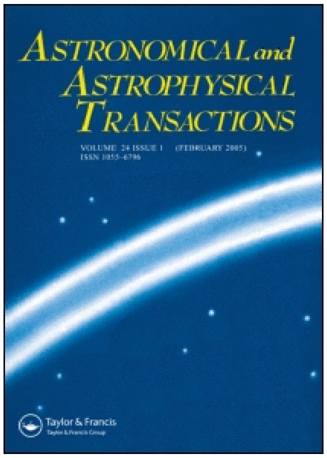

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Chaotic orbits of distant stars

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Chaotic orbits of distant stars

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A disk galaxy model with a dense nucleus and additional perturbing terms is used for the study of properties of orbits of distant stars. Our numerical experiments show that the majority of distant stars are in chaotic orbits. There are distant stars displaying regular orbits as well. A number of distant stars are ejected to the halo on approaching the dense and massive nucleus. A polynomial relationship exists between the mass of the nucleus and the critical angular momentum of the distant stars. Numerical calculations suggest that the majority of distant stars spend their orbital time in the halo where it is easy to be observed. We present evidence that the dominant term for driving stars to distant orbits is the presence of the dense nucleus combined with the perturbation caused by nearby galaxies. The origin of young OB stars observed in the halo is also discussed.

Keywords: Orbits; Distant stars; Dense nucleus; Galactic models

1. Introduction

Observations show that most of the distant stars are observed in the halo and they are old. Nevertheless a number of scientists claim that young O, B and A stars have been identified in the halo (see [1]). Then the question arises: Do we have a low level of star formation in the halo or, the observed young stars are disk stars ejected from the galactic plane? If so, what mechanism have driven these stars in galactocentric distances of about 30 kpc or more? One of the targets of this work is to try to provide and answer to the above two interesting questions. In order to do this we use a disk-halo potential with a dense nucleus. This potential is similar to the potential used by Carlberg and Innanen [2]. The dynamical model is

$$V(r, z) = -\frac{M}{\{[\alpha + (z^2 + h^2)^{1/2}]^2 + r^2 + b^2\}^{1/2}} - \frac{M_n}{(r^2 + z^2 + c_n^2)^{1/2}}, \tag{1}$$

where r, z are cylindrical co-ordinates, M, M_n, is the mass of the disk and nucleus, respectively, α is the scale length of the disk, h corresponds to disk scale height, b is the core radius of the halo component while, c_n, is the scale length of the nucleus. To the potential (1) we add

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some additional terms describing perturbation from nearby galaxies. Then, the total potential becomes

$$V_{tot}(r, z) = V(r, z) + kr^3 + \lambda(r^2 + \beta z^2)^2, \tag{2}$$

where k, λ, β are parameters. Our choice is justified by the following lines of arguments: it is well known that in galaxies with dense massive nuclei (see [3–5]) low angular momentum stars, moving near the galactic plane when passing near the nucleus, are scattered to much higher scale heights displaying chaotic motion. In our previous work, we had tried to present the properties of low angular momentum stars and to find connections between the chaotic motion and some basic physical quantities such as angular momentum and mass of the nucleus. What is new here is the study of distant orbits. By the term "distant orbit" we mean orbits that reach large galactocentric distances, of order of 30 *kpc* or larger. In order to have a better estimation of distant orbits, we agree to call distant orbits all orbits obtaining galactocentric distances more than 15 *kpc*. This is the reason for introducing the additional terms in equation (2). As we shall see later, those additional terms are the basic factor for driving stars to distant orbits.

On this basis we can say that, apart from the search for origin of the young stars, observed in the halo, as it was mentioned before, there are some more reasons for doing this research. Those reasons are: (i) to find a relationship between the critical value of angular momentum (that is the maximum value of the angular momentum L_z for which stars are driven in to the halo displaying a distant chaotic orbits, for a given value of the mass of nucleus M_n) and M_n, (ii) to explain this relationship using elementary theoretical arguments.

In section 2 we present the behavior of orbits in the potential (2). Particularly, we study the orbits of distant stars and try to explain the presence of OB stars into the halo at large galactocentric distances. In the same section, a relationship between the critical value of the angular momentun L_{zc} and M_n is given. In section 3 we present a theoretical explanation for the relatioship between L_{zc} and M_n. We close with a discussion which is given in section 4.

Our investigation is mostly based on the numerical integration of the equations of motion, which are

$$\ddot{r} = -\frac{\partial V_{eff}}{\partial r}, \; \ddot{z} = -\frac{\partial V_{eff}}{\partial z}, \tag{3}$$

where the dot indicates derivative with respect to time, while

$$V_{eff} = \frac{L_z^2}{2r^2} + V_{tot}(r, z), \tag{4}$$

is the effective potential, describing motion in the (r, z) meridian plane, and L_z is the conserved component of the angular momentum. A system of galactic units is used, where the unit of length is 1 *kpc*, the unit of time is 0.97746×10^8 *yr* and the unit of mass is $2.325 \times 10^7\ M_\odot$. The velocity unit is 10 *km/s* while G is equal to unity. Our test particle is a star of mass $= 1$. Therefore, the energy unit (per unit mass) is $100\ (km/s)^2$. In these units, we use the values $M = 12000$, $b = 8\,kpc$, $\alpha = 3$, $h = 0.1\,kpc$, and $c_n = 0.25\,kpc$. The mass of nucleus, M_n, is treated as a parameter.

2. Orbit calculations

In order to study the dynamical behavior of the system we use the two dimensional Hamiltonian,

$$H = \frac{1}{2}(p_r^2 + p_z^2) + V_{eff}(r, z) = E, \tag{5}$$

where p_r, p_z are the momenta, per unit mass, conjugate to r and z, while E is the star's energy.

A good description of the total set of orbits in the Hamiltonian (5) can be obtained using the $r - p_r$ $z = 0$, $p_z > 0$ Poincare phase plane. Figure 1 shows the structure of this plane, when $k = -0.35$, $\lambda = 0.01$, $\beta = 0.01$. The value of the energy is $E = -1150$, the value of the mass of nucleus is $M_n = 400$ and the value of the angular momentun is $L_z = 20$.

One observes a large chaotic sea and areas of regular motion. There are two distinct areas of regular orbits: (i) orbits that stay inside the main body of the galaxy, (ii) orbits that stay out of the main body of the galaxy. On the other hand, we observe that all chaotic orbits are orbits that can go to far galactocentric distances, that is all chaotic orbits are distant orbits. Note that the phase plane in figure 1 corresponds to a test particle of low angular momentun.

Figure 2 shows a plot between the critical value of the angular momentum L_{zc} and the mass of the nucleus M_n. The values of parameters are $k = -0.35$, $\lambda = 0.01$, $\beta = 0.01$. In order to find the relationship, we calculated numerically a large number of orbits. Orbits were started at $r = 8.5\,kpc$, $z = 0$ with zero radial velocity. Crosses indicate the numerically found values, while the solid line is the best fit. Here we find that the mass of nucleus M_n has a second degree polynomial dependence on L_{zc}. Indeed the best fit line can be represented by the equation

$$M_n = 0.062L_{zc}^2 + 0.404L_{zc} + 6.920, \tag{6}$$

which is a second degree polynomial in L_{zc}. Orbits with values of parameters on the lower part of the $[L_{zc} - M_n]$ plane, including the line, give distances, that is, all chaotic orbits while orbits with values of parameters on the upper part of the plane lead to regular orbits that stay inside the main body of the galaxy.

Observing figure 2 we see that $L_z = 20$ is the critical angular momentun corresponding to a mass of the nucleus about $M_n = 50$, while in figure 1 we have a massive nucleus with $M_n = 400$. In other words, the combination of low angular momentum and a massive nucleus produce large chaotic regions. Numerical calculations, not shown here, indicate that the chaotic sea decreases as the value of angular momentum increases, for a given value of M_n, while the chaotic regions increase as the mass of the nucleus increases for a constant value of L_z.

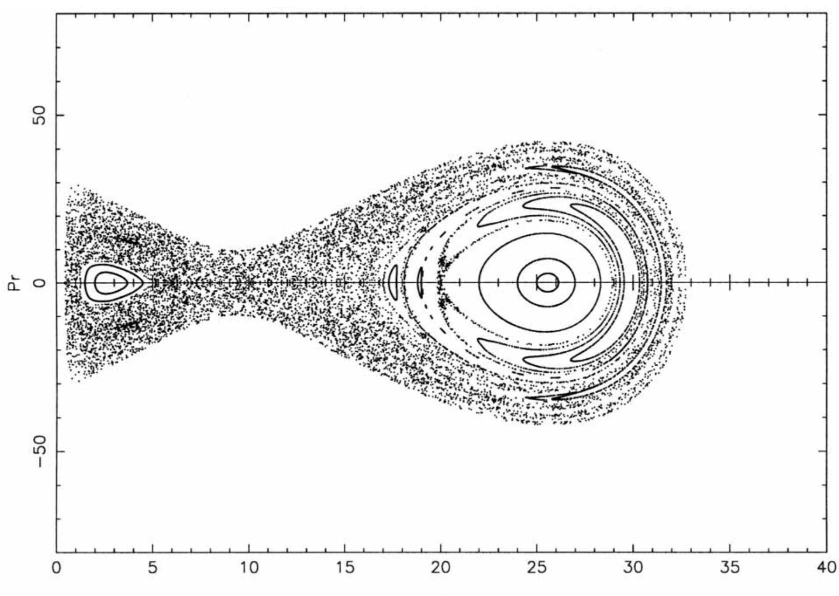

Figure 1. The structure of the $r - p_r$ phase plane, when $k = -0.35$, $\lambda = 0.01$, $\beta = 0.01$. The value of the energy is $E = -1150$, $M_n = 400$ and the value of the angular momentun is $L_z = 20$.

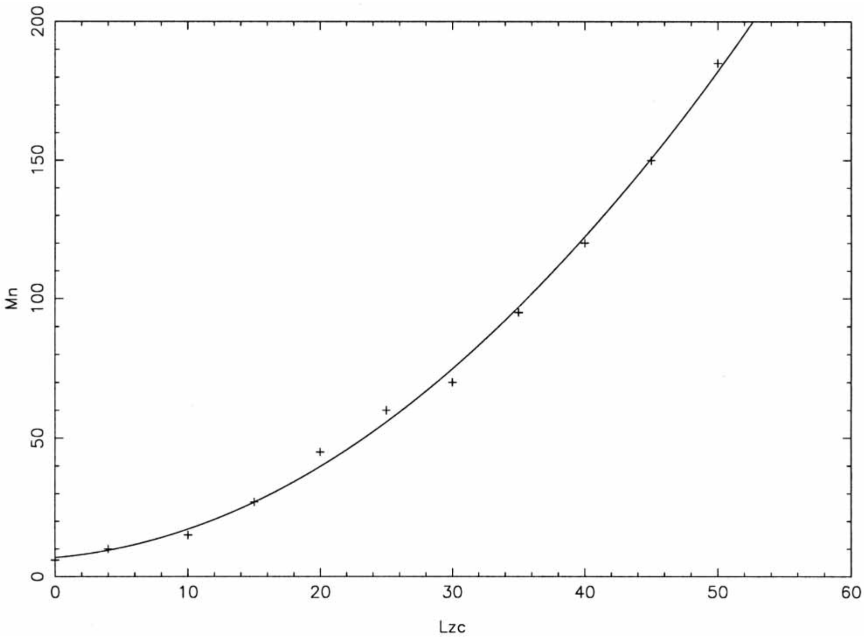

Figure 2. Relationship between L_{zc} and M_n. Details are given in text.

Figure 3 shows a regular orbit that stays near the central region of the galaxy. The initial conditions are $r = 2\,kpc$, $z = p_r = 0$. The value of p_z is found from the energy integral. The values of parameters are as in figure 1. The orbit was calculated for 50 time units. One observes that although the orbit stays in the central galactic region it does not approach the nucleus. Figure 4 shows a distant chaotic orbit. Here, the initial conditions are $r = 8.5\,kpc$, $z = p_r = 0$,

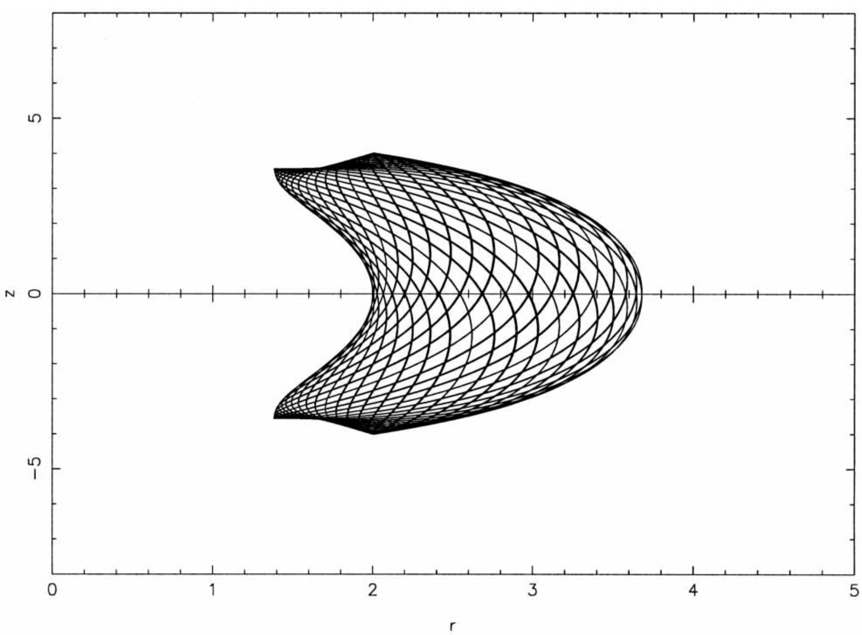

Figure 3. A regular orbit that stays near the central region of galaxy. Initial conditions are $r = 2\,kpc$, $z = p_r = 0$. The values of other parameters are as in figure 1.

while the value of p_z is found using the energy integral. The values of the other parameters are $k = -0.35$, $\lambda = 0.01$, $\beta = 0.01$. The value of the energy is $E = -1150$, $M_n = 150$ and the value of the angular momentun is $L_z = 44$. The orbit was calculated for 80 time units. It is interesting to observe that, inside the main body of the galaxy, the orbit stays near the galactic plane while at large galactocentric distances it gains considerable height, that is, it spends most of its orbital time into the halo. The orbit shown in figure 5 is a regular orbit which spends all its orbital time at large galactocentric distances. In other words, this orbit stays out of the main galactic body and spends most of its orbital time into the halo. The values of the parameters are as in figure 1. The initial conditions are $r = 23\,kpc$, $z = p_r = 0$, while the value of p_z is found using energy integral. It is of interest to note that this orbit goes higher into the halo than the orbit shown in figure 4.

All numerical calculations strongly suggest that in order, for a star, to display a distant chaotic orbit three factors must be present. The first is that we must have a galaxy model with a massive nucleus. The second is that the galaxy must have nearby companions. The third factor is that the star must possess a low angular momentun. The first and the last are very well expressed by relationship (6) while the second is expressed by the presence of the extra perturbing terms in equation (2). Numerical work, not given here, indicates that in a galaxy without a nucleus the number of stars in distant orbits is small, while all orbits in the galaxy are regular.

Let us now go to make some comments for the existence of distant stars in the halo. The mechanism, responsible for the presence of all distant stars in the halo, is the perturbation from the nearby galaxies. In a galaxy model without this extra perturbation, that is when $k = -\lambda = 0$, there are no stars in distant orbits. On the other hand, in a galaxy model (2), where both, the perturbing terms and the massive nucleus are present the possibility of a star to have been ejected from the galactic plane is evident. Numerical calculation not given here suggest that the time needed for a star to be ejected from the galactic plane in a distant orbit is

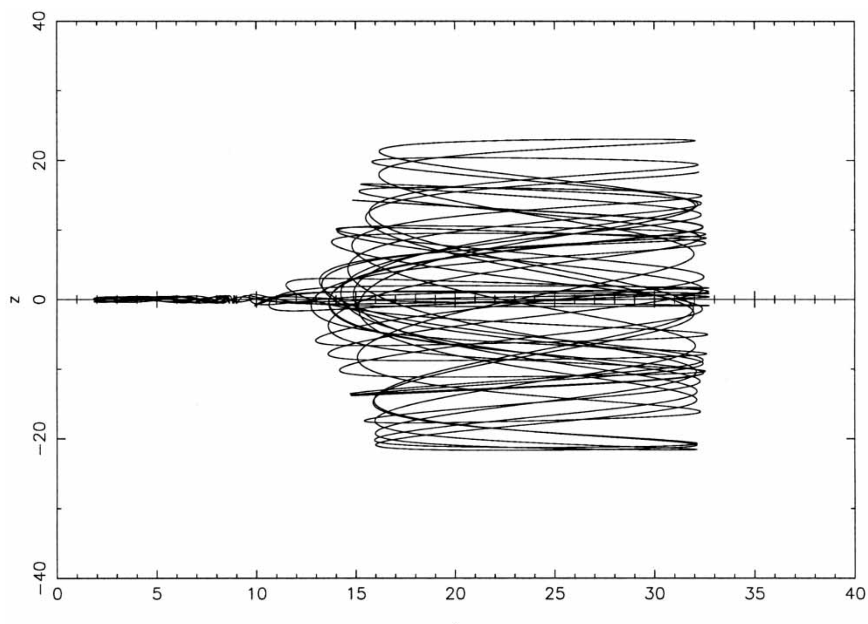

Figure 4. A distant chaotic orbit. The initial conditions are $r = 8.5\,kpc$, $z = p_r = 0$. The values of the other parameters are $k = -0.35$, $\lambda = 0.01$, $\beta = 0.01$. The value of the energy is $E = -1150$, $M_n = 150$ and the value of the angular momentun is $L_z = 44$.

of the order of 50 time units, that is about 5×10^9 *yr* while the time needed for a distant star to return again to the main body of the galaxy is about 300 time units, that is about 3×10^{10} *yr*. On this basis it is clear that the number of distant stars in the galaxy model (2) must be increasing.

Our model suggest that a large number of distant stars are disk stars scattered to the halo in chaotic orbits. We would like to make clear that all these stars are not young OB stars. The majority of those stars are old stars. It is well known that the central bulges of spiral galaxies contain old stars, while the disks of spirals contain mix of old and young stars. On the other hand, given the fact that the age of B stars is of order of several time 10^7 *yr*, it seems more possible for those stars, to have been formed into the halo. Another explanation, for the presence of the small number of OB stars at large z-coordinates, could be the evolution in binaries and the mass exchange through a Roche lobe flow [6, 7].

Of course one cannot forget the possibility, where stars are trapped in distant orbits (see figure 5). These stars are old halo stars, probably RR-Lyrae stars. This argument is strongly supported by observational data were faint RR-Lyrae stars were discovered at distances up to 33 *kpc* from the galactic centre (see [8–10]).

3. Theoretical approach

We now go to find the form of equation (6) using some theoretical arguments together with numerical evidence. Responsible for driving the star in a far galactocentric distant is the F_r force near the nucleus. On approaching the nucleus there is a change in the star's momentun in the r direction given by

$$m\Delta \upsilon_r = \langle F_r \rangle \Delta t, \tag{7}$$

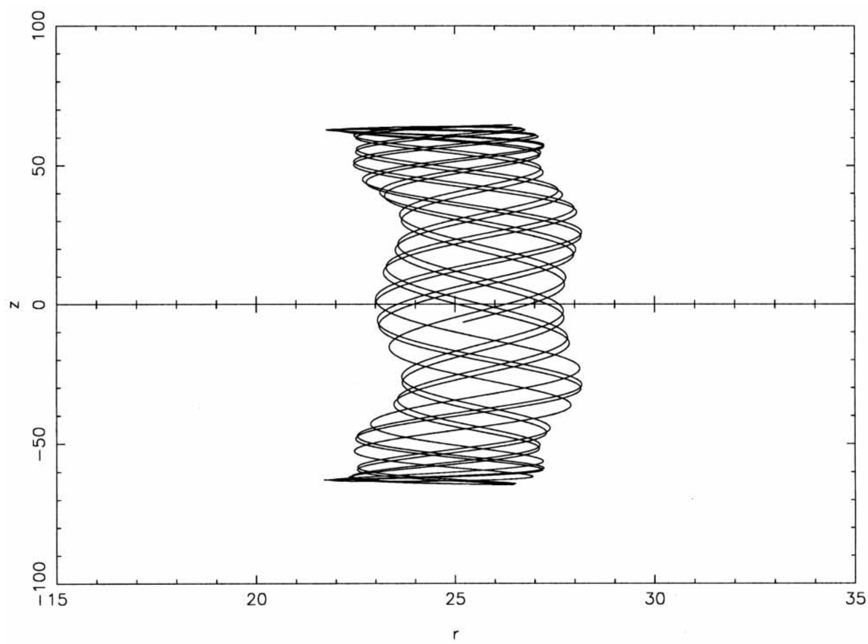

Figure 5. A regular distant orbit. The values of the parameters are as in figure 1. The initial conditions are $r = 23\,kpc$, $z = p_r = 0$.

where m is the mass of the star, $\langle F_r \rangle$ is the average force acting along the r direction near the nucleus and Δt is the duration of the encounter. It was observed that the star is driven far from the galactic centre after several passes by the nucleus. Let us assume that the star goes to a distant orbit after n ($n > 1$) when the total change in the momentum in the r-direction is of order of $-m\upsilon_\phi$, where υ_ϕ is the tangential velocity of the star near the nucleus. Thus we have

$$m \sum_{i=1}^{n} \Delta \upsilon_{ri} \approx \langle F_r \rangle \sum_{i=1}^{n} \Delta t_i. \tag{8}$$

If we set

$$\sum_{i=1}^{n} \Delta \upsilon_{ri} = -\upsilon_\phi = -\frac{L_{zc}}{r}, \ \sum_{i=1}^{n} \Delta t_i = T_c, \ m = 1, \tag{9}$$

in equation (8), we find

$$-\frac{L_{zc}}{r T_c} = \langle F_r \rangle. \tag{10}$$

Elementary numerical calculation show that, near the nucleus, where $r = r_0 < 1, z \approx 0$ the $\langle F_r \rangle$ force is repulsive and can be written in the form

$$\langle F_r \rangle \approx \frac{L_{zc}^2}{r_0^3} - \frac{M_n r_0}{(r_0^2 + c^2)^{3/2}}, \tag{11}$$

because the other terms are negligible near the nucleus. Inserting this value of $\langle F_r \rangle$ in (10) and rearranging, we find

$$M_n \approx \left[\frac{L_{zc}^2}{r_0^4} + \frac{L_{zc}}{T_c r_0^2} \right] (r_0^2 + c_n^2)^{3/2} \approx \alpha_1 L_{zc}^2 + \alpha_2 L_{zc}, \tag{12}$$

where α_1, α_2 are constants. This is because the values of r_0 and T_c is about the same for the range of values of M_n and L_z used in figure 2. Equation (12) gives a polynomial relationship between M_n and L_{zc} but it is not complete. Numerical experiments show that, when $L_{zc} \to 0$, there must be a minimum value of $M_n = M_{n0}$ in order to drive the star to a distant orbit (see diagram in figure 2). Adding this additional term in (12), we obtain

$$M_n \approx \alpha_1 L_{zc}^2 + \alpha_2 L_{zc} + M_{n0}. \tag{13}$$

The form of equation (13) is exactly the same as the numerically found relationship (6).

4. Discussion

In this work we have try to study the properties of orbits of distant stars. In order to do this, we have adopted a galactic model with a disk halo and a dense nucleus. This flattened potential is a generalization of the potential used by Miyamoto and Nagai [11]. A large number of galactic models are available in the literature (see [12–14]). In addition to the above global models one must not forget the local galactic models made up of harmonic oscillators (see [15–17] and references therein). Interesting information on galactic models can be found in the work of Caldwall and Ostriker [18].

Our numerical experiments have shown that a relationship exists between the critical value of the angular momentum and the mass of the nucleus for the distant chaotic orbits. This relationship is not linear (see [3]) but it can be expressed as a two degree polynomial in terms of the critical angular momentum. The relationship was also found using some theoretical arguments.

It is evident that almost all distant stars are old stars. The presence of a few young OB stars in the halo is a very interesting observation that needs to be clarified. If we take into account that all those stars have small ages, we have to accept two possible explanations. The first explanation might be that those stars have been formed into the halo, while a second could be the evolution in binaries and the mass exchange through a Roche lobe flow. We believe that more observation data are needed, in order to have a better idea about the origin of the distant OB stars.

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References

[1] K. Croswell, D.W. Latham, B.W. Carney, W. Schuster and L. Aguilar, Astron. J. **101**(6) 2078 (1991).
[2] R.G. Carlberg and K.A. Innanen, A.J. **94** 666 (1987).
[3] N.D. Caranicolas and K.A. Innanen, A.J. **103** 1308 (1991).
[4] N.D. Caranicolas, A&SS **246** 15 (1997).
[5] N.D. Caranicolas and N.J. Papadopoulos, A&A. **399** 957 (2003).
[6] R.Q. Huang and R.E. Taam, A&A **236** 107 (1990).
[7] R.Q. Huang, A&A **422** 981 (2004).
[8] A. Saha, Ap. J. **283** 580 (1984).
[9] A. Saha, Ap. J. **289** 310 (1985).
[10] A. Saha, J. Oke, Ap. J. **285** 688 (1984).
[11] W. Miyamoto and R. Nagai, Publ. Astron. Soc. Jpn. **27** 533 (1975).
[12] M. Clutton-Brock, K.A. Innanen and K.A. Papp, A&SS **47** 299 (1976).
[13] J.N. Bahcall, M. Schmidt and R.M. Soneira, Ap. J. Lett. **258** L23 (1980).
[14] D. Richstone, Ap. J. **252** 496 (1982).
[15] A. Deprit, Cel. Mech. **51** 202 635 (1991).
[16] A. Elipe, Phys. Rev. E. **61**(6) (1999).
[17] N.D. Caranicolas, J. Ap. Astron. **22** 309 (2001).
[18] J.A.R. Caldwell and J.P. Ostriker, Ap. J. **251** 61 (1981).