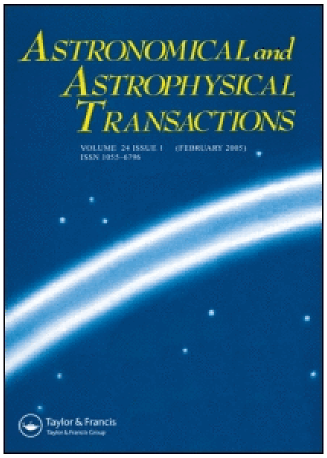

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Self-similar and self-affine structures in the observational data on solar activity

O. Chumak [a]
[a] Sternberg Astronomical Institute, Moscow, Russia

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Self-similar and self-affine structures in the observational data on solar activity

O. CHUMAK*

Sternberg Astronomical Institute, Universitetskij Prospekt 13, Moscow 119899, Russia

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Stochastic self-similarity and self-affinity in the data on the solar activity phenomena is discussed in this paper. The following results are given and discussed. Firstly, the area–perimeter method was applied to obtain the fractal dimension values of solar spot umbras and of equal-intensity field lines in solar active regions (ARs). The fractal dimension value $D_0 = 1.35 \pm 0.03$ has been obtained for sunspot umbras. Field lines of equal intensity have different values of fractal dimensions for north and south polarities. Secondly, it is shown that the fractal dimension extracted from correlation between the magnetic flux and its cross-sectional area (*i.e.* a photosphere) is temporally invariant for an AR, but the fractal dimensions can be considerably different for various ARs. Thirdly, the multifractal nature of ARs magnetic fields was confirmed by a study of the scaling properties of their Renyi entropy. Fourthly, the R/S analysis was used for the Wolf series with purpose of estimating the long-time 'memory' structure of this series. Time periods with different frequency bands were found in the series. Fifthly, the Renyi entropy and the Higuchi method were used for the study of solar X-ray flux variations. It is shown that the fractal dimensions obtained with this algorithm can be used as a good X-ray index.

Keywords: Solar activity; Entropies; Fractals

1. Introduction

Today the notions of entropy, information, fractals and fractal dimensions are used in all natural sciences, economics, medicine and other disciplines. Owing to their universality and outstanding effectiveness in the study of complicated systems and processes of various natures, these notions and the corresponding methods have gained great popularity in the last few decades. Hundreds of special papers devoted only to fractal time series analysis are published every year. The entropy–fractal parameterization of complexity systems is the second important field for application of these methods. In this paper we discuss manifestations of stochastic self-similarity and self-affinity in such solar activity phenomena as sunspots, active regions (ARs), magnetic fields and time series of some solar activity

*Email: chumak@sai.msu.ru

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indices. Other examples of the applications of fractal analysis in solar physics can be found in the book by Mogilevsky [1] and in the excellent review by Zeleny and Milovanov [2] (and references therein).

2. Contour–area correlation in solar active regions

The simplest method for obtaining the Hausdorf dimension is the 'contour-area' method. The value of this dimension allows us to estimate the complexity of a flat structure in a quantitative sense. Lines of equal intensity of any physical parameter are suitable data for the application of this method [3]. On the Sun, sunspots are good objects for such analysis. The fractal dimension of sunspot umbras is obtained with the 'area-perimeter' method [4]. It is assumed in this method that the area S of the flat figure correlates with length L of its boundary according to the power law $S = AL^Q$. The Hausdorf dimension of the figure is $D_0 = 2/Q$ [5]. The correlation between $\log S$ and $\log L$ gives for solar sunspot umbras the value $Q = 1.48$ and $D_0 = 1.35 \pm 0.03$. We shall discuss these result below.

Another case of application of this method in ARs is their radial magnetic field. We can try to find correlations between the lengths of the equal field values B_0 of north and south polarities and the areas corresponding to these lines. If these structures are self-similar, then the Hausdorf dimension gives us a quantitative estimation of the complexity of the magnetic field configurations of north and south polarities.

As an example, figure 1 shows the realization of this idea for the large AR 9077 which produced the famous flare known as the 'Bastille flare'. We used 106 magnetograms (longitudinal component) of the AR which were acquired from 10 July 2002 to 17 July 2002 in Huairou (PR China). All data were reduced to the centre of the solar disc as usual. We take $B_0 = 200$ G. We can see in the figure that the complexity of the structural field of south polarity distinctly differs from that of north polarity.

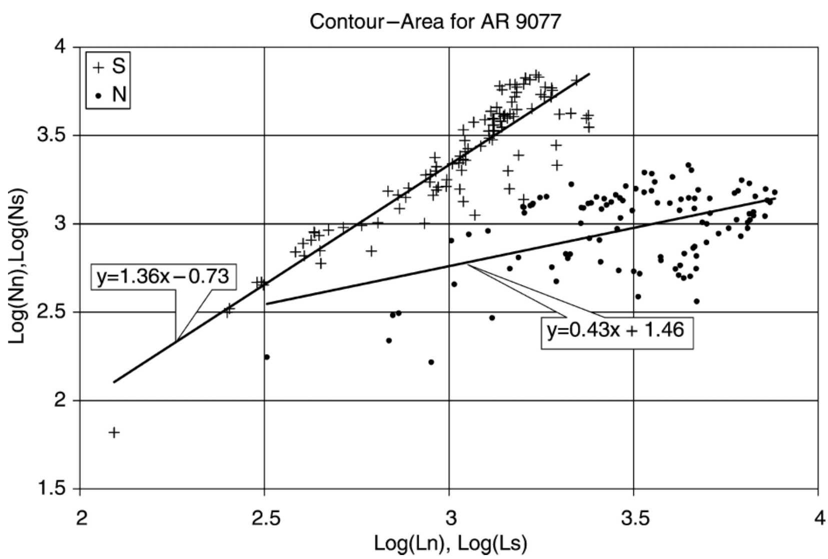

Figure 1. Contour–area correlation for NOAA 9077.

3. Area–flux correlation in solar active regions

It is known that in the simple case, correlation between the magnetic flux and its cross-sectional area is expressed by the linear law $F \propto S$, where F is the flux and S is the area inside the flat current contour that generates the flux. However, in complicated cases, such as solar ARs, magnetic fluxes consist of large numbers of elementary fluxes and the correlation formula should be written as $F \propto S^Q$, where Q can be some fraction. The following questions need to be answered. Is the Q constant or a function of time? Are the Q values the same or different for various ARs? Are the Q values the same for north and south polarities or not? Are the Q values the same for strong and weak fields or not? The correlation between the magnetic flux and its cross-sectional area (*i.e.*, a photosphere) for solar ARs has been discussed by Chumak and Chumak [6]. In [6] one can find the answers to some of these questions. It was shown that, as a rule, Q does not change with time but varies for different ARs. So one can use Q as the parameter for quantitative classification of ARs. It was shown also that Q can have different values for north and south polarities in the same AR. This unexpected result indicates the structural difference of fields of north and south polarities.

Now we have new confirmation of this result. Figure 2 shows the correlation between the logarithms of the north and south polarity areas (in pixels) and the logarithms of the corresponding polarity fluxes (in gauss pixels) for eight ARs (NOAA 8525, 8550, 8592, 8598, 8594, 8599, 8602 and 8635). For all the ARs the magnetometric data that were used were obtained when an AR passed the central meridian. From this figure we can see the large differences between the fractal dimensions of north polarity ($Q_n = 2.48$) and south polarity ($Q_s = 0.89$).

4. Multifractal nature of the magnetic fields of active regions

The multifractal nature of the magnetic fields of ARs can be found both with the area–perimeter method and with the Renyi entropy analysis. These can be seen in figures 3 and 4 respectively.

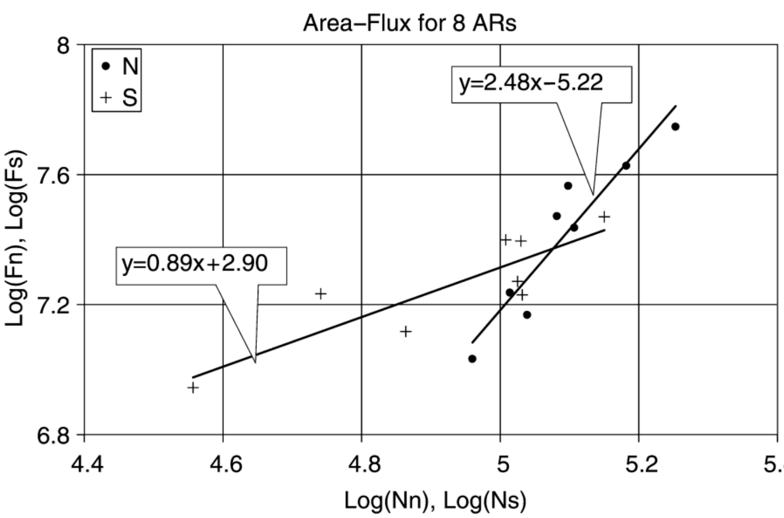

Figure 2. Area–flux correlation for ARs eight.

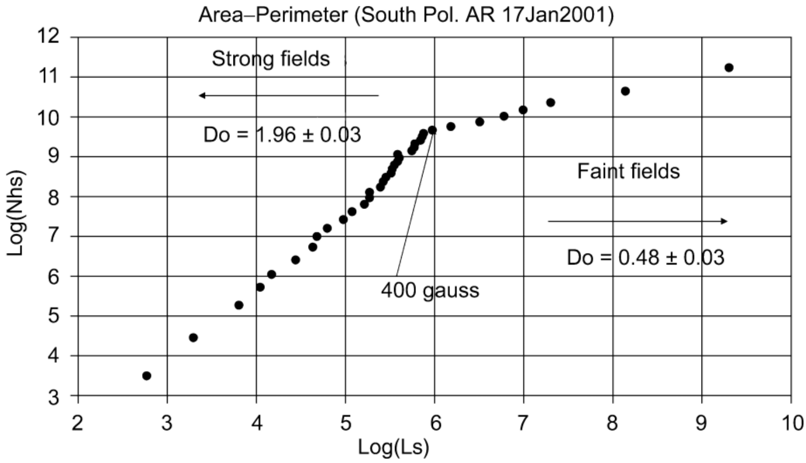

Figure 3. Example of an area–perimeter diagram.

An example of an area–perimeter diagram for various intensity magnetic fields of south polarity of AR 6659 (17 January 2001) is shown in figure 3. We can see from this figure that D_0 for strong fields is close to 2.0 and greater than D_0 for sunspot umbras. For weak fields (less than 400 G), $D_0 < 0.5$. So we can observe two scaling laws with fraction exponents simultaneously and should draw conclusions about the multifractal (two-fractal) nature of the magnetic fields of ARs. This conclusion is supported independently by calculations of the Renyi entropy [5, 7].

Figure 4 demonstrates the correlation between the Renyi 2 entropy and the logarithm of the inverse value of grating period $1/e$ for the same data. We can see the distinctive break near $\log(1/e)$ ≈3.0–3.5. This corresponds to about 1.25 Mm in absolute units and gives us the value of the scale constant for scaling differences between strong and weak fields. The field break-point constant is 400 G, as we can see from the previous figure.

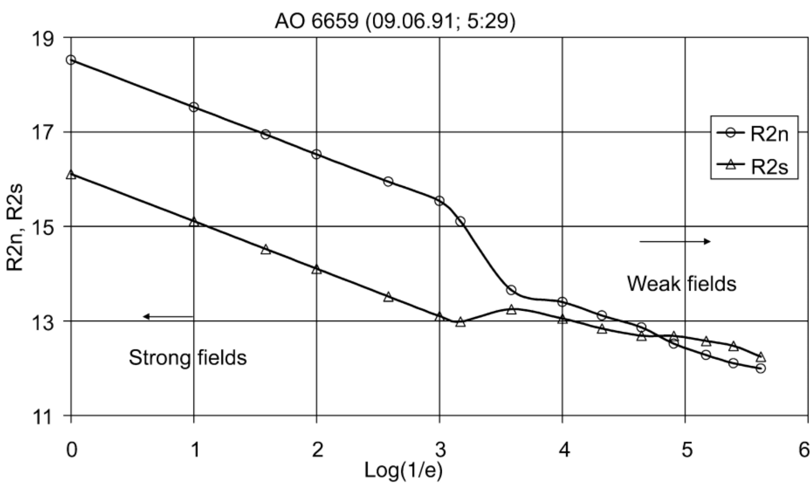

Figure 4. Correlation between the Renyi 2 entropy and the logarithm of the inverse value of grating period.

5. The R/S analysis for the Wolf series

Scaling laws are well known in time series analyses. There are many specific methods for analysing various time series. One of these methods is the Hurst R/S analysis. This method makes it possible to find the persistent and non-persistent frequency bands in the time series and to estimate the time horizon. We used this method for the Wolf time series analysis. A similar analysis can be found in the book by Feder [8]. We have used more data and have obtained some new results. Figure 5 shows the R/S diagram for the Wolf number. Here N is the number of months. Some parts with different tilts of the R/S diagram were obtained. We single out three of these parts: firstly, for $\log(N) \approx 2.0$–2.35 (8.4–18.7 years) with a tilting angle tangent of about 0.63; secondly, for $\log(N) \approx 0.8$–1.6 (0.5–3.3 years); thirdly, for $\log(N) \approx 1.4$–2.75 (25–47 years). For $\log(N) > 2.75$ the tilting angle became near to zero. The larger the tilting angle, the better is the row persistency and the more predictable is the time series behaviour. So we can conclude that for the Wolf number series there are a few time intervals with good predictability and a few with not so good predictability and the time horizon lies near 47 years.

6. Study of solar X-ray flux variations with the Renyi and the Higuchi method

The Renyi entropy method can be used for time series analysis of the same problems as the Higuchi method [9]. We analyse the curve as well as the shore of Great Britain as studied by Richardson. Using the number of covers we realize scaling and calculate the Renyi entropies. Further we can find the Renyi fractal dimension spectrum from the correlation between the Renyi entropies and the logarithm of the inverse value of grating period. The correlation dimension D_2 is the same as can be obtained using the Higuchi method. We use this method for a solar X-ray flux time series (which has been acquired with the GOES satellites) analysis. We found great variations in the fractal dimension values for these data. Large differences in

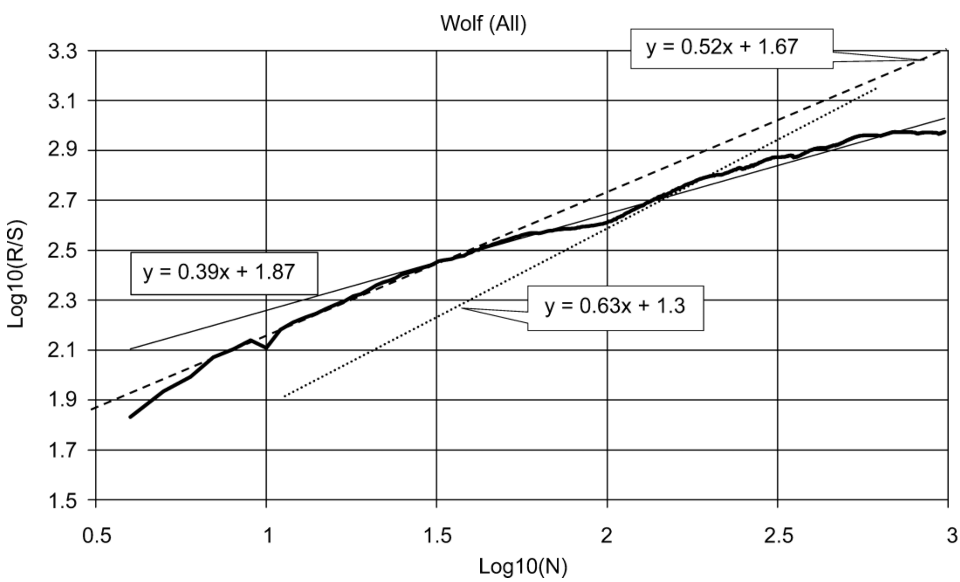

Figure 5. R/S diagram for the Wolf number series.

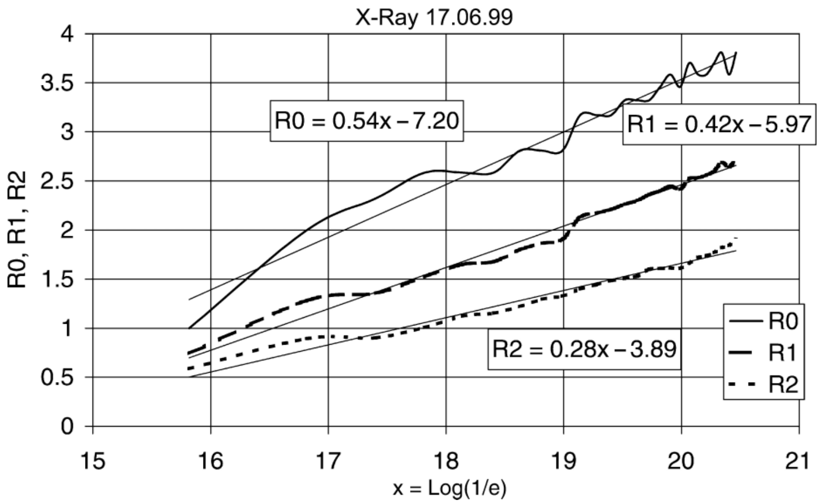

Figure 6. Renyi diagram for solar X-ray fluxes (17 June 1999).

the dimension values can be found even for two successive days. So we can conclude that the fractal dimensions obtained in this way can be used as a good and very sensitive quantitative indices for the solar X-ray flux. In figures 6 and 7 we show correlations between $\log(1/e)$ and the first three Renyi entropies. The corresponding equations for the trend lines are shown in the diagrams. The tilting angle tangents of the trend lines (the first term of the trend-line equations) gives the corresponding dimensions. The trend-line equations are shown in the figures. Comparing figure 6 and figure 7 we can see a large difference in the dimension values for two successive days. So we can conclude that the fractal dimensions obtained in this way can be used as a good quantitative index for the solar X-ray flux.

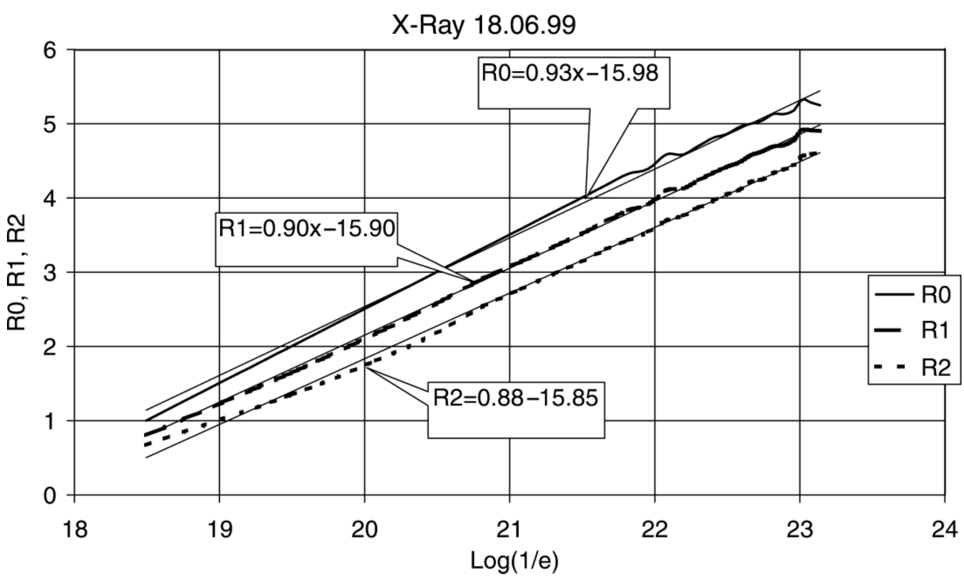

Figure 7. Renyi diagram for solar X-ray fluxes (18 June 1999).

7. Discussion

Numerous scaling laws, similar to those above, indicate the existence of self-similarity structures in various solar activity events. The study of such laws using the observational database provides a step towards understanding the physical nature of solar activity. So the obtained value of the fractal dimension of sunspot umbras is of interest in the sense that it corresponds to the structures known in the percolation theory under the name 'elastic skeleton' (or framework). Such structures can appear in a wide range of scales: from laboratory turbulence, on scales of the order of 10 m up to clusters of galaxies with scales of about 10 Mpc [10].

The magnetic fields observed in the solar photosphere are generated by the dynamo process in the non-stationary convection of the solar plasma [11]. The framework structures in the upper layers of the Sun are clusters of convection cells. Some of these can penetrate through all the convection zone. The magnetic moments in such a cluster can be correlated between themselves, approximately as is realized in the Ising model [3]. A correlation of the magnetic moments leads to increasing magnetic flux density in such aggregates. So, the initial weak chaotic field in such structures can be sharply enhanced and non-equilibrium structure becomes self-sustaining.

The cells located on the penetrating branches of the cluster are supported by constant energy inflow. The cells located on dead-end branches will be dampered. The whole complex of penetrating branches of the cluster is called the skeleton (framework). Skeletons are stable structures in percolating media. The skeletons are fractal objects. A fractal dimension equal to $4/3$ corresponds to 'elastic skeletons', that is skeletons in which the shortest routes connect the most distant points [10]. The fractal dimension of sunspots which we obtained (see section 2) rather corresponds to the 'elastic skeletons'. It should be noted that the appearance of penetrating clusters is a critical phenomenon and may be connected with specific phase transitions.

The multifractal nature of the magnetic fields of ARs (see section 4) indicates the existence of two different kinds of hydrodynamic motion which can generate a magnetic field. This could be of interest to dynamo theory.

Scaling analysis is of interest also from a practical point of view because it can provide useful information both for forecasting and for parameterizations (see sections 5 and 6).

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