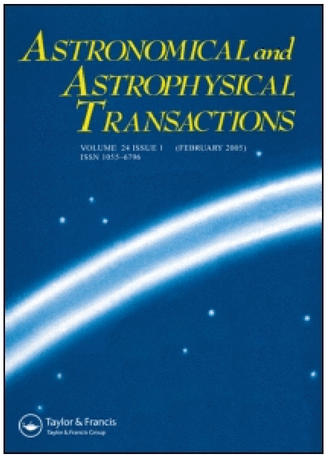

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Quasi-quantization of the orbits in the Solar System

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Quasi-quantization of the orbits in the Solar System

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The motion of the planets, asteroids and comets in the Solar System are controlled by gravity and are well explained in classical mechanics by Newton's and Kepler's laws. Using these laws, one can easily calculate some of the unknown variables such as the period or the angular momentum of the planets. However, neither Newton's laws nor Kepler's laws explain the relationship between the angular momentum of the orbits in the Solar System. The purpose of this paper is to demonstrate that the specific angular momentum (angular momentum per unit mass) of any planet in the Solar System can be expressed in terms of the specific angular momentum of the planet Mercury and, consequently, to prove that the semi-major axis of any planet, asteroid or comet can be expressed as a multiple of the semi-major axis of Mercury.

Keywords: Solar System; Planets; Angular momentum; Quantization

1. Introduction

It is generally believed that a series of events caused the formation of the Solar System about 4.6 billion years ago [1]. The infinitesimal rocks and dust that made up our planets secured themselves in nearly circular orbits, while the leftover materials belonging to the same orbits made their homes in an elliptical fashion. It is very possible that some of the events that made up our Solar System were chaotic [2]. However, if the Solar System was destined to survive as a stable system, it may be possible to explain these regularities with some mathematical laws.

The regularities that govern the Solar System are analogous but not exactly the same as the atomic model that some scientists have tried to model it after [3, 4]. Just like the electrons orbiting the nucleus in the atomic model, in the planetary system the planets orbit the Sun. However, the electromagnetic laws governing the atomic structure are not the same as the gravitational laws; hence, it might not be possible to design mathematical models based on the model for the hydrogen atom. At the same time, for a relatively stable system such as the Solar System, it would make sense if some mathematical laws could describe the regularities of this system.

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2. Quasi-quantization

The orbits in a planetary system are quantized if the eccentricities and inclinations of the orbits are made fixed. In reality, however, the eccentricities and the inclinations of the orbits are not fixed, as each orbit contains many bodies of different eccentricities and inclinations, and hence the orbits are not quantized in the same manner as for the atomic model.

Another main difference between the planetary model and the atomic model is that an electron 'jumps' from one orbit to another emitting photon. In the planetary model, if a planet or an asteroid is destined to move from one orbit to another, it will do so by increasing its eccentricity so much that eventually its orbit overlaps the next orbit and adopts the next orbit as its home orbit. This movement normally happens over a long astronomical time period and occurs because of a stronger gravitational force in the nearby orbit. Examples of these movements of asteroids can be seen between the orbit of the asteroid belt and the orbit of Jupiter, where a large number of asteroids that belong to the asteroid belt are attracted to the orbit of Jupiter. This is the main reason why some individual asteroids do not appear to fit our model.

3. The specific angular momentum and the semi-major axis

A planet in the Solar System moves in an elliptical orbit about the Sun under the influence of Newton's gravitational attraction between the planet and the Sun, where the Sun is in one of the focal points of the orbit, obeying Kepler's laws [1].

The angular momentum L of a planet moving in an elliptical orbit can be expressed in terms of the products of the mass m of the planet, the orbital velocity v of the planet and the semi-latus rectum l. Similarly, the specific angular momentum J (or angular momentum L/m per unit mass), can be expressed in terms of the product of the semi-latus rectum and the velocity of the planet. Since the Sun does not exert a torque on any planet, the angular momentum of a planet remains constant at all times [1].

It is only possible for a planet to move in an orbit for which its specific orbital angular momentum J is N multiples of Mercury's specific orbital angular momentum J_0. That is,

$$J_n = N J_0, \tag{1}$$

with N defined as

$$\mathrm{N} = \left(1 + \frac{k(1-e_n)n^{\ln(n)}}{1-e_0^2}\right)^{0.5}, \tag{2}$$

where $k = 0$ for Mercury's orbit, $k = 1$ for other orbits, e_n is the orbital eccentricity of the nth planet, e_0 is the eccentricity of Mercury and n is the orbital number (with $n = 0$ for Mercury, $n = 1$ for Venus, etc.).

Consider the nth planet with mass m_n revolving in an orbit about the Sun with mass M, where $M \gg m_n$, with semi-latus rectum $\boldsymbol{l}_n$. The net torque exerted by the Sun on the planet is

$$\boldsymbol{\tau}_n = \boldsymbol{l}_n \times \boldsymbol{F}_n = \boldsymbol{l}_n \times \frac{\mathrm{d}\boldsymbol{p}_n}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{l}_n \times \boldsymbol{p}_n) - (\boldsymbol{v}_n \times \boldsymbol{p}_n) = \frac{\mathrm{d}\boldsymbol{L}_n}{\mathrm{d}t} = 0,$$

where $\boldsymbol{\tau}_n$ is the torque, $\boldsymbol{F}_n$ is the gravitational force, $\boldsymbol{p}_n$ is the linear momentum, $\boldsymbol{L}_n$ is the angular momentum of the planet and $\boldsymbol{v}_n$ is the orbital velocity of the planet (note that

$\boldsymbol{v}_n \times \boldsymbol{p}_n = 0$). Also, we assumed that the centre of mass between the nth planet and the Sun is the Sun's centre (since $M \gg m_n$). Solving for the above equation yields

$$\boldsymbol{L}_n = m_n(\boldsymbol{l}_n \times \boldsymbol{v}_n) = \boldsymbol{C}_n,$$

where $\boldsymbol{C}_n$ is a constant. Since the mass of the planet is constant, the specific angular momentum $\boldsymbol{J}_n$ can be expressed as

$$\boldsymbol{J}_n = \boldsymbol{l}_n \times \boldsymbol{v}_n = c_n.$$

At the distance of semi-latus rectum, taking the direction of the velocity of the orbit to be perpendicular to the radius of the orbit, we can write the magnitudes of the above equation as

$$l_n v_n = c_n, \tag{3}$$

where c_n is a constant. Expressing the constant in the above equation in terms of the specific angular momentum J_0 of Mercury (where $J_0 = l_0 v_0$) and the orbital number N defined in equations (1) and (2) yields

$$v_n^2 l_n^2 = N^2 l_0^2 v_0^2. \tag{4}$$

According to Newton's laws, the condition for a planet revolving around the Sun is

$$\frac{m_n v_n^2}{l_n} = \frac{GMm_n}{l_n^2}$$

or

$$v_n^2 l_n^2 = GMl_n, \tag{5}$$

where G is the gravitational constant. By substituting equation (5) into equation (4), we obtain

$$l_n = N^2 l_0. \tag{6}$$

Now, let us express the semi-latus rectum of the orbit in terms of the semi-major axis a_n and the eccentricity e_n of the orbit, that is

$$l_n = a_n(1 - e_n^2). \tag{7}$$

Since the planetary orbits are not exactly coplanar, we project the above equation into the actual orbit by multiplying it by the secant of the inclination i_n:

$$l_n = a_n(1 - e_n^2)\sec(i_n). \tag{8}$$

Substituting equations (2) and (8) into equation (6) yields

$$a_n(1 - e_n^2)\sec(i_n) = a_0(1 - e_0^2)\sec(i_0)\left(1 + \frac{k(1 - e_n)n^{\ln(n)}}{1 - e_0^2}\right)$$

or, in terms of the semi-major axis a_n,

$$a_n = a_0\left(\frac{1 - e_0^2}{1 - e_n^2} + \frac{kn^{\ln(n)}}{1 + e_n}\right)\frac{\cos(i_n)}{\cos(i_0)}, \tag{9}$$

where $k = 0$ if $n = 0$, and $k = 1$ if $n > 0$. a_0, e_0 and i_0 are the semi-major axis, eccentricity and inclination respectively of the planet Mercury.

Since the eccentricity can take different values for the same quantum number n, the orbits degenerate; this is analogous to Sommerfeld's model for the hydrogen atom [5].

4. Results

Table 1 shows the observed values and the theoretically calculated values of the nearly circular semi-major axis of the orbits in the Solar System. It should be noted that the percentage difference between the theoretical and observed values for all orbits but that of Uranus is less than 5%. Since we have observed asteroids as far as orbital number $n = 17$ (table 2) (note that $n = 0$ for Mercury's orbit), the present author has given some mythical names to the orbits of the asteroid belts or planets yet to be found. After $n = 8$ for Neptune, the author has named the asteroid belts on undiscovered planets as follows: $n = 9$, Plutinos (asteroids named after Pluto with nearly circular orbits); $n = 10$, Mithra (the ancient Persian god of wide pastures who has 1000 ears and 10 000 eyes) [8]; $n = 11$, Nut (the ancient Egyptian sky god) [9]; $n = 12$, Loki (the Viking manipulator god) [10].

We noticed that for every nearly circular orbit in the Solar System there are many asteroids that belong to the same orbit but have very eccentric orbits. Some of these asteroids, however,

Table 1. The observed and the theoretically calculated semi-major axes of the nearly circular orbits.

Orbit number	Orbital name	Eccentricity	Inclination (deg)	Observed a (AU)	Theoretical a (AU)	Difference (%)
0	Mercury	0.206[a]	7.00[a]	0.387[a]	0.387	0.0
1	Venus	0.007[a]	3.39[a]	0.723[a]	0.760	5.0
2	Earth	0.017[a]	0.00[a]	1.000[a]	0.994	0.6
3	Mars	0.093[a]	1.85[a]	1.524[a]	1.569	2.9
4	Ceres	0.079[a]	10.58[a]	2.767[a]	2.798	1.1
5	Jupiter	0.048[a]	1.31[a]	5.203[a]	5.333	2.5
6	Saturn	0.054[a]	2.48[a]	9.537[a]	9.537	0.0
7	Uranus	0.047[a]	0.77[a]	19.191[a]	16.799	12.5
8	Neptune	0.009[a]	1.77[a]	30.069[a]	29.553	1.7
9	Plutinos	0.068 ± 0.056[b]	6.1 ± 6.8[b]	44.236 ± 1.285[b]	45.738	3.4
10	Mithra	0.039	1.65		75.7	
11	Nut	0.039	1.65		118	
12	Loki	0.039	1.65		181	

[a]From [6].
[b]From [7].

Table 2. The observed (mean ± standard deviation (SD)) and the theoretically calculated semi-major axes of the asteroid belts.

n	Known asteroid belt	Known number of asteroids[a]	Mean eccentricity ± SD[a]	Mean inclination ± SD[a]	Mean semi-major axis ± SD[a]	Theoretical semi-major axis	Difference (%)
1	Venoid	26	0.456 ± 0.102	12.1 ± 6.8	0.779 ± 0.050	0.723	7.2
2	Earthoid	68	0.362 ± 0.115	14.2 ± 10.5	0.856 ± 0.034	0.866	1.1
3	Marsoid	401	0.483 ± 0.166	15.2 ± 11.1	1.407 ± 0.234	1.319	6.3
4	Asteroid	362	0.635 ± 0.103	14.3 ± 13.6	2.195 ± 0.304	2.186	0.4
5	Jupitoid	51	0.619 ± 0.113	28.1 ± 17.5	3.163 ± 0.595	3.368	6.5
6	Saturnoid	6	0.446 ± 0.203	11.3 ± 4.8	7.260 ± 0.880	7.014	3.4
7	Uranoid	8	0.557 ± 0.175	17.9 ± 17.7	11.036 ± 1.883	11.028	0.1
8	Neptunoid	16	0.322 ± 0.131	12.3 ± 8.3	22.937 ± 1.876	22.168	3.4
9	Plutoid	156	0.199 ± 0.071	10.8 ± 8.0	39.387 ± 0.672	40.301	2.3
10	Mithroid	37	0.382 ± 0.70	13.1 ± 8.6	55.064 ± 3.160	55.596	1.0
11	Nutoid	13	0.583 ± 0.079	13.3 ± 8.8	77.058 ± 6.457	75.875	1.5
12	Lokioid	9	0.698 ± 0.102	21.2 ± 10.9	101 ± 12	104	2.4

[a]From [7].

Table 3. The observed and the theoretical semi-major axes of some minor planets and Trojan asteroids.

Orbital number	Minor planet	Eccentricity[a]	Inclination[a] (deg)	Observed[a] a (AU)	Theoretical a (AU)	Difference (%)
4	Ceres	0.079	10.6	2.767	2.798	1.1
5	Jupiter Trojans[b]	0.073	12.2	5.201	5.104	1.9
7	Chiran	0.383	6.9	13.704	12.780	6.7
9	Pluto	0.244	17.2	39.236	37.793	3.7
9	Quaoar	0.037	8.0	43.370	46.881	8.1
16	Sedna	0.859	11.9	538	449	16.5

[a]From [7].
[b]Jupiter Trojans: 1594 asteroids; $e = 0.073 \pm 0.041$, $i = 12.2° \pm 7.7°$, $a = 5.201 \pm 1.9$ AU [6].

Table 4. The observed and the theoretically calculated semi-major axes of some comets.

Orbital number	Comet	Eccentricity[a]	Inclination[a] (deg)	Observed[a] a (AU)	Theoretical a (AU)	Difference (%)
3	Encke	0.847	11.8	2.21	1.98	10.2
5	D'arrest	0.614	19.5	3.49	3.60	3.2
5	Borrelly	0.624	30.3	3.61	3.29	8.8
5	Giacobini	0.706	31.8	3.52	3.22	8.4
5	Churyumov–Gerasimenko	0.632	7.1	3.51	3.78	7.7
5	West	0.540	30.5	3.45	3.36	2.5
7	Crommelin	0.919	29.0	9.20	9.94	8.1
7	Tempel–Tuttle	0.906	17.5	10.33	10.60	2.6
8	Halley	0.967	17.8	17.94	19.73	10.0
19	Hyakutake	0.999784	55.1	1165	1145	1.7

[a]From [7].

have similar semi-major axes and eccentricities for their orbits and can be lumped into a so-called asteroid belt. We observed that almost all planets have at least one asteroid belt that share their orbits with the more circular planetary orbits (two orbits could belong to the same orbital number n, even though their semi-major axes are different. This is due to the difference in the eccentricities and inclinations of the bodies within that orbital number). Table 2 shows these asteroid belts. The names chosen for these groups of asteroids are the same as the planets belonging to the same orbit except that the suffix–oid is added to resemble the word asteroid. Again, it should be noted that the percentage difference between the average semi-major axes of these asteroids and the theoretical values for the most part are less than 5%. The list of some asteroid belts are given in the Appendix, tables A1–A9; since the list of Marsoid, asteroid and Plutoid belts as well as Plutinos are too long, the author has not provided these lists in this paper.

Tables 3 and 4 show some of the most popular and well-known minor planets and comets respectively.

5. Discussion

Even though the large majority of the asteroids (over 3500) fit their given orbits very well, there are some asteroids that do not appear to fit a given orbit with an acceptable percentage difference between their observed semi-major axis and the theoretical values. This is mainly due to the movement of the asteroids from a less-bounded gravitational orbit to a more favorite orbit because of the stronger presence of the gravitational attraction. The majority of these asteroids are in the asteroid belt where they are attracted to the orbit of Mars (nearby orbit but weaker gravitationally) and Jupiter's orbit (farther, but stronger gravitationally).

Comets in our Solar System also can be categorized into orbits. From the 300 comets examined [7], 209 of the comets (70%) have low calculation errors. The majority of these comets belong to Jupiter's orbit (65%) and Saturn's orbit (15%). Also 91 comets (30%) fit the orbits with a higher calculated error (more than 15%). These comets are in transition from their less gravitationally attractive orbit to another stable and more gravitationally attractive orbit. Just like some of the asteroids that are in orbital transition from the asteroid belt to Jupiter's orbit, 77% of the unfitted comets are on either side of Jupiter's orbit and are in orbital transition to Jupiter's orbit. So, if we consider the comets that belong to Jupiter's orbit, and those that are in transition to Jupiter's orbit, we find that approximately 70% of comets are linked to Jupiter's orbit. Clearly Jupiter's gravity is in control of the majority of known comets.

6. Conclusion

The theory presented in this paper would in no way indicate whether a planet exits in a given orbit. However, it will give the position of an orbit for given values of the eccentricity and the inclination. From the pattern in the Solar System, one can predict that, apart from all the eccentric and incline orbits, in each orbital number n, there exits a near circular stable orbit. In these nearly circular orbits, there may either be an asteroid belt (such as the orbits of Plutinos) or a planet. Here, the author has taken the average values of the eccentricities and the inclinations of the circular orbits and applied them to find the semi-major axis of some of these planets and asteroid belts. These values are given in table 1. So, perhaps one of the reasons that no planets have been found yet beyond the orbit of Pluto (if we consider Pluto as a planet) is because the next orbit is close to twice the distance of Pluto from the Sun.

It would be of interest to test the theory on other planetary systems. Since most newly discovered planetary systems are planets of Jupiter's size, one can use this theory to predict the orbits of the smaller bodies (particularly around the inhabitable zone) within those planetary systems.

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References

[1] J. Bennett, M. Donahue, N. Schneider and M. Voit, *The Cosmic Prospective*, 2nd edition (Addison-Wesley, San Francisco, CA, 2002), pp. 197, 122, 123, 136.
[2] R. Malhotra, M. Holman and T. Ito, Proc. Natn. Acad. Sci. USA **98** 12 342 (2001).
[3] L. Nottale, G. Schumacher and J. Gay, Astron. Astrophys. **322** 1018 (1997).
[4] A. Rubcic and J. Rubcic, Fizika B **7** 1 (1998).
[5] R. Eisberg and R. Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*, 2nd edition (Wiley, New York, 1985), pp. 114–115.
[6] National Space Science Data Center, NASA, http://nssdc.gsfc.nasa.gov/ (2001).
[7] Minor Planet Center, IAU, http://cfa-www.harvard.edu/cfa/ps/mpc.html (2004).
[8] R. Castleden, *An Exploration of Life in Neolithic Britain 4700–2000 BC* (Routledge, London, 1990), p. 236.
[9] K. Hart, *Ritual* (Cambridge University Press, Cambridge, 1999), p. 157.
[10] D. Miller, *The Epic Hero* (The John Hopkins University Press, Baltimore, MD, 2000), p. 243.

Appendix

List of some asteroid belts are given in tables A1–A9.

Table A1. Members of the Venoid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] a (AU)	Theoretical a (AU)	Difference (%)
2003 FK1	23.4	0.486	0.707	0.690	2.5
2001 CP36	10.6	0.407	0.714	0.712	0.2
2002 VE68	9.0	0.411	0.724	0.717	1.0
2001 CK32	8.1	0.383	0.725	0.713	1.7
2003 KO2	23.5	0.511	0.727	0.700	3.7
1989 VA	28.8	0.595	0.729	0.721	1.1
2000 BM19	6.9	0.359	0.741	0.711	4.1
1998 XX2	6.9	0.366	0.742	0.712	4.1
2000 AZ93	8.6	0.360	0.747	0.708	5.2
2001 WF49	18.2	0.373	0.751	0.682	9.2
1994 WR12	6.8	0.397	0.757	0.717	5.2
2002 GQ	10.7	0.378	0.767	0.706	7.9
2002 JX8	4.3	0.306	0.771	0.709	8.1
2003 TL4	12.1	0.382	0.776	0.704	9.3
2004 FH	3.5	0.319	0.780	0.710	8.9
2003 UC20	3.8	0.337	0.781	0.712	8.9
1998 TU3	5.4	0.484	0.787	0.747	5.0
2003 NZ6	18.4	0.496	0.793	0.717	9.5
2000 SP43	10.4	0.467	0.811	0.731	9.8
2000 EM26	3.9	0.470	0.816	0.743	8.9
1999 YK5	16.7	0.558	0.829	0.759	8.4
2000 ED14	13.8	0.567	0.835	0.776	7.0
1999 LT7	9.1	0.572	0.855	0.793	7.2
2000 SY2	19.2	0.643	0.859	0.826	3.9
2000 FO10	14.3	0.595	0.859	0.797	7.2
1999 JD6	17.0	0.633	0.883	0.824	6.6

[a]From [7].

Table A2. Members of the Earthoid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] a (AU)	Theoretical a (AU)	Difference (%)
2003 CP20	25.6	0.322	0.741	0.806	8.8
2000 GD2	32.1	0.477	0.758	0.771	1.8
2002 AY1	29.9	0.438	0.779	0.781	0.2
2001 XU1	27.2	0.546	0.797	0.836	4.9
2002 UA31	30.7	0.487	0.799	0.786	1.7
2002 XS90	34.1	0.242	0.809	0.749	7.4
2003 HT42	4.9	0.262	0.815	0.897	10.1
2000 HB24	2.7	0.430	0.816	0.898	10.1
1999 HF1	25.7	0.462	0.819	0.817	0.3
1998 ST27	21.0	0.530	0.819	0.870	6.2
2002 CC14	12.5	0.401	0.820	0.874	6.6
2001 BE10	17.5	0.369	0.823	0.852	3.5

(*continued*)

Table A2. Continued.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] a (AU)	Theoretical a (AU)	Difference (%)
2002 FW1	6.6	0.341	0.824	0.887	7.6
2002 RW25	1.3	0.286	0.825	0.897	8.7
2000 RH60	19.6	0.551	0.826	0.888	7.5
2003 SD220	8.5	0.210	0.828	0.902	8.9
1997 UH9	25.5	0.475	0.830	0.821	1.1
1978 RA	15.8	0.437	0.832	0.866	4.1
2002 VV17	9.7	0.437	0.837	0.888	6.0
2002 AB2	13.3	0.388	0.841	0.870	3.5
1976 UA	5.9	0.450	0.844	0.898	6.5
2002 LT38	6.2	0.314	0.845	0.889	5.2
2000 QP	34.7	0.463	0.847	0.745	12.0
2003 HB	18.1	0.381	0.850	0.849	0.1
1998 VF32	24.0	0.446	0.852	0.824	3.2
2000 AC6	4.7	0.286	0.853	0.894	4.8
2001 BB16	2.0	0.172	0.854	0.922	8.0
2000 WP19	7.7	0.289	0.854	0.889	4.1
2002 AU4	17.2	0.374	0.856	0.853	0.3
2003 LN6	0.6	0.210	0.857	0.912	6.4
2000 YS134	3.5	0.225	0.857	0.906	5.8
2002 DB4	16.6	0.370	0.858	0.856	0.3
2003 FU3	13.1	0.394	0.859	0.871	1.4
2000 CH59	3.3	0.423	0.863	0.897	3.9
2002 CW11	3.1	0.226	0.865	0.907	4.8
1997 NC1	16.7	0.209	0.866	0.874	0.9
2000 UH11	32.2	0.422	0.870	0.760	12.7
2003 GQ22	17.0	0.182	0.872	0.880	0.9
2001 XY10	31.0	0.387	0.872	0.766	12.1
2003 AF23	23.2	0.426	0.875	0.826	5.6
2001 HC	23.7	0.499	0.875	0.841	3.9
2001 TX44	15.2	0.546	0.875	0.907	3.7
2002 BN	27.8	0.547	0.875	0.832	4.9
1998 VR	21.8	0.318	0.876	0.830	5.2
2003 AK18	7.4	0.384	0.876	0.886	1.2
1993 VD	2.1	0.551	0.876	0.942	7.6
2002 NN4	5.4	0.434	0.877	0.896	2.2
2000 AF6	2.7	0.411	0.878	0.895	2.0
2003 YX1	5.8	0.267	0.879	0.895	1.9
1998 HE3	3.4	0.441	0.879	0.900	2.4
2000 WC1	17.4	0.263	0.880	0.859	2.3
2004 EU9	28.4	0.502	0.880	0.809	8.1
2002 AX1	33.0	0.542	0.880	0.787	10.6
2004 BY1	3.6	0.222	0.884	0.907	2.6
2000 UK11	0.8	0.248	0.885	0.903	2.1
2001 UP	7.7	0.287	0.885	0.889	0.4
2003 SW130	3.7	0.304	0.885	0.893	0.9
2004 DA53	5.1	0.329	0.885	0.890	0.5
2001 FO127	7.2	0.160	0.886	0.920	3.8
2003 RU11	4.6	0.183	0.889	0.917	3.1
2003 WT153	0.4	0.181	0.890	0.920	3.4
2003 GS	12.0	0.219	0.893	0.890	0.4
2000 EB14	11.7	0.499	0.896	0.899	0.3
1996 BG1	3.8	0.281	0.897	0.896	0.1
1998 DG16	16.2	0.358	0.897	0.857	4.4
2003 YR1	29.2	0.450	0.898	0.788	12.2
2004 ER21	8.0	0.170	0.899	0.915	1.7
2002 JW15	11.8	0.266	0.899	0.881	2.0

[a]From [7].

Table A3. Members of the Jupitoid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] *a* (AU)	Theoretical *a* (AU)	Difference (%)
1997 MS	55	0.728	1.938	2.182	12.6
1998 KK56	25.7	0.506	3.185	3.564	11.9
1998 KO3	54.5	0.771	2.596	2.240	13.7
1998 XM4	62.7	0.417	1.657	1.891	14.1
2000 FL1	43.5	0.521	2.715	2.852	5.0
2000 KB	56.3	0.798	2.336	2.175	6.9
2000 KE41	50.4	0.865	3	2.723	9.2
2000 YG29	18.9	0.695	3.166	3.586	13.3
2001 RA42	22.1	0.532	3.231	3.628	12.3
2001 SK276	20.7	0.591	3.195	3.594	12.5
2001 UO16	25.2	0.53	3.263	3.545	8.7
2001 WN15	57.1	0.833	2.274	2.204	3.1
2001 XP1	39.3	0.751	2.895	2.961	2.3
2002 AB29	46.5	0.758	2.534	2.641	4.2
2002 JB9	46.7	0.785	2.717	2.665	1.9
2002 JC68	28.7	0.54	3.019	3.424	13.4
2002 KG4	27.6	0.663	2.941	3.362	14.3
2002 TW55	59.4	0.664	2.117	1.931	8.8
2002 WZ2	51.4	0.884	2.46	2.788	13.3
2003 DA16	45.7	0.625	2.593	2.663	2.7
2003 JC11	26.5	0.559	3.055	3.471	13.6
2003 KP2	45.1	0.7	2.737	2.676	2.2
2003 SJ5	39.1	0.509	2.677	3.066	14.5
2003 WB8	20.2	0.551	3.253	3.650	12.2
2004 BB103	55.9	0.622	1.907	2.139	12.2
1984 BC	21.4	0.534	3.495	3.643	4.2
1984 WE1	19.8	0.505	3.645	3.723	2.1
1992 AB	40.8	0.554	3.282	2.941	10.4
1992 EB1	21.5	0.57	3.381	3.597	6.4
1994 JC	31	0.515	3.374	3.378	0.1
1998 BC34	17.2	0.562	3.314	3.702	11.7
2000 BK2	6.6	0.558	3.525	3.854	9.3
2000 CA13	1.4	0.563	3.714	3.873	4.3
2000 GQ132	30.1	0.531	3.265	3.389	3.8
2000 KD41	5.5	0.589	3.369	3.827	13.6
2000 OG44	7.3	0.581	3.87	3.822	1.2
2000 SB1	22.2	0.54	3.344	3.615	8.1
2000 XO8	12.1	0.662	4.147	3.710	10.5
2000 YN30	22.4	0.58	3.922	3.563	9.1
2001 TX16	8.1	0.598	3.585	3.797	5.9
2001 WX1	23.3	0.572	3.79	3.548	6.4
2002 KJ8	4.9	0.593	3.369	3.827	13.6
2002 LJ27	17.7	0.5	3.353	3.777	12.7
2002 RQ28	9.4	0.568	3.716	3.816	2.7
2002 VP94	13.8	0.62	4.003	3.707	7.4
2003 BM1	11.7	0.529	3.942	3.838	2.6
2003 BU35	14.9	0.536	3.74	3.778	1.0
2003 UL12	19.7	0.698	3.303	3.569	8.1
2003 UR267	8.4	0.506	3.85	3.913	1.6
2003 WR25	9	0.71	3.354	3.748	11.7
5025 P-L	6.2	0.895	4.201	4.594	9.4

[a]From [7].

Table A4. Members of the Saturnoid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] *a* (AU)	Theoretical *a* (AU)	Difference (%)
2000 QJ46	4.4	0.673	5.839	6.443	10.3
2000 VU2	13.8	0.553	6.918	6.568	5.1
1998 HO121	12	0.587	7.119	6.516	8.5
2003 CC22	6.4	0.432	7.393	7.166	3.1
2000 GM137	15.8	0.122	7.888	8.656	9.7
1998 SG35	15.6	0.309	8.4	7.511	10.6

[a]From [7].

Table A5. Members of the Uranoid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] *a* (AU)	Theoretical *a* (AU)	Difference (%)
2004 DA62	52.2	0.471	7.767	7.461	3.9
1999 RG33	34.9	0.772	9.386	8.720	7.1
2001 YK61	12.3	0.694	10.683	10.625	0.5
2000 EC98	4.3	0.455	10.759	12.259	13.9
1998 QJ1	23.5	0.813	11.263	9.711	13.8
1999 UG5	5.3	0.386	11.819	12.795	8.3
2004 CJ39	3.6	0.482	12.959	12.069	6.9
1977 UB	6.9	0.382	13.654	12.791	6.3

[a]From [7].

Table A6. Members of the Neptunoid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] *a* (AU)	Theoretical *a* (AU)	Difference (%)
1992 AD	24.7	0.574	20.423	17.501	14.3
2002 CA249	6.4	0.385	20.713	21.563	4.1
2003 CO1	19.7	0.478	20.955	19.211	8.3
2003 QP112	31.2	0.329	21.129	19.309	8.6
1999 HD12	10.1	0.583	21.322	18.869	11.5
2003 UY292	8.6	0.272	21.864	23.286	6.5
2003 QC112	16.7	0.213	22.046	23.625	7.2
2002 DH5	22.4	0.37	22.211	20.271	8.7
2002 VR130	3.5	0.35	23.063	22.195	3.8
2002 GZ32	15	0.221	23.188	23.672	2.1
1996 RX33	9.4	0.204	23.868	24.511	2.7
2000 CO104	3.1	0.151	24.266	25.926	6.8
2003 QN112	7.9	0.333	25.115	22.295	11.2
1995 DW2	4.1	0.25	25.15	23.892	5.0
2002 PQ152	9.4	0.196	25.609	24.671	3.7
2001 KF77	4.4	0.239	26.062	24.089	7.6

[a]From [7].

Table A7. Members of the Mithroid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] *a* (AU)	Theoretical *a* (AU)	Difference (%)
2003 UY117	7.5	0.412	55.248	55.412	0.3
2000 FE8	5.9	0.408	55.937	55.749	0.3
2002 CZ248	5.2	0.382	56.634	56.846	0.4
2002 GP32	1.6	0.426	55.767	55.331	0.8
1998 WA31	9.5	0.427	55.034	54.556	0.9
2000 YW134	19.8	0.293	58.263	57.348	1.6
2000 SR331	4.3	0.44	55.557	54.671	1.6
2000 EE173	5.9	0.548	49.995	50.833	1.7
1999 DE9	7.6	0.424	56.121	54.942	2.1
2001 KC77	12.9	0.36	55.334	56.526	2.2
2002 GG32	14.7	0.346	55.233	56.666	2.6
2001 XQ254	7.1	0.443	55.803	54.295	2.7
2000 PE30	18.4	0.341	54.158	55.792	3.0
2002 CY224	15.7	0.352	54.405	56.151	3.2
1999 HB12	13.1	0.419	56.109	54.172	3.5
2002 GZ31	1.1	0.369	55.64	57.604	3.5
2002 JR146	13.1	0.382	53.398	55.596	4.1
2002 GX32	13.9	0.375	53.135	55.687	4.8
2003 QE112	4.2	0.42	52.538	55.432	5.5
2000 SM331	12	0.437	57.358	53.737	6.3
2003 QB92	3.4	0.57	53.735	50.326	6.3
2000 CQ105	19.6	0.395	57.476	53.281	7.3
2001 FM194	28.6	0.371	54.62	50.512	7.5
1999 CC158	18.7	0.278	54.299	58.403	7.6
2000 YC2	19.9	0.38	58.278	53.749	7.8
2002 TC302	35.1	0.293	55.197	49.867	9.7
2002 GA32	15.1	0.329	52.177	57.273	9.8
2001 KE77	20.6	0.332	50.324	55.405	10.1
1999 RJ215	19.7	0.411	58.878	52.656	10.6
2000 SU331	3.5	0.474	59.86	53.490	10.6
2000 CM114	19.7	0.409	60.134	52.729	12.3
1999 HW11	17.2	0.26	53.006	59.733	12.7
2001 KG76	1.5	0.344	51.796	58.649	13.2
1999 CV118	5.5	0.293	53.153	60.671	14.1
1998 XY95	6.7	0.425	64.7	55.012	15.0
2000 AF255	30.9	0.257	48.984	53.781	9.8
1999 KR16	24.8	0.306	49.076	54.786	11.6

[a]From [7].

Table A8. Members of the Nutoid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] *a* (AU)	Theoretical *a* (AU)	Difference (%)
2003 QZ91	27.4	0.652	64.52	66.424	3.0
2000 SQ331	5.4	0.707	69.807	72.203	3.4
2001 KV76	15.3	0.511	70.199	78.703	12.1
2002 CX154	15.9	0.48	72.821	80.087	10.0
2000 PF30	6.3	0.501	75.939	81.632	7.5
2001 FK194	8.8	0.633	76.325	74.764	2.0
2000 PH30	8.1	0.5	76.451	81.362	6.4
2001 KZ76	25.7	0.516	78.997	73.285	7.2
2003 QH91	3.6	0.713	80.467	72.144	10.3
2001 XT254	0.5	0.567	82.692	78.739	4.8
2000 CP105	19.4	0.585	83.164	73.450	11.7
1996 TL66	24	0.58	83.325	71.358	14.4
2001 FZ173	12.7	0.628	87.042	74.022	15.0

[a]From [7].

Table A9. Members of the Lokioid belt.

Name of asteroid	Eccentricity[a]	Inclination[a] (deg)	Observed[a] *a* (AU)	Theoretical *a* (AU)	Difference (%)
1999 DG8	40	0.598	82.215	90.275	9.8
1999 CY118	25.5	0.624	92.145	104.698	13.6
1999 TD10	6	0.872	95.703	101.102	5.6
2000 OM67	23.4	0.598	97.336	108.153	11.1
1999 RZ215	25.6	0.692	100	100.523	0.5
2002 GB32	15	0.624	100	112.046	12.0
2003 FX128	22.3	0.828	104	95.942	7.7
1999 CZ118	27.7	0.679	117	99.429	15.0
2000 PJ30	5.7	0.767	122	106.427	12.8

[a]From [7].