mathrm{d}x + (\nabla_{\boldsymbol{u}}\Pi + \nabla_{\boldsymbol{u}}E[\boldsymbol{u}] + \nabla_{\dot{\boldsymbol{u}}}D[\dot{\boldsymbol{u}}], \delta\boldsymbol{u})_\Omega = 0. \tag{5}$$

Using equation (2) and the independence of variables of variations and on the basis of principle (5) we obtain the differential equations of the problem given by

$$\nabla_{\boldsymbol{u}}E[\boldsymbol{u}] + (\nabla_{\dot{\boldsymbol{u}}}\Pi[\boldsymbol{u}], \delta\boldsymbol{u})_\Omega = 0, \tag{6}$$

$$\begin{aligned}
(\nabla_{\boldsymbol{u}}E[\boldsymbol{u}], \delta\boldsymbol{u})_\Omega = &- \int_\Omega \frac{E}{2(1+\nu)}\left(\frac{1}{1-2\nu}\nabla(\nabla\cdot\boldsymbol{u}) + \Delta\boldsymbol{u}\right)\delta\boldsymbol{u}\,\mathrm{d}v \\
&+ \int_{\partial\Omega}\sum_{i=1}^{3}\frac{E\nu}{(1+\nu)(1-2\nu)}\nabla\cdot\boldsymbol{u}\gamma_i + \frac{E\nu}{2(1+\nu)}\left(\frac{\partial\boldsymbol{u}}{\partial x_i}\boldsymbol{n} + \Delta u_i\,\boldsymbol{n}\right)\delta u_i\,\mathrm{d}v.
\end{aligned} \tag{7}$$

Here, $\boldsymbol{n} = (\gamma_1, \gamma_2, \gamma_3)$ is a vector of the external normal to the boundary $\partial\Omega$ of the domain Ω. Now equation (6) can be presented in the following form:

$$-\frac{E}{2(1+\nu)}\left(\frac{1}{1-2\nu}\nabla(\nabla\cdot\boldsymbol{u}) + \Delta\boldsymbol{u}\right) = \gamma\delta\left(-\frac{\boldsymbol{r}}{r^3} + \frac{\boldsymbol{\rho}}{r^3} - \frac{3\boldsymbol{r}(\boldsymbol{r},\boldsymbol{\rho})}{r^5}\right), \quad r \in \Omega. \tag{8}$$

The boundary conditions are given by the following basic assumptions in the considered problem.

(i) The displacements of the mantle particles on the core surface are equal to zero.
(ii) The tension on the external surface of the mantle is equal to zero.

Boundary conditions (i) and (ii) are represented by the equations:

$$\boldsymbol{u}|_{r=r_0} = \boldsymbol{0}, \tag{9}$$

$$\frac{E\nu}{(1+\nu)(1-2\nu)}\nabla\cdot\boldsymbol{u}\,\gamma_i + \frac{E\nu}{2(1+\nu)}\left(\frac{\partial\boldsymbol{u}}{\partial x_i}\boldsymbol{n} + \Delta u_i\,\boldsymbol{n}\right)\Bigg|_{r=r_1} = 0 \quad (i = 1, 2, 3), \tag{10}$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right), \qquad \Delta = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right).$$

Remark In equations (8)–(10) the basic reference system used is the Cartesian reference system connected with the core, which is characterized by translational displacements and is considered to be an inertial reference system. We neglect here possible non-inertial dynamic effects in the mantle deformations. In the case of translational relative displacements of the mantle and core with a constant relative velocity these equations are correct and exact.

Taking into account the relations between the standard elastic parameters from section 2.1, namely:

$$\mu = \frac{E}{2(1+\nu)}, \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \nu = \frac{\lambda}{2(\lambda+\mu)}, \quad E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}, \tag{11}$$

the equations of the boundary problem for determination of displacement vector can be described in the form:

$$(\lambda+\mu)\nabla(\nabla\cdot\boldsymbol{u}) + \mu\,\Delta\boldsymbol{u} = \gamma\delta\left(\frac{\boldsymbol{r}}{r^3} - \frac{\boldsymbol{\rho}}{r^3} + \frac{3\boldsymbol{r}(\boldsymbol{r},\boldsymbol{\rho})}{r^5}\right), \tag{12}$$

$$\boldsymbol{u}|_{r=r_0} = \boldsymbol{0}, \tag{13}$$

$$X_\nu|_{r=r_1} = 0, \quad Y_\nu|_{r=r_1} = 0, \quad Z_\nu|_{r=r_1} = 0, \tag{14}$$

where:

$$\begin{aligned} X_\nu| &= \frac{\mu}{r}\left(\frac{\lambda}{\mu}x\,\nabla\cdot\boldsymbol{u} + \boldsymbol{r}\,\nabla u + \frac{\partial}{\partial x}(\boldsymbol{r}\cdot\boldsymbol{u}) - u\right), \\ Y_\nu| &= \frac{\mu}{\boldsymbol{r}}\left(\frac{\lambda}{\mu}y\,\nabla\cdot\boldsymbol{u} + \boldsymbol{r}\,\nabla v + \frac{\partial}{\partial y}(\boldsymbol{r}\cdot\boldsymbol{u}) - v\right), \\ Z_\nu| &= \frac{\mu}{r}\left(\frac{\lambda}{\mu}z\,\nabla\cdot\boldsymbol{u} + \boldsymbol{r}\,\nabla w + \frac{\partial}{\partial z}(\boldsymbol{r}\cdot\boldsymbol{u}) - w\right). \end{aligned} \tag{15}$$

3. Solution of the elasticity problem

Equations (13)–(15) are the boundary conditions: on the surface of the core, displacements of particles of mantle are absent (equation (13)); the component of tension normal to the external surface of planet is equal to zero (equations (14) and (15)).

Equation (8) is linear and its solution can be presented as a superposition of two solutions:

$$\boldsymbol{u} = \boldsymbol{u}^{(-1)} + \boldsymbol{u}^{(-2)}, \tag{16}$$

where $\boldsymbol{u}^{(-1)}$ is a solution of the equation:

$$(\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}^{(-1)}) + \mu\, \Delta\boldsymbol{u}^{(-1)} = \gamma\rho\left(\frac{\boldsymbol{r}}{r^3}\right) \tag{17}$$

satisfying the boundary conditions (13)–(15), and $\boldsymbol{u}^{(-2)}$ is a solution of the equation:

$$(\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}^{(-2)}) + \mu\, \Delta\boldsymbol{u}^{(-2)} = \gamma\delta\left(-\frac{\boldsymbol{\rho}}{r^3} + \frac{3\boldsymbol{r}(\boldsymbol{r}, \boldsymbol{\rho})}{r^5}\right) \tag{18}$$

also satisfying the boundary conditions (13)–(15). The sum of solutions (16) also satisfies the boundary conditions on the external and inner surfaces of the mantle.

3.1 *Solution of equation (8) (n = −1)*

Let us consider a volume spherical function $V_n = \gamma/r$ of the order $n = -1$. Equation (8) can be presented in the form of a system of three scalar equations:

$$\begin{aligned}
(\lambda + \mu)\frac{\partial}{\partial x}(\nabla \cdot \boldsymbol{u}^{(-1)}) + \mu\, \Delta u^{(-1)} + \delta\frac{\partial V_{-1}}{\partial x} &= 0,\\
(\lambda + \mu)\frac{\partial}{\partial y}(\nabla \cdot \boldsymbol{u}^{(-1)}) + \mu\, \Delta v^{(-1)} + \delta\frac{\partial V_{-1}}{\partial y} &= 0,\\
(\lambda + \mu)\frac{\partial}{\partial z}(\nabla \cdot \boldsymbol{u}^{(-1)}) + \mu\, \Delta w^{(-1)} + \delta\frac{\partial V_{-1}}{\partial z} &= 0,\\
\boldsymbol{u}^{(-1)} = (u^{(-1)}, v^{(-1)}, w^{(-1)}).
\end{aligned} \tag{19}$$

The particular solution of equations (8) is known for arbitrary volume spherical functions. In particular, for $n = -1$ on the base of the general equations (4.3) and (4.4) in [14] we obtain:

$$\begin{aligned}
u_1^{(-1)} &= \frac{\delta}{2(\lambda + 2\mu)} r^2 \frac{\partial V_{-1}}{\partial x},\\
v_1^{(-1)} &= \frac{\delta}{2(\lambda + 2\mu)} r^2 \frac{\partial V_{-1}}{\partial y},\\
w_1^{(-1)} &= \frac{\delta}{2(\lambda + 2\mu)} r^2 \frac{\partial V_{-1}}{\partial z}.
\end{aligned} \tag{20}$$

The solution (20) must be added to the solution of the homogeneous equation of the theory of elasticity:

$$(\lambda + \mu)\, \nabla(\nabla \cdot \boldsymbol{u}_{\mathbf{0}}^{(-1)}) + \mu\, \Delta u_0^{(-1)} = 0, \tag{21}$$

which is known and has the following form:

$$u_0^{(-1)} = F_{-1}(r)\frac{\partial V_{-1}}{\partial x} + G_{-1}(r)V_{-1}x,$$
$$v_0^{(-1)} = F_{-1}(r)\frac{\partial V_{-1}}{\partial y} + G_{-1}(r)V_{-1}y \tag{22}$$
$$w_0^{(-1)} = F_{-1}(r)\frac{\partial V_{-1}}{\partial z} + G_{-1}(r)V_{-1}z$$

The functions $F_{-1}(r)$, and $G_{-1}(r)$ are defined by the general equations (4.13) in [14], and in the case $n = -1$ we obtain:

$$F_{-1}(r) = c_4^{(-1)}r + c_3^{(-1)} - \frac{3\lambda + 5\mu}{6(\lambda + 2\mu)}c_2^{(-1)}r^3 - c_1^{(-1)}r^2,$$
$$G_{-1}(r) = c_4^{(-1)}r^{-1} - \frac{\lambda + \mu}{2(\lambda + 2\mu)}c_2^{(-1)}r - c_1^{(-1)}. \tag{23}$$

where $c_i^{(-1)}$ are constants determined from the boundary conditions.

Taking into account the simple relations:

$$V_{-1}x = -r^2\frac{\partial V_{-1}}{\partial x}, \quad V_{-1}y = -r^2\frac{\partial V_{-1}}{\partial y}, \quad V_{-1}z = -r^2\frac{\partial V_{-1}}{\partial z}, \tag{24}$$

we present the solution of the system (8) in the following forms:

$$u^{(-1)} = u_0^{(-1)} + u_1^{(-1)} = \left(F_{-1}(r) - r^2G_{-1}(r) + \frac{\delta}{2(\lambda + 2\mu)}r^2\right)\frac{\partial V_{-1}}{\partial x}c_i^{(-1)},$$
$$v^{(-1)} = v_0^{(-1)} + v_1^{(-1)} = \left(F_{-1}(r) - r^2G_{-1}(r) + \frac{\delta}{2(\lambda + 2\mu)}r^2\right)\frac{\partial V_{-1}}{\partial y}, \tag{25}$$
$$w^{(-1)} = w_0^{(-1)} + w_1^{(-1)} = \left(F_{-1}(r) - r^2G_{-1}(r) + \frac{\delta}{2(\lambda + 2\mu)}r^2\right)\frac{\partial V_{-1}}{\partial z}.$$

In accordance with general theory, the components of tension on the spherical surface of radius r corresponding to solution (14) of the homogeneous equation (13) are defined by equation (4.18) in [14] in which we have:

$$\omega = V_{-1},$$
$$P_{-1}(r) = -2c_4^{(-1)} - \frac{4c_3^{(-1)}}{r} + \frac{\mu}{3(\lambda + 2\mu)}c_2^{(-1)}r^2 + c_1^{(-1)}r, \tag{26}$$
$$Q_{-1}(r) = -2\mu c_4^{(-1)}r^{-1} + \frac{\mu(\lambda + \mu)}{\lambda + 2\mu}c_2^{(-1)}r + \mu c_1^{(-1)}.$$

The components of the tension on the sphere of radius corresponding to solution (12) are defined by equations (4.23) of [14] for which:

$$n = -1, \quad C = \frac{\delta}{2(\lambda + 2\mu)}. \tag{27}$$

The components of the tension on the sphere of radius corresponding to solution (29) are defined by equations (4.24) of [14]. Taking into account the properties of the function V_n we obtain the following expressions:

$$X_\nu = \left(\mu P_{-1}(r) - rQ_{-1}(r) + \frac{\delta\lambda}{\lambda+2\mu}r\right)\frac{\partial V_{-1}}{\partial x},$$
$$Y_\nu = \left(\mu P_{-1}(r) - rQ_{-1}(r) + \frac{\delta\lambda}{\lambda+2\mu}r\right)\frac{\partial V_{-1}}{\partial y}, \tag{28}$$
$$Z_\nu = \left(\mu P_{-1}(r) - rQ_{-1}(r) + \frac{\delta\lambda}{\lambda+2\mu}r\right)\frac{\partial V_{-1}}{\partial z}.$$

The boundary conditions (5) will be equivalent to the following two equations:

$$F_{-1}(r) - r^2G_{-1}(r) + \frac{\delta}{2(\lambda+2\mu)}r^2\bigg|_{r=r_0} = 0,$$
$$\mu P_{-1}(r) - rQ_{-1}(r) + \frac{\delta\lambda}{\lambda+2\mu}r\bigg|_{r=r_1} = 0. \tag{29}$$

From equations (23) we obtain the relation:

$$F_{-1}(r) - r^2G_{-1}(r) = c_3^{(-1)} - \frac{\mu}{3(\lambda+2\mu)}c_2^{(-1)}r^3. \tag{30}$$

This means that solution (25) does not contain the constants $c_1^{(-1)}$ and $c_4^{(-1)}$.

From equations (26) we obtain a system of equations for determination of the constants $c_2^{(-1)}$ and $c_3^{(-1)}$:

$$c_3^{(-1)} - \frac{\mu}{3(\lambda+2\mu)}c_2^{(-1)}r_0^3 + \frac{\delta}{2(\lambda+2\mu)}r_0^3 = 0,$$
$$-\frac{4\mu}{r_1}c_3^{(-1)} - \frac{\mu(3\lambda+2\mu)}{3(\lambda+2\mu)}c_2^{(-1)}r_1^2 + \frac{\delta\lambda}{\lambda+2\mu}r_1 = 0. \tag{31}$$

Solving the linear equations (31) we find that:

$$c_2^{(-1)} = -\frac{3\delta(\lambda+2\mu)}{\mu\Delta_d}\left(2\mu r_0^2 + \lambda r_1^2\right), \tag{32}$$
$$c_3^{(-1)} = -\frac{\delta}{2\Delta_d}r_0^2r_1^2\left[2\lambda r_0 - (3\lambda+2\mu)r_1\right],$$

where

$$\Delta_d = -(\lambda+2\mu)[4\mu r_0^3 + (3\lambda+2\mu)r_1^3].$$

From equations (27), (29) and (36) finally we obtain:

$$\boldsymbol{u} = -\gamma\delta\left(d_1 + d_2r^2 + d_3r^3\right)\frac{\boldsymbol{r}}{r^3}, \tag{33}$$

where the constant coefficients d_i are determined by the formulae:

$$d_1 = \frac{1}{2\Delta_d}r_0^2r_1^2[2\lambda r_0 - (3\lambda+2\mu)r_1],$$
$$d_2 = \frac{1}{2(\lambda+2\mu)}, \tag{34}$$
$$d_3 = -\frac{1}{\Delta_d}(2\mu r_0^2 + \lambda r_1^2).$$

In special dimensionless notation the solutions (33) and (34) can be presented as:

$$\boldsymbol{u} = K_{\Delta_c}\left(\left(A_0 + A_2\zeta^2 + A_3\zeta^3\right)\frac{\boldsymbol{r}}{r}\zeta^{-2}r_1\right), \tag{35}$$

where:

$$\begin{aligned} A_0 &= -\frac{\lambda d_1}{r_1^2} = -\frac{1}{2\Delta_d}\lambda r_0^2[2\lambda r_0 - (3\lambda + 2\mu)r_1], \\ A_2 &= -\lambda d_2 = -\frac{\lambda}{2(\lambda + 2\mu)}, \\ A_3 &= -\lambda r_1 d_3 = \frac{1}{\Delta_d}\lambda r_1(2\mu r_0^2 + \lambda r_1^2), \\ \Delta_d &= -(\lambda + 2\mu)[4\mu r_0^3 + (3\lambda + 2\mu)r_1^3]. \end{aligned} \tag{36}$$

Here, $\zeta = r/r_1$ is the dimensionless radius of the mantle particle, $K_{\Delta_c} = f\Delta m_c/\lambda r_1$ is the constant dimensionless coefficient and Δm_c is the superfluous mass of the moving core.

3.2 *Solution of equation (8) (n = −2)*

In this case, equations (8) can be represented as:

$$\begin{aligned} (\lambda + \mu)\frac{\partial}{\partial x}(\nabla \cdot \boldsymbol{u}^{(-2)}) + \mu\,\Delta u^{(-2)} + \delta\frac{\partial V_{-2}}{\partial x} &= 0, \\ (\lambda + \mu)\frac{\partial}{\partial y}(\nabla \cdot \boldsymbol{u}^{(-2)}) + \mu\,\Delta v^{(-2)} + \delta\frac{\partial V_{-2}}{\partial y} &= 0, \\ (\lambda + \mu)\frac{\partial}{\partial z}(\nabla \cdot \boldsymbol{u}^{(-2)}) + \mu\,\Delta w^{(-2)} + \delta\frac{\partial V_{-2}}{\partial z} &= 0, \\ \boldsymbol{u}^{(-2)} = (u^{(-2)}, v^{(-2)}, w^{(-2)}). \end{aligned} \tag{37}$$

Here $V_{-2} = \gamma(\boldsymbol{r}, \boldsymbol{\rho})/r^3$ is a harmonic and homogeneous volume function of the order $n = -2$.

Particular solution of equations (37) in accordance with equations (4.3) and (4.4) of [14] for $n = -2$ is defined by formulae similar to equations (24):

$$\begin{aligned} u_1^{(-2)} &= \frac{\delta}{2(2\lambda + 5\mu)}r^2\frac{\partial V_{-2}}{\partial x}, \\ v_1^{(-2)} &= \frac{\delta}{2(2\lambda + 5\mu)}r^2\frac{\partial V_{-2}}{\partial y}, \\ w_1^{(-2)} &= \frac{\delta}{2(2\lambda + 5\mu)}r^2\frac{\partial V_{-2}}{\partial z}. \end{aligned} \tag{38}$$

The solution of the corresponding homogeneous equation of elasticity theory, namely:

$$(\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}_{\boldsymbol{0}}^{(-2)}) + \mu\,\Delta \mathrm{u}_0^{(-2)} = 0, \tag{39}$$

is described by equations (4.5) of [14] in which $\omega = V_{-2}$ and the functions are:

$$\begin{aligned} F_{-2}(r) &= c_4^{(-2)}r^3 + c_3^{(-2)} - \frac{2\lambda + 3\mu}{5(\lambda + 2\mu)}c_2^{(-2)}r^5 + \frac{\lambda + 3\mu}{2(\lambda + 2\mu)}c_1^{(-2)}r^2, \\ G_{-2}(r) &= 3c_4^{(-2)}r - \frac{\lambda + \mu}{\lambda + 2\mu}c_2^{(2)}r^3 + \frac{2\lambda + 5\mu}{\lambda + 2\mu}c_1^{(-2)}. \end{aligned} \tag{40}$$

From the solution of equations (39) we obtain the following:

$$\begin{aligned} u^{(-2)} &= u_0^{(-2)} + u_1^{(-2)} = \left(F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2\right)\frac{\partial V_{-2}}{\partial x} + G_{-2}(r)V_{-2}x, \\ v^{(-2)} &= v_0^{(-2)} + v_1^{(-2)} = \left(F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2\right)\frac{\partial V_{-2}}{\partial y} + G_{-2}(r)V_{-2}y, \\ w^{(-2)} &= w_0^{(-2)} + w_1^{(-2)} = \left(F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2\right)\frac{\partial V_{-2}}{\partial z} + G_{-2}(r)V_{-2}z. \end{aligned} \tag{41}$$

In vector form the solution (41) can be described as follows:

$$\boldsymbol{u}^{(-2)} = \gamma\left[\left(F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2\right)\left(\frac{\boldsymbol{\rho}}{r^3} - \frac{3\boldsymbol{r}(\boldsymbol{r}\cdot\boldsymbol{\rho})}{r^5}\right) + G_{-2}(r)\frac{\boldsymbol{r}(\boldsymbol{r}\cdot\boldsymbol{\rho})}{r^3}\right] \tag{42}$$

or

$$\begin{aligned} \boldsymbol{u}^{(-2)} = \gamma \Bigg\{ &\left(F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2\right)\frac{\boldsymbol{\rho}}{r^3} \\ &+ \left[G_{-2}(r)r^2 - 3\left(F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2\right)\right]\frac{\boldsymbol{r}(\boldsymbol{r}\cdot\boldsymbol{\rho})}{r^5}\Bigg\}. \end{aligned} \tag{43}$$

In accordance with general equations (4.18) and (4.23) of [14] the components of tension on the surface of the mantle are represented by the expressions:

$$\begin{aligned} X_\nu &= \mu\left(P_{-2}(r) - \frac{2\delta}{2\lambda+5\mu}r\right)\frac{\partial V_{-2}}{\partial x} + \left(Q_{-2}(r) - \frac{2\delta(\lambda+\mu)}{2\lambda+5\mu}\right)V_{-2}\frac{x}{r}, \\ Y_\nu &= \mu\left(P_{-2}(r) - \frac{2\delta}{2\lambda+5\mu}r\right)\frac{\partial V_{-2}}{\partial y} + \left(Q_{-2}(r) - \frac{2\delta(\lambda+\mu)}{2\lambda+5\mu}\right)V_{-2}\frac{y}{r}, \\ Z_\nu &= \mu\left(P_{-2}(r) - \frac{2\delta}{2\lambda+5\mu}r\right)\frac{\partial V_{-2}}{\partial z} + \left(Q_{-2}(r) - \frac{2\delta(\lambda+\mu)}{2\lambda+5\mu}\right)V_{-2}\frac{z}{r}, \end{aligned} \tag{44}$$

where

$$\begin{aligned} P_{-2}(r) &= -\frac{6}{r}c_3^{(-2)} - \frac{3\lambda+2\mu}{5(\lambda+2\mu)}c_2^{(-2)}r^4 - \frac{\mu}{\lambda+2\mu}c_1^{(-2)}r, \\ Q_{-2}(r) &= -3c_1^{(-2)}\mu, \\ c^{(-2)} &= \frac{\delta}{2(2\lambda+5\mu)}. \end{aligned} \tag{45}$$

The boundary condition (10) will be equivalent to the following system of two equations:

$$\begin{aligned} \gamma\left(F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2\right)\frac{1}{r^3} &= 0, \\ G_{-2}(r) - \frac{3}{r^2}\left(F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2\right) &= 0, \end{aligned} \tag{46}$$

or

$$F_{-2}(r) + \frac{\delta}{2(2\lambda+5\mu)}r^2 = 0, \quad G_{-2}(r) = 0. \tag{47}$$

In particular, for $r = r_0$ we obtain:

$$F_{-2}(r_0) + \frac{\delta}{2(2\lambda + 5\mu)} r_0^2 = 0,$$
$$G_{-2}(r_0) = 0,$$

and for $r = r_1$, from equations (45), we find that:

$$\begin{aligned} P_{-2}(r_1) - 4c^{(-2)} r_1 &= 0, \\ Q_{-2}(r_1) - 4c^{(-2)}(\lambda + \mu) &= 0. \end{aligned} \tag{48}$$

Substituting equations (47) and (48) in equations (45), we obtain the algebraic system of equations for determination of the constants:

$$\begin{aligned} c_4^{(-2)} r_0^3 + c_3^{(-2)} - \frac{2\lambda + 3\mu}{5(\lambda + 2\mu)} c_2^{(-2)} r_0^5 + \frac{\lambda + 3\mu}{2(\lambda + 2\mu)} c_1^{(-2)} r_0^2 + \frac{\delta}{2(2\lambda + 5\mu)} r_0^2 &= 0, \\ 3c_4^{(-2)} r_0 - \frac{\lambda + \mu}{\lambda + 2\mu} c_2^{(-2)} r_0^3 + \frac{2\lambda + 5\mu}{\lambda + 2\mu} c_1^{(-2)} &= 0, \\ -\frac{6}{r_1} c_3^{(-2)} - \frac{3\lambda + 2\mu}{5(\lambda + 2\mu)} c_2^{(-2)} r_1^4 - \frac{\mu}{\lambda + 2\mu} c_1^{(-2)} r_1 - \frac{2\delta}{2\lambda + 5\mu} r_1 &= 0, \\ -3c_1^{(-2)} \mu - \frac{2\delta(\lambda + \mu)}{2\lambda + 5\mu} &= 0. \end{aligned} \tag{49}$$

Solving the system (49) (see appendix A) we obtain the following expressions:

$$\begin{aligned} c_1^{(-2)} &= -\frac{2\delta(\lambda + \mu)}{3\mu(2\lambda + 5\mu)}, \\ c_2^{(-2)} &= \frac{5\delta}{3\Delta_d} (\lambda + 2\mu)[(\lambda + 4\mu) r_0^2 - 2\mu r_1^2], \\ c_3^{(-2)} &= -\frac{\delta}{18\Delta_d} (\lambda + 4\mu) r_0^2 r_1^2 [4\mu r_0^3 + (3\lambda + 2\mu) r_1^3], \\ c_4^{(-2)} &= \frac{\delta}{9 r_0 \Delta_d} (\lambda + \mu)[9(\lambda + 4\mu) r_0^5 - 10\mu r_0^3 r_1^2 + 2(3\lambda + 2\mu) r_1^5], \\ \Delta_d &= \mu(\lambda + 2\mu)\left[2(\lambda + 4\mu) r_0^5 + (3\lambda + 2\mu) r_1^5\right]. \end{aligned} \tag{50}$$

Now from equations (40) we obtain the following polynomial equations:

$$F_{-2}(r) + \frac{\delta}{2(2\lambda + 5\mu)} r^2 = a_1 + a_2 r^2 + a_3 r^3 + a_4 r^5,$$
$$G_{-2}(r) r^2 - 3\left(F_{-2}(r) + \frac{\delta}{2(2\lambda + 5\mu)} r^2\right) = a_5 + a_6 r^2 + a_7 r^5,$$

where

$$\begin{aligned}
a_1 &= c_3^{(-2)},\\
a_2 &= \frac{\lambda+3\mu}{2(\lambda+2\mu)}c_1^{(-2)} + \frac{\delta}{2(2\lambda+5\mu)},\\
a_3 &= c_4^{(-2)},\\
a_4 &= -\frac{2\lambda+3\mu}{5(\lambda+2\mu)}c_2^{(-2)},\\
a_5 &= -3c_3^{(-2)},\\
a_6 &= \frac{2\lambda+5\mu}{\lambda+2\mu}c_1^{(-2)} - 3\frac{\lambda+3\mu}{2(\lambda+2\mu)}c_1^{(-2)} - \frac{3\delta}{2(2\lambda+5\mu)}\\
&= \frac{\lambda+\mu}{2(\lambda+2\mu)}c_1^{(-2)} - \frac{3\delta}{2(2\lambda+5\mu)},\\
a_7 &= -\frac{\lambda+\mu}{\lambda+2\mu}c_2^{(-2)} + 3\frac{2\lambda+3\mu}{5(\lambda+2\mu)}c_2^{(-2)} = \frac{\lambda+4\mu}{5(\lambda+2\mu)}c_2^{(-2)}.
\end{aligned} \tag{51}$$

Finally, the solution (42) and (43) of the problem can be represented by the formula:

$$\boldsymbol{u}^{(2)} = \gamma\delta\left((a_1 + a_2r^2 + a_3r^3 + a_4r^5)\frac{1}{r^3}\boldsymbol{\rho} + (a_5 + a_6r^2 + a_7r^5)\frac{1}{r^5}(\boldsymbol{r}\cdot\boldsymbol{\rho})\boldsymbol{r}\right), \tag{52}$$

where the coefficients a_i (equations (51)) are transformed to the following forms:

$$\begin{aligned}
a_1 &= -\frac{1}{18\Delta_a}(\lambda+4\mu)r_0^2r_1^2[4\mu r_0^3 + (3\lambda+2\mu)r_1^3],\\
a_2 &= -\frac{\lambda}{6\mu(\lambda+2\mu)},\\
a_3 &= \frac{1}{9r_0\Delta_a}(\lambda+\mu)[9(\lambda+4\mu)r_0^5 - 10\mu r_0^3r_1^2 + 2(3\lambda+2\mu)r_1^5],\\
a_4 &= -\frac{1}{3\Delta_a}(2\lambda+3\mu)[(\lambda+4\mu)r_0^2 - 2\mu r_1^2],\\
a_5 &= -3a_1,\\
a_6 &= -\frac{\lambda+4\mu}{6\mu(\lambda+2\mu)},\\
a_7 &= \frac{1}{3\Delta_a}(\lambda+4\mu)[(\lambda+4\mu)r_0^2 - 2\mu r_1^2],\\
\Delta_a &= \mu(\lambda+2\mu)\left[2(\lambda+4\mu)r_0^5 + (3\lambda+2\mu)r_1^5\right].
\end{aligned} \tag{53}$$

3.3 *Full solution of the problem*

Here, we present the final formulae describing the full solution of the equation of the elasticity problem (equation (8)) with the boundary conditions (9) and (10). In the dimensionless notation

$$\zeta = \frac{r}{r_1}, \quad K_{\Delta_C} = \frac{f\,\Delta m_c}{\lambda r_1}, \tag{54}$$

the full solution of the considered problem (16) on the basis of equations (35), (36), (51) and (53) can be described in the following final form:

$$\begin{aligned}\boldsymbol{u} = K_{\Delta c}\bigg[&\left(-\frac{\lambda d_1}{r_1^2} - \lambda d_2\zeta^2 - \lambda r_1 d_3\zeta^3\right)\frac{\boldsymbol{r}}{r}\zeta^{-2}r_1\\ &+\left(\frac{\lambda a_1}{r_1^2} + \lambda a_2\zeta^2 + \lambda r_1 a_3\zeta^3 + \lambda r_1^3 a_4\zeta^5\right)\zeta^{-3}\boldsymbol{\rho}\\ &+\left(\frac{\lambda a_5}{r_1^2} + \lambda a_6\zeta^2 + \lambda r_1^3 a_7\zeta^5\right)\zeta^{-3}\frac{(\boldsymbol{r}\cdot\boldsymbol{\rho})}{r^2}\boldsymbol{r}\bigg]\end{aligned}$$

or

$$\begin{aligned}\boldsymbol{u} = K_{\Delta c}\bigg(&\left(A_0 + A_2\zeta^2 + A_3\zeta^3\right)\frac{\boldsymbol{r}}{r}\zeta^{-2}r_1 + \left(B_0 + B_2\zeta^2 + B_3\zeta^3 + B_5\zeta^5\right)\zeta^{-3}\boldsymbol{\rho}\\ &+\left(C_0 + C_2\zeta^2 + C_5\zeta^5\right)\zeta^{-3}\frac{(\boldsymbol{r}\cdot\boldsymbol{\rho})}{r^2}\boldsymbol{r}\bigg),\end{aligned} \tag{55}$$

where

$$\begin{aligned}
A_0 &= -\frac{\lambda d_1}{r_1^2} = -\frac{1}{2\Delta_d}\lambda r_0^2[2\lambda r_0 - (3\lambda + 2\mu)\, r_1],\\
A_2 &= -\lambda d_2 = -\frac{\lambda}{2(\lambda + 2\mu)},\\
A_3 &= -\lambda r_1 d_3 = \frac{1}{\Delta_d}\lambda r_1(2\mu r_0^2 + \lambda r_1^2),\\
B_0 &= \frac{\lambda a_1}{r_1^2} = -\frac{1}{18\Delta_a}\lambda(\lambda + 4\mu)r_0^2[4\mu r_0^3 + (3\lambda + 2\mu)r_1^3],\\
B_2 &= \lambda a_2 = -\frac{\lambda^2}{6\mu(\lambda + 2\mu)},\\
B_3 &= \lambda r_1 a_3 = \frac{r_1}{9r_0\Delta_a}\lambda(\lambda + \mu)[9(\lambda + 4\mu)r_0^5 - 10\mu r_0^3 r_1^2 + 2(3\lambda + 2\mu)r_1^5],\\
B_5 &= \lambda r_1^3 a_4 = -\frac{1}{3\Delta_a}\lambda r_1^3(2\lambda + 3\mu)[(\lambda + 4\mu)r_0^2 - 2\mu r_1^2],\\
C_0 &= \frac{\lambda a_5}{r_1^2} = \frac{1}{6\Delta_a}\lambda(\lambda + 4\mu)r_0^2\left[4\mu r_0^3 + (3\lambda + 2\mu)r_1^3\right],\\
C_2 &= \lambda a_6 = -\frac{\lambda(\lambda + 4\mu)}{6\mu(\lambda + 2\mu)},\\
C_5 &= \lambda r_1^3 a_7 = \frac{1}{3\Delta_a}\lambda(\lambda + 4\mu)r_1^3\left[(\lambda + 4\mu)r_0^2 - 2\mu r_1^2\right],\\
\Delta_d &= -(\lambda + 2\mu)[4\mu r_0^3 + (3\lambda + 2\mu)r_1^3],\\
\Delta_a &= \mu(\lambda + 2\mu)\left[2(\lambda + 4\mu)r_0^5 + (3\lambda + 2\mu)r_1^5\right].
\end{aligned} \tag{56}$$

The first term in the solution (55) and (56) describes the central radial deformations caused by the superfluous mass of the core in its concentric position. As these static deformations take place for a non-moveable core we exclude them from consideration.

4. Deformations of the Earth's surface due to displacements of the core

4.1 *Vector of displacement of the Earth's mantle particles*

The solution (54) and (55) of the problem of the theory of elasticity obtained in sections 1 and 2 is applicable to analysis of the deformations of different celestial bodies, considered as a core-mantle system. Here we shall give a preliminary study of the possible deformations of the Earth due to small relative displacements of the core and mantle.

Let us consider a reduced two-layer model of the Earth. We shall model the core of the Earth as a rigid homogeneous sphere. The mantle is also homogeneous and spherical in an undeformed state. The basic values of parameters of the considered model of the Earth are:

$$\begin{aligned} &\mu = 1.80,\ \lambda = 2.57 \times 10^{11}\,\mathrm{Nm}^{-2}, \\ &\delta = 4.44\,\mathrm{g/cm}^{-3},\ \mathrm{m}_{\oplus}/\Delta m_{\mathrm{c}} = 5.1760,\ K_{\Delta_{\mathrm{c}}} = 0.208\,33, \\ &r_0 = 3480\,\mathrm{km},\ r_1 = 6371\,\mathrm{km},\ \varsigma_0 = r_1/r_0 = 0.5462. \end{aligned} \tag{57}$$

In equations (57), δ is the value of the mean density of the Earth's mantle, and r_0 and r_1 are the mean radii of the core and the mantle respectively of the Earth. We shall describe the elastic properties of the mantle by the mean values of the Lame coefficients μ and λ. Δm_{c} is the superfluous mass of the moving core. $\mathrm{m}_{\oplus}$ is the Earth's mass. $K_{\Delta\mathrm{c}}$ is the dimensionless fundamental parameter of the problem. The superfluous mass of the core is $\Delta m_{\mathrm{c}} = 0.1932 m_{\oplus}$ and all the other parameters of the model (57) of the Earth were determined on the basis of the standard Earth model PREM [15]. For given values of the parameters (57) of the problem from the basic equations (56) we obtain the following numerical values of the coefficients A_i, B_i and C_i $(i = 0, 1, 2, 3)$:

$$\begin{aligned} &A_0 = -0.042\,322\,4, && A_2 = -0.208\,266, && A_3 = -0.121\,592, && \\ &B_0 = -0.038\,157, && B_2 = -0.099\,119, && B_3 = 0.402\,043, && B_5 = 0.045\,425, \\ &C_0 = 0.114\,472, && C_2 = -0.376\,807, && C_3 = 0, && C_5 = -0.042\,106. \end{aligned} \tag{58}$$

4.2 *Lengthening of the parallels in the southern hemisphere and shortening of the parallels in the northern hemisphere of the Earth due to displacement of the Earth's core to the North pole*

On the basis of the values of the parameters (57) and (58) and the equations of the solution (55) and (56) for $\zeta = 1$ we obtain the following expression for the displacement vector of the particles of the mantle's surface:

$$\boldsymbol{u} = 0.064\,777\,273\boldsymbol{\rho} - 0.063\,576\,461\frac{(\boldsymbol{r}\cdot\boldsymbol{\rho})}{r^2}\boldsymbol{r}. \tag{59}$$

The parallel component (projection of the displacement vector on the plane of the parallel) is determined by the following expression:

$$u_{\mathrm{p}} = -\rho 0.063\,576\,461 \sin\varphi\cos\varphi,$$

which means that the length of the circle of parallel with latitude φ changes its dependence of the core displacement ρ according to the law:

$$\delta L_{\mathrm{p}} = 2\pi u_{\mathrm{p}} = -\rho 0.3994 \sin\varphi\cos\varphi. \tag{60}$$

In the case of secular drift of the core along the polar axis of the Earth with a constant velocity $\dot{\rho}$, the length of the corresponding parallel will change with the velocity:

$$\delta\dot{L}_{\mathrm{p}} = 2\pi\dot{u}_p = -\dot{\rho}0.399\,463\sin\varphi\cos\varphi.$$

In particular, the variation in the circle of the parallel $\varphi = -\pi/4$ is determined to be

$$\delta\dot{L}_{\mathrm{p}}(-\pi/4) = -\dot{\rho}0.199\,731. \qquad (61)$$

Taking the observed value of parallel $-\pi/4$ lengthening from the Chinese results [11, 12] as $\delta\dot{L}_{\mathrm{p}}(-\pi/4) = 1.6$ from the basic equation (61) we obtain an evaluation of the velocity of the core drift [16, 17]:

$$\dot{\rho} = 8.0\,\mathrm{cm\,year}^{-1} \qquad (62)$$

The velocity of the core drift (62) is in agreement with the predicted value of the velocity of the centre of the Earth drift obtained in [10]. In reality the displacement of the superfluous mass of the core relative to the mantle (2) generates the secular drift of the Earth's centre of mass with velocity υ_{c} which can be evaluated from the simple formula

$$\upsilon_{\mathrm{c}} = \dot{\rho}\frac{\Delta m_{\mathrm{c}}}{m_{\oplus}} = 1.55\,\mathrm{cm\,year}^{-1}. \qquad (63)$$

Corresponding evaluations of the centre of mass drift obtained in [10] give $\upsilon_{\mathrm{c}} = 1.7$–$2.5\,\mathrm{cm\,year}^{-1}$. The above-mentioned values of the drift in the Earth's centre of mass are also confirmed by the space geodesy data of the last few decades [18–20] which testify to the existence of a similar drift with a velocity of about 1–2 $\mathrm{cm\,year}^{-1}$ with the main tendency of motion in the northern direction. Of course the core drift and centre of mass drift occurred in the same direction towards the North Pole.

5. Conclusions

In this paper the phenomenon of the deformations of the Earth's mantle and its surface caused by gravitational action of the moveable core has been described in an analytical form as a result of the solution of the corresponding problem of the theory of elasticity. The inversion phenomenon of the contraction of the northern hemisphere and expansion of the southern hemisphere of the Earth predicted in a series of papers has Barkin [1–4] has been obtained, confirming the the space geodesy data. So, the circles of the middle parallels in the southern hemisphere lengthen and in the northern hemisphere symmetrically shorten. Practical confirmation of the secular drift of the Earth's centre of mass in a northern direction predicted earlier [10] has been obtained.

Acknowledgements

The authors are grateful to Professor V.G. Vilke for his help with and useful discussions on this paper. The work has been accepted by supported by the Russian Foundation for Basic Reseach under grant N 02-05-64176.

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Appendix 1. Solution of algebraic system (49) from section 3.2

Substituting equations (47) and (48) in equation (45), we obtain the following algebraic system of equations for determination of the constants c_i:

$$c_4 r_0^3 + c_3 - \frac{2\lambda + 3\mu}{5(\lambda + 2\mu)} c_2 r_0^5 + \frac{\lambda + 3\mu}{2(\lambda + 2\mu)} c_1 r_0^2 + \frac{\delta}{2(2\lambda + 5\mu)} r_0^2 = 0, \tag{A1}$$

$$3c_4 r_0 - \frac{\lambda + \mu}{\lambda + 2\mu} c_2 r_0^3 + \frac{2\lambda + 5\mu}{\lambda + 2\mu} c_1 = 0, \tag{A2}$$

$$-\frac{6}{r_1} c_3 - \frac{3\lambda + 2\mu}{5(\lambda + 2\mu)} c_2 r_1^4 - \frac{\mu}{\lambda + 2\mu} c_1 r_1 - \frac{2\delta}{2\lambda + 5\mu} r_1 = 0, \tag{A3}$$

$$-3c_1\mu - \frac{2\delta(\lambda + \mu)}{2\lambda + 5\mu} = 0. \tag{A4}$$

From equation (A4) we obtain:

$$c_1 = -\frac{2\delta(\lambda + \mu)}{3\mu(2\lambda + 5\mu)}. \tag{A5}$$

Using solution (A5), we transform equation (A3) to the following reduced form:

$$\frac{6}{r_1}c_3 + \frac{3\lambda + 2\mu}{5(\lambda + 2\mu)}c_2 r_1^4 + \frac{\mu}{\lambda + 2\mu}c_1 r_1 + \frac{2\delta}{2\lambda + 5\mu}r_1 = 0,$$

and therefore

$$\begin{aligned}\frac{6}{r_1}c_3 + \frac{3\lambda + 2\mu}{5(\lambda + 2\mu)}c_2 r_1^4 &= -\frac{\mu}{\lambda + 2\mu}\left(-\frac{2}{3}\right)\frac{\delta(\lambda + \mu)}{\mu(2\lambda + 5\mu)}r_1 - \frac{2\delta}{2\lambda + 5\mu}r_1\\ &= \frac{\delta}{(\lambda + 2\mu)(2\lambda + 5\mu)}\left(\frac{2}{3}(\lambda + \mu) - 2(\lambda + 2\mu)\right)r_1\\ &= -\frac{2\delta}{3(\lambda + 2\mu)}r_1.\end{aligned}$$

Now we have reduced equation (A3) to:

$$\frac{6}{r_1}c_3 + \frac{3\lambda + 2\mu}{5(\lambda + 2\mu)}c_2 r_1^4 = -\frac{2\delta}{3(\lambda + 2\mu)}r_1, \tag{A6}$$

from which we obtain:

$$c_3 = -\frac{3\lambda + 2\mu}{30(\lambda + 2\mu)}c_2 r_1^5 - \frac{\delta}{9(\lambda + 2\mu)}r_1^2. \tag{A7}$$

Now we transform equations (A1) and (A2), substituting for the constants c_1 and c_3. Equation (A1) will be:

$$\begin{aligned}c_4 r_0^3 - \frac{3\lambda + 2\mu}{30(\lambda + 2\mu)}c_2 r_1^5 - \frac{\delta}{9(\lambda + 2\mu)}r_1^2 - \frac{2\lambda + 3\mu}{5(\lambda + 2\mu)}c_2 r_0^5\\ - \frac{\lambda + 3\mu}{2(\lambda + 2\mu)}\frac{2\delta(\lambda + \mu)}{3\mu(2\lambda + 5\mu)}r_0^2 + \frac{\delta}{2(2\lambda + 5\mu)}r_0^2 = 0.\end{aligned}$$

So

$$c_4 r_0^3 - Ac_2 = F, \tag{A8}$$

where

$$A = \frac{1}{30(\lambda + 2\mu)}\left[(3\lambda + 2\mu)r_1^5 + 6(2\lambda + 3\mu)r_0^5\right], \tag{A9}$$

$$F = \frac{\delta}{9(\lambda + 2\mu)}r_1^2 + \frac{\lambda + 3\mu}{2(\lambda + 2\mu)}\frac{2\delta(\lambda + \mu)}{3\mu(2\lambda + 5\mu)}r_0^2 - \frac{\delta}{2(2\lambda + 5\mu)}r_0^2$$

$$F = \frac{\delta}{\mu(\lambda + 2\mu)(2\lambda + 5\mu)}\left[\frac{1}{9}\mu(2\lambda + 5\mu)r_1^2 + \left(\frac{1}{3}(\lambda + 3\mu)(\lambda + \mu) - \frac{1}{2}\mu(\lambda + 2\mu)\right)r_0^2\right].$$

Taking into account the relation:

$$\begin{aligned}\frac{1}{3}(\lambda + 3\mu)(\lambda + \mu) - \frac{1}{2}\mu(\lambda + 2\mu) &= \frac{1}{6}\left[2(\lambda + 3\mu)(\lambda + \mu) - 3\mu(\lambda + 2\mu)\right]\\ &= \frac{1}{6}\lambda(2\lambda + 5\mu),\end{aligned}$$

we obtain the following reduced expression:

$$F = \frac{\delta}{18\mu(\lambda + 2\mu)} \left(3\lambda r_0^2 + 2\mu r_1^2\right). \tag{A10}$$

Equation (A2) can be transformed to:

$$3c_4 r_0 - \frac{\lambda + \mu}{\lambda + 2\mu} c_2 r_0^3 = -\frac{2\lambda + 5\mu}{\lambda + 2\mu} \left(-\frac{2\delta(\lambda + \mu)}{3\mu(2\lambda + 5\mu)}\right)$$

or together with equation (A8) we have:

$$3c_4 r_0 - \frac{\lambda + \mu}{\lambda + 2\mu} c_2 r_0^3 = \frac{2\delta(\lambda + \mu)}{3\mu(\lambda + 2\mu)}, \qquad c_4 r_0^3 - A c_2 = F. \tag{A11}$$

From equations (A11) it follows that:

$$c_4 r_0^3 = \frac{\lambda + \mu}{3(\lambda + 2\mu)} c_2 r_0^5 + \frac{2\delta(\lambda + \mu)}{9\mu(\lambda + 2\mu)} r_0^2 = F + A c_2.$$

The equation for c_2 becomes:

$$\left(\frac{\lambda + \mu}{3(\lambda + 2\mu)} r_0^5 - A\right) c_2 = F - \frac{2\delta(\lambda + \mu)}{9\mu(\lambda + 2\mu)} r_0^2. \tag{A12}$$

Here,

$$\Delta = \frac{\lambda + \mu}{3(\lambda + 2\mu)} r_0^5 - A = -\frac{1}{30(\lambda + 2\mu)} \left[(3\lambda + 2\mu) r_1^5 + 6(2\lambda + 3\mu) r_0^5\right] + \frac{\lambda + \mu}{3(\lambda + 2\mu)} r_0^5$$

or

$$\Delta = -\frac{1}{30(\lambda + 2\mu)} \left[(3\lambda + 2\mu) r_1^5 + 2(\lambda + 4\mu) r_0^5\right].$$

Using

$$F = \frac{\delta}{18\mu(\lambda + 2\mu)} \left(3\lambda r_0^2 + 2\mu r_1^2\right),$$

the right-hand side of equation (A12) will be:

$$F - \frac{2\delta(\lambda + \mu)}{9\mu(\lambda + 2\mu)} r_0^2 = \frac{\delta}{18\mu(\lambda + 2\mu)} \left(3\lambda r_0^2 + 2\mu r_1^2\right) - \frac{2\delta(\lambda + \mu)}{9\mu(\lambda + 2\mu)} r_0^2$$

or

$$F - \frac{2\delta(\lambda + \mu)}{9\mu(\lambda + 2\mu)} r_0^2 = \frac{\delta}{18\mu(\lambda + 2\mu)} \left[-(\lambda + 4\mu) r_0^2 + 2\mu r_1^2\right].$$

Now from equations (A11) and (A12) we obtain the solution for the constant c_2:

$$c_2 = \frac{F - [2\delta(\lambda + \mu)/9\mu(\lambda + 2\mu)] r_0^2}{[(\lambda + \mu)/3(\lambda + 2\mu)] r_0^5 - A},$$

$$c_2 = \frac{\delta}{18\mu(\lambda + 2\mu)} [-(\lambda + 4\mu) r_0^2 + 2\mu r_1^2] \Big/ \left(-\frac{1}{30(\lambda + 2\mu)} [(3\lambda + 2\mu) r_1^5 + 2(\lambda + 4\mu) r_0^5]\right)$$

and finally,

$$c_2 = \frac{5\delta}{3\Delta_d} [(\lambda + 4\mu) r_0^2 - 2\mu r_1^2] \quad \Delta_d = \mu \left[(3\lambda + 2\mu) r_1^5 + 2(\lambda + 4\mu) r_0^5\right]. \tag{A13}$$

A.1 *Determination of c_3*

From equation (A7), namely:

$$c_3 = -\frac{3\lambda + 2\mu}{30(\lambda + 2\mu)} c_2 r_1^5 - \frac{\delta}{9(\lambda + 2\mu)} r_1^2 \tag{A14}$$

and from equation (A13), we obtain:

$$c_3 = -\frac{3\lambda + 2\mu}{30(\lambda + 2\mu)} \frac{5\delta}{3\Delta_d} [(\lambda + 4\mu) r_0^2 - 2\mu r_1^2] r_1^5 - \frac{\delta}{9(\lambda + 2\mu)} r_1^2$$

$$c_3 = -\frac{\delta}{18(\lambda + 2\mu)\Delta_d} (\lambda + 4\mu) r_0^2 r_1^2 [4\mu r_0^3 + (3\lambda + 2\mu) r_1^3]. \tag{A15}$$

A.2 *Determination of c_4*

From equation (A11), namely:

$$3c_4 r_0 - \frac{\lambda + \mu}{\lambda + 2\mu} c_2 r_0^3 = \frac{2\delta(\lambda + \mu)}{3\mu(\lambda + 2\mu)},$$

where (equation (A13)):

$$c_2 = \frac{5\delta}{3\Delta_d}[(\lambda + 4\mu) r_0^2 - 2\mu r_1^2], \quad \Delta_d = \mu\left[(3\lambda + 2\mu) r_1^5 + 2(\lambda + 4\mu) r_0^5\right],$$

we determine

$$3c_4 = \frac{\lambda + \mu}{3r_0(\lambda + 2\mu)} \frac{5\delta}{3\Delta_d} [(\lambda + 4\mu) r_0^2 - 2\mu r_1^2] r_0^3 + \frac{2\delta(\lambda + \mu)}{9 r_0 \mu(\lambda + 2\mu)},$$

and therefore:

$$c_4 = \frac{\delta}{9 r_0 (\lambda + 2\mu)\Delta_d} (\lambda + \mu)[9(\lambda + 4\mu) r_0^5 - 10\mu r_0^3 r_1^2 + 2(3\lambda + 2\mu) r_1^5]. \tag{A16}$$

The final formulae for calculations are, from equations (A5), (A13), (A15) and (A16),

$$c_1 = -\frac{2\delta(\lambda + \mu)}{3\mu(2\lambda + 5\mu)},$$

$$c_2 = \frac{5\delta}{3\Delta_d}[(\lambda + 4\mu) r_0^2 - 2\mu r_1^2],$$

$$c_3 = -\frac{\delta}{18(\lambda + 2\mu)\Delta_d} (\lambda + 4\mu) r_0^2 r_1^2 [4\mu r_0^3 + (3\lambda + 2\mu) r_1^3],$$

$$c_4 = \frac{\delta}{9 r_0 (\lambda + 2\mu)\Delta_d} (\lambda + \mu)[9(\lambda + 4\mu) r_0^5 - 10\mu r_0^3 r_1^2 + 2(3\lambda + 2\mu) r_1^5],$$

$$\Delta_d = \mu\left[(3\lambda + 2\mu) r_1^5 + 2(\lambda + 4\mu) r_0^5\right].$$