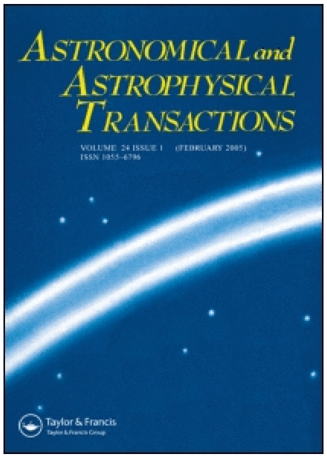

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A comparative study of the evolution of the geographical ideas and measurements until the time of Eratosthenes

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A comparative study of the evolution of the geographical ideas and measurements until the time of Eratosthenes

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We perform a comparative study of the evolution of the most important methods for geographical, cartographical and astronomical measurements developed by ancient Greek scientists and philosophers until the time of Eratosthenes. It seems that the novel geometrical method invented by Eratosthenes for the measurement of the size of the Earth did not appear suddenly but was the final outcome of long-lasting intellectual activity. It is shown that Anaximander, Pytheas, Eudoxus of Cnidus, Dicaearchus, Aristotle and Archimedes, the most famous ancient Greek philosopher astronomers and geographers before Eratosthenes, affected his thinking and contributed to his discovery. Furthermore, we briefly describe and explain the method of Eratosthenes, its significance, new errors that intruded into Eratosthenes' measurement, as well as its application for the determination of the shape and the size of the Earth.

Keywords: Geographical ideas; Geographical measurements; Eratosthenes

1. Introduction

Since very early ancient philosophers, geographers, astronomers and mathematicians using the gnomon have attempted to measure the geographical latitude of a place, the size of the Earth and the distances between various places in order to make a map of the then known world. The most well known among these is without doubt Eratosthenes. There were several researchers before him who contributed to the development of geography and cartography. Eratosthenes, by applying a clever geometric method, was the first who managed to measure the meridian of the Earth with precision. His method, which was again based on the gnomon, emerged from a gradual development of ideas and the use of the gnomon for various geographical and astronomical measurements.

In previous work, we studied and clarified the contribution of Posidonius of Rhodes to geography and cartography. Also, we analysed and explained, using the celestial sphere, his astronomical method for the measurement of a meridian of the Earth, mentioned the errors of his method and pointed out its paramount importance even today (see [1] (p. 406)).

The aim of the present paper is to perform a comparative study of the evolution of the geographical ideas and measurements developed by ancient Greek scientists and philosophers

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until the time of Eratosthenes. It seems that the novel geometrical method for the measurement of Earth introduced by Eratosthenes did not arise suddenly but was the final outcome of long-lasting intellectual activity. Anaximander, Pytheas, Eudoxus of Cnidus, Dicaearchus, Aristotle and Archimedes the most famous of the philosophers, geographers and astronomers, were the predecessors of Eratosthenes, who affected his thinking and contributed to Eratosthenes' discovery. Finally, we briefly describe and explain the method of Eratosthenes for measuring the shape and the size of the Earth, as well as the errors inherent in Eratosthenes' measurement and we point out the applications of his method to cartography, geography and astronomy.

2. The evolution of the ancient Greek geographical ideas and measurements

2.1 *Anaximander of Miletus*

Anaximander (611–546 BC) was a philosopher, geographer, cartographer and astronomer. He was a student and successor of Thales of Miletus (Suida). He is considered as the first cartographer, since he was the first who attempted to draw a map of the entire known world (*oicoumene, οἰκουμέενη*) at that time, in which the dimensions of land and sea were given. Strabo [2] (1.1.1) mentioned Anaximander among the eminent geographers. A synopsis of what various ancient authors wrote about him follows.

Diogenes Laertius [3] (2.1-2) (see also [4]) said that 'he [Anaximander] discovered the gnomon, which he set up on *skiatheron* [sundials] in Lacaedaimona [Sparta], and which showed *tropes* [solstices] and *isemeries* [equinoxes]. He also constructed sundial'. Certain authors, however, have disputed this (see [5] (p. 61)). Diogenes also wrote that '[Anaximander] first evaluated the perimeter of land and sea and also constructed the sphere'.

The geographer, philosopher and writer Agathemerus [6] (I, 1) (3rd century AD) (see also [7]) agreed that he was the first who dared to draw a map of oicoumene.

Simplicius in his book *Comments on Aristotle's 'On the Heaven'* (*Σχόλια εὶς τό περί Οὺρανοῦ τοῦ Ἀριστοτέλους*) [8] (II, 10, 1, p. 471) (see also [9]) wrote: 'Anaximander first measured sizes and distances of the planets from the Earth.'

Finally, in Suida's [10] dictionary we find: 'He discovered the equinox and solstices and clocks . . . and the gnomon he imported and the whole outline of geometry he showed. He wrote many works, among which *On the Nature of Earth's Period*, and *On the Fixed Stars* and *The Celestial Sphere.*' Something similar has also been mentioned by Eusebius [11] (Praeparatio evan. x, 14, 11).

Dicks [12] (p. 20) claimed that Anaximander was the first important geographer who draw the first map of the world and had knowledge of the oblique course of the Sun and the Moon (p. 23). Also Heath [13] (Vol. I, p. 139) wrote that Anaximander used the gnomon to determine the solstices. According to certain authors (see, for example, [13] (Vol. I, pp. 139 and 140) and [14] (Vol. I, paragraph 41)), Anaximander was also a mathematician. Many of the contemporary authors considered him as the first mathematical physicist with mathematical intuition, although not all of them shared this opinion (see, for example, [5] (p. 60)). The use of the gnomon which was crucial in Anaximander's work necessitated the knowledge of mathematics.

2.2 *Aristotles' figure*

The most ancient attempt to estimate the Earth's circumference that we know of is that mentioned by Aristotle (384–322 BC) in his book *Περί Οὺρανοῦ* [15] (B, XIV, 15, 298α).

In this book he wrote: '*Καί τῶν μαθηματικῶν δέ ὅσοι τό μέγεθος ἀναλογίζεσθαι πειρῶνται τῆς περιφερείας είς τετταράκοντα λέγουσιν εἶναι μυριάδας.'Εξ ὧν τεκμαιρομένοις οὐ μόνον* ... (And those of the mathematicians who try to calculate the length of the circumference of the Earth say that it is forty myriads (400 000) *stadia* (stades). From these they assume that not only ....' If we accept the length of a *stadium* (stade) as equal to the Olympic stade, that is 185 m (the stade of Eratosthenes of 157.5 m was not known at that time), then 400 000 stades × 185 m = 74 000 km. The value given by Aristotle is almost twice the real value, which is 40 074 km at the equator and 39 942 km at the poles (the length of the meridian).

Aristotle did not mention where he found this value. In his text he did not say whether this estimation (calculation) had been made by a mathematician or not, nor did he mention any name.

We note that Aristotle was contemporary to two eminent mathematicians, astronomers and philosophers, Archytas of Tarentum (440–360 BC) and Eudoxus of Cnidus (408–355 BC). Horace [16] in a verse of his book (I, ode 28, 1–6), which reads

Te maris et terrae numeroque carentis harenae
mensorem cohibent, Archyta,
pulveris exigui prope latum parva Matinum
munera nec quidquam tibi prodest
aerias temptasse domos animoque rotundum
percurrisse polum morituro,

mentioned Archytas as a 'measurer of the Earth and sea' and in his verses quoted the astronomical research of Archytas. Archytas made important discoveries in astronomy and mathematics. Eudoxus of Cnidus was a student of Plato and, like Archytas, he wrote many books and made many important discoveries in astronomy and mathematics. Therefore the number given by Aristotle could be attributed to one of them. However, in his text, Aristotle did not attribute the value to one mathematician but to more than one and for this reason he did not mention any name. We could therefore make the conjecture that it was both these two mathematicians who adopted the value of 400 000 stades and who were implied by Aristotle.

2.3 *The basic idea for the measurement*

The methods developed by Dicaearchus, Eratosthenes and Posidonius of Rhodes for the determination of the size of the Earth are based on the following idea.

The initial rough assumption is that the Earth has the shape of a sphere and that two places T_1 and T_2 are found on the same terrestrial meridian. Also, the distance between the two places T_1 and T_2 is known (in stades or kilometres). The vertical lines $KT_1\Sigma_1$ and $KT_2\Sigma_2$ that pass from these places form an epicentric angle ω which is equal to the difference in the geographical latitudes of the two places. These lines also define the celestial meridian passing from the points Σ_1 and Σ_2, which is precisely above the terrestrial meridian (figure 1).

From a well-known theorem of geometry we know that the ratio of two arcs of two concentric circles which correspond to the same epicentric angle (or in equal epicentric angles of two different circles) is equal to the ratio of the respective radii. Thus, the following equality holds:

$$\frac{\mathrm{arc}(\mathrm{T}_1\mathrm{T}_2)}{2\pi R_{\mathrm{E}}} = \frac{<(\Sigma_1\Sigma_2)^\circ}{\text{celestial meridian} = 360^\circ}, \tag{1}$$

where R_{E} is the radius of the Earth.

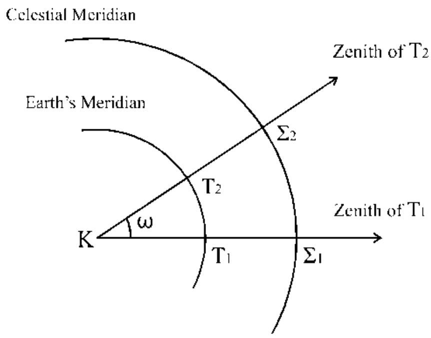

Figure 1. The angle ω between T_1 and T_2 as measured from the centre of the Earth K. $\Sigma_1\Sigma_2$ is an arc of the celestial meridian and T_1T_2 is an arc of the Earth's meridian.

If we calculate in some way the distance $\Sigma_1\Sigma_2$ (in degrees), that is the epicentric angle ω, then, provided that the distance T_1T_2 is known, we can immediately find from equation (1) the length of the terrestrial meridian (that is the Earth's radius and circumference). An estimate for the arc $\Sigma_1\Sigma_2$ was not easy in antiquity because one should find two stars Σ_1 and Σ_2 which pass through the vertical (zenith) of the two places T_1 and T_2 respectively at the same time, something rather rarely observed. In order to overcome this difficulty, various methods had been developed by Dicaearchus, Eratosthenes and Posidonius.

2.4 *The use of a gnomon*

The gnomon in its original form (before its use in sundials) was a vertical (upright) stick (see [13] (Vol. I, p. 78)) directed along the force of gravity and consequently vertical to the Earth's surface; the Earth had been considered to be spherical since the time of Pythagoras. Anaximander was the first to introduce the gnomon (see [3] (2.1-2) and [10]). From very early days, this device had been used for the measurement of the geographical latitude of a place, for the determination of places of the same latitude and the mapping of parallels and for the measurement of the obliquity of the ecliptic (see [17] and [18] (Synt. ii, 6), see also [19] (pp. 43–45, 725–727 and 746–748)).

In a particular place the ratio of the length of the gnomon's shadow on the soil at midday (on the day) of the equinox (the upper culmination of the Sun during the equinox) to the gnomon's length gives the tangent of the angle β (see figure 2 later) between the gnomon and the direction of the solar rays. This angle equals the geographical latitude of the place because at the equinox the Sun is on the celestial equator and the solar rays directed to the Earth's surface are parallel to the level of the celestial and terrestrial equator.

By measuring the shadows of the gnomons in two cities at the same time and, provided that the ratios of the lengths of the shadows to the respective heights of the gnomons were equal, ancient astronomers managed to determine places on the same geographical parallel.

For the calculation of the obliquity of the ecliptic they measured at the upper culmination of the Sun the ratio of the length of the shadow to the height of the stick when the Sun was at the summer and winter solstices respectively. Then they determined the corresponding angles β (figure 2). Consequently, they were able to find the two angles formed between the direction of the solar rays at the equinox and the direction of the solar rays during the two solstices which are equal and finally to determine the obliquity of the ecliptic. Strabo is one of the main sources of geographical information.

Eudoxus of Cnidus used the gnomon for several observations; for example, he determined the longest day at certain geographical latitudes. Also Pytheas, who was a navigator, geographer and astronomer (see [20] (p. 150)) contemporary to Aristotle and Alexander the Great, was the first to measure with accuracy the geographical latitude of Marseilles (Massilia). When the Sun was at its zenith at the summer solstice (the summer tropic), the solar rays were parallel to the ecliptic plane. So at places of the summer tropic the gnomon did not throw a shadow, while in Marseilles the solar rays formed an angle of $19^\circ\ 12'$ with it (see [2] (2.5.8, 1.4.c63, 2.5.41), [12] (p. 179) and [20] (p. 150)). Thus, the difference between the latitudes of Marseilles and the summer solstice (the summer tropic) was $\omega = 19^\circ\ 12'$ instead of $19^\circ\ 50'$, which is the true current value. If we take into consideration that the obliquity of the ecliptic at that time was $23^\circ\ 43'$, then the geographical latitude of Marseilles calculated by Pytheas was $42^\circ\ 55'$. Compared with the ancient estimate of 24°, which was at that time acceptable, the latitude was $43^\circ\ 12'$. According to Eratosthenes' value of $23^\circ\ 51'$ the corresponding latitude was $43^\circ\ 03'$ (see [12] (p. 178)). The actual modern value is $43^\circ\ 17'$. Pytheas also found the relation between the geographical latitude and the duration of the longest day, as well as the altitude of the Sun at the winter solstice. His observations and knowledge led to the first use of geographical parallels of latitude for the determination of places where the same astronomical phenomena can be observed. Thus, he set the basis of the scientific use of parallels in cartography (see [20] (p. 151)).

Dicaearchus' contribution to geography and cartography was also important since he was the first to determine the first parallel of the Earth. This parallel was a reference line from east to west. He had the idea of drawing a parallel line to the equator, which he considered as the main parallel of latitude bisecting the known world of that time. This parallel passed through the Straits of Gibraltar and of Messina (Messana), from Athens, Rhodes and the mountains of Taurus (see [2] (2.5.42, 2.1.24, 2.1.1), [12] (p. 30) and [20] (p. 152)).

Eratosthenes studied the work of Eudoxus, Dicaearchus, Pytheas, Aristotle, Archimedes and others, where he found much information. Using the gnomon, he estimated the latitudes of various places and determined the geographical parallels of the Earth. He then used these measurements to construct a map of the world. Eratosthenes also used the gnomon for the measurement of the obliquity of the ecliptic, which he found to be equal to $23^\circ\ 51'$ (see [17] (Synt. i, 22, 67) and [12] (p. 40)) (see also [21]). Before Eratosthenes, the obliquity of the ecliptic was considered to be equal to 24°. This value is quoted by Eudemus of Rhodes in his book *History of Astronomy* (see [22] (p. 1980) and [23] (p. 241)). This fragment leaves us with uncertainty as to whether Oenopides of Chios (who was slightly later than Anaxagoras, around 500–430 BC) was the one who found the obliquity of the ecliptic to be equal to 24°. This value was known at the time of Eudoxus of Cnidus. Heath [13] (Vol. I, p. 174) and [24] (p. 131) reported that it does not seem that Oenopides actually measured the obliquity of the ecliptic. Dicks [5] (p. 214) wrote that no true value is attributed to Oenopides for the obliquity of the ecliptic (Eratosthenes was the first to conduct a precise measurement) but it is likely that the approximate value of 24° originated from Oenopides. Others (see, for example, [25] (41,7)) have reported that it was Oenopides who discovered the ecliptic as the oblique Sun's path and who possibly knew that the obliquity of the ecliptic was approximately 24°.

2.5 *Archimedes' figure – Dicaearchus*

Archimedes (287–212 BC) mentioned that the length of the meridian of the Earth is 300 000 stades without, however, mentioning in particular the source from which he took this value (see [26] (Vol. II, p. 246)). There has been some discussion with regard to the origin of this figure, which is likely to belong to Dicaearchus (see [27] (p. 270)).

The following arguments support the conjecture that it was Dicaearchus who calculated this value.

(1) Dicaearchus (350–285 BC) came from Messana (Messina), Sicily. He spent the larger part of his life in the Peloponnese and particularly in Sparta. He was a student of Aristotle and schoolmate of Theophrastus and Eudemus of Rhodes. He was a mathematician, astronomer, geographer, cartographer and philosopher. Many geographers and authors such as Strabo, Eratosthenes, Posidonius of Rhodes, Geminus, Polybius, Plutarch, Stephanus Byzantius, Agathemerus, Athenaeus, Pliny and others mention Dicaearchus in their books. The fact that they made comments on his results in geography justifies the importance of his work. Strabo [2] (1.1.1) classified Dicaearchus among the eminent geographers and philosophers who gave a strong impetus to the growth of geography. Strabo [2] (book 2, 104–106, 170) often referred to Dicaearchus and particularly to the calculations that he made in order to determine distances between various places in the Mediterranean. Polybius also referred to the measurements of Dicaearchus who is also known for his several treatises on philosophy, literature, history and politics. Most important, however, are his several geographical studies, as that on the period of the Earth (*Γῆς Περίοδος*), which contained geographical tables and a map and measurements of the mountains of Peloponnese [10]. Unfortunately, his work has been lost and later writers have rescued only extracts. Nevertheless, it is fair to say that Dicaearchus was the only renowned geographer and cartographer of the time of Archimedes (see [12] (pp. 30 and 122) and [20] (pp. 152–153)).

(2) Dicaearchus measured the height of many mountains of Peloponnese, but also of the rest of Greece, for example the height of Attavyros of Rhodes, which he found smaller than 8 stades = 1260 m, the heights of Kyllene, which he found to be 15 stades (see [2] and [28] (p. 171)), and of Pelion, which he found to be 381 m and which he considered insignificant compared with the size of the Earth (see [28] (ii, p. 162) and [30] (p. 173)).

(3) The foundation of the capital of the state of Thrace, Lysimachia, had taken place in 309–306 BC by the king of Thrace Lysimachos (see [31] (I, 9, 8)); hence the calculation was made later than that time.

(4) Another argument is provided by the information (see [30] (p. 173)) which we obtain from Cleomedes [32] (I, 8, p. 78) about the distance of the cities Lysimachia and Syene with respect to the constellations of Draco and Cancer respectively. According to Cleomedes, who obtained the information mainly from Posidonius, Dicaearchus should have assumed the following.

   (a) The cities Syene of Upper Egypt (near the modern city Aswan Dam) and Lysimachia of Thrace lie on the same meridian.
   (b) The head of the constellation Draco, which is formed by the stars β, γ, ξ and ν at the peaks of a quadrilateral, and particularly Gamma Draconis, which is the southernmost and most brilliant star of Draco's head, passes through Lysimachia's zenith, while Syene's zenith passes from the constellation Cancer.
   (c) The distance of the two cities was 20 000 stades according to *bematistai* (step measurers).
   (d) The difference between the declinations of these stars was considered to be 24° but it was not reported how this estimate was made. Consequently the arc of the celestial meridian between the two cities (figure 1) is $\Sigma_1\Sigma_2 = 24°$, that is one fifteenth of the entire celestial meridian. Therefore, the epicentric angle $\Sigma_1\hat{K}\Sigma_2 = 360°/15$ and thus the difference between the latitudes of Syene and Lysimacheia was one fifteenth of the

Earth's meridian. Consequently, from equation (1) it is found that

$$2\pi R_{\mathrm{E}} = 20\,000 \text{ stades} \times 15 = 300\,000 \text{ stades}.$$

The diameter of the Earth is, for $\pi = 3$, 100 000 stades. If we take into consideration that the Olympic stade was 185 m, then the meridian is 55 500 km and the radius 9250 km.

(5) For the calculation of the diameter of the Earth, the value of π was taken to be equal to 3. Archimedes was the first who approximated the value of π by a geometrical method between the limits $3\frac{10}{71} = 3.140\,845 < \pi < 3\frac{1}{7} = 3.142\,857$ (see [14] (paragraph 41, p. 74) and [13] (Vol. I, p. 232)) and that should not be dated before about 267 BC (when he should have been at least 20 years old). So the value of 100 000 stades or 300 000 stades, for the diameter or the circumference of the Earth respectively, should be dated before the calculation of the value of π by Archimedes. Therefore, the calculation of the circumference of the Earth should have been made between 309 BC and 267 BC approximately, the time when Dicaearchus was alive.

(6) According to Strabo [2] (2.4.2) and also Polybius, Dicaearchus was the first to calculate the distances of many places in the Mediterranean area, the knowledge of which was necessary for mapping the world. Also he was the first to trace a main parallel of latitude approximately bisecting the known world, which he used in his map. The calculation, therefore, of the size of the Earth as well as of the distances of various places was necessary for Dicaearchus in order to create his map.

2.5.1 Errors of the method. The current value for the length of the equator of the Earth is 40 074 km, while that of its meridian is 39 942 km. We observe, therefore, that the error is important, roughly 39%. This is due to the following main factors.

(1) The two cities were not on the same meridian.

(2) Their distance was not 20 000 stades. Eratosthenes, using his method, found that it was 13 000 stades (see [33] (p. 34)).

(3) The difference between the latitudes of the two cities was not 24° because, firstly, the latitude of Lysimachia was roughly 40° 36′ while the latitude of Syene was 24° 05′, giving a difference in their latitudes of 16° 31′, secondly, the declination of γ Draconis was at that time approximately 53° 11′ (see [30] (p. 174)) and, thirdly, Syene was not precisely on the Tropic of Cancer. As a consequence, the stars of the constellations of Cancer and Draco were not exactly at zenith as seen from Syene and Lysimacheia respectively. As far as the errors (a)–(d), which we mention in section 3.2 and were introduced into this estimation, their contribution is very small compared with the general error of 39%.

3.1 *The method of Eratosthenes of Cyrene*

Strabo considered Eratosthenes (276–194 BC) as the first eminent geographer of antiquity; in addition, it is well known that he was also a mathematician, astronomer, philosopher and an outstanding orator. He even occupied himself with a poetry and literature study – *philology*, a term that he invented himself. He had a wide knowledge; this is the reason why the king of Egypt Ptolemy III the Benefactor invited him to assume the directorship of the famous Library of Alexandria.

Eratosthenes wrote several works on geographical, astronomical, mathematical and philosophical topics but also contributed to poetry and literature. From his astronomical books the most well known is *Catasterismoi.* In one of his works he calculated the obliquity of the ecliptic without the use of telescope with an amazing accuracy, using the gnomon. He found the value 23° 51′ (see also [21]). At that time (about 230 BC) the real value was about 23° 43′, while today it is approximately 23° 27′. While he was the director of the library of Alexandria, he wrote two important works. The first was the *Measurement of the Size of the Earth*, and the other was *Geographica*; both have been lost. The second work consisted of three books and contained a description of the whole known world at his time. Fragments of this work have been saved in the writings of later geographers, such as Polybius, Hipparchus, Posidonius, Ptolemy, Strabo, Cleomedes and others, who found much information but also criticized it.

The method of Eratosthenes for the estimation of the size of the Earth was simple and brilliant (see [20] (p. 154)). As is evident from the preceding comments, the novel geometrical method invented by Eratosthenes did not appear suddenly, as a peculiar mathematical method, but as a result of the gradual development of ideas and geographical knowledge, as well as of the use of gnomon for geographical, astronomical and cartographic measurements until the time of Eratosthenes. Anaximander, Pytheas, Eudoxus of Cnidus, Dicaearchus, Aristotle and Archimedes, the most famous ancient Greek philosophers, geographers and astronomers, were the predecessors of Eratosthenes, who affected his thinking and contributed to Eratosthenes' discovery.

According to Cleomedes [32] (i, 10, p. 100), Eratosthenes assumed the following.

(i) The Earth is spherical.
(ii) Alexandria and Syene (a town in southern Egypt) were on the same terrestrial–celestial meridian and, since meridians are great circles in the sky, the circles of the Earth that lie directly under them are also great circles.
(iii) The distance between Syene and Alexandria is 5000 stades.
(iv) The rays of sunlight reaching various places on the surface of the Earth are parallel to each other; that is, the solar rays falling on Alexandria and Syene are parallel.
(v) Syene lies on the summer tropic, the Tropic of Cancer.
(vi) Straight lines intersecting parallel lines form equal opposing angles.
(vii) The arcs of circles corresponding to equal angles have the same length ratio to their respective circumferences.

Eratosthenes had the brilliant idea of measuring the length of the shadow of a vertical stick (a gnomon) in both Syene and Alexandria – the latter being about 800 km to the north of the former – at the local noon (the moment of the upper culmination of the Sun) of the summer solstice, in order to find the difference in the Sun's altitude at the two cities. He observed that the solar rays were falling vertically on the surface of the Earth in Syene, and therefore the Sun was at the zenith (figure 2), the end of the vertical line KS, which is equivalent to assumption (v). More specifically, he discovered that at noon and at the summer solstice the Sun threw no shadow in the city in an area of 300 stades diameter, while at the same time it was also culminating in Alexandria and its rays were forming an angle β with the gnomon, that is with the vertical line KA. By measuring the length of the stick's shadow and since the length of the stick was known, he determined the angle β. Hence, the Sun was to the south of the Alexandria's zenith by an angle β. This angle is presumably equal (assumption (vi)) to the angle ω formed by the vertical KS of Syene and the vertical KA of Alexandria, which in turn equals the difference between the geographical latitudes of the two cities: $\varphi_A - \varphi_S = \Delta\varphi = \omega$. KS and KA also define the celestial meridian passing over these cities, which lies exactly over

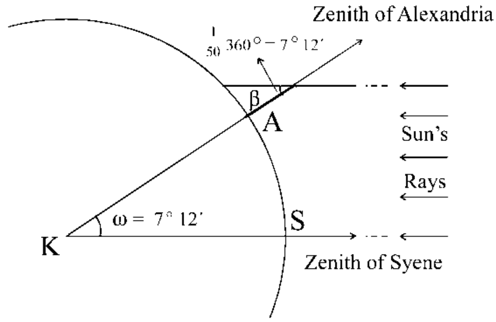

Figure 2. Eratosthenes' geometric method of measuring the circumference of the Earth.

the terrestrial meridian. Thus, from the equation (1) we have

$$\frac{\text{arc (SA)}}{2\pi R_{\text{E}}} = \frac{\angle(\text{SA})^\circ}{\text{celestial meridian} = 360^\circ}, \tag{2}$$

where 360° is the angle of the whole meridian. The angle of the arc SA is measured by means of the epicentric angle ω. Therefore, since the length of the arc SA is known in stades, equation (2) gives immediately the length of the terrestrial meridian (and the radius of the Earth).

Eratosthenes measured the angle $\beta = \omega$ and found that it was equal to 7.2° or 7° 12′, with rather good accuracy. This value is exactly one fiftieth of the circle and so it corresponds to one fiftieth of the terrestrial meridian. The length of the meridian corresponding to the angle ω is 5000 stades, equal to the distance between Syene and Alexandria (figure 2); hence from equation (2) this distance must be one fiftieth of the length of the circumference of the Earth. Consequently, the circumference of the Earth is $C = 5000\,\text{stades} \times 50 = 250\,000\,\text{stades}$. If we adopt the value 1 stade = 157.5 m (see [34] (p. 364)) for the stade used by Eratosthenes, then the circumference (meridian) of the Earth is 39 375 km. Alternatively, if we consider a later value by Eratosthenes for the circumference of the Earth, namely 252 000 stades, this is 39 690 km. This value is very close to the true value of the circumference, which ranges from 40 075 km (equatorial) to 39 942 km (polar). The corresponding radius of the Earth according to Eratosthenes is $R_{\text{E}} = 6281$ km or, if we adopt the value 252 000 stades, $R_{\text{E}} = 6317$ km. Thus, the radius calculated by Eratosthenes is only about 40 km smaller than the true value of the polar radius, which is 6357 km, and about 61 km smaller than the true value of the equatorial radius (6378 km). This estimation by Eratosthenes is a good approximation of the true value.

3.2 *The errors that affected the accuracy of the method*

Eratosthenes' method is geometrical and is based on astronomical observations. The errors that occur in the measurements are independent of this method. These errors were not known at his time and this is perhaps why he did not take some of them into consideration. In the following we briefly present the errors of Eratosthenes' measurement many of which appear for the first time in the literature.

(a) The rotation of Earth around its axis causes an additional error, which is different from that due to the geoid shape of the Earth, because the direction and the magnitude of the gravitational field intensity $\overline{g}$ are influenced by its rotation.
(b) The measurement of the length of the shadow thrown by the gnomon is not accurate, as happens also with a sundial, because of the existence of a penumbra.

(c) In Syene and Alexandria, because of the phenomenon of atmospheric or astronomical diffraction, the solar rays are not parallel.
(d) The Earth in antiquity was considered at best a perfect sphere, while in reality it is flattened towards its poles; that is, it has the shape of a geoid.
(e) The distance between Syene and Alexandria was not exactly 5000 stades, but approximately 800 km.
(f) Syene was not exactly placed on the Tropic of Cancer in the age of Eratosthenes, but slightly to the north of it, since back then the inclination of the ecliptic was $23°\ 43'$, while Eratosthenes had determined that it was about $23°\ 51'$, as stated earlier. With the value of $23°\ 43'$, Syene is located not less than $8'$ to the north of the Tropic of Cancer while for the modern value of $23\ °27'$ it is about $24'$.
(g) Syene, with a geographical longitude of about $32°\ 53'$, and Alexandria (area of the Museum), with a geographical longitude of about $29°\ 55'$, do not lie on the same meridian, the latter being around $\Delta\lambda \approx 3°$ to the west.

The value of 250 000 stades for the circumference of the Earth was given by Cleomedes [32] (i.10, p. 100), while Strabo [2] (2.5.7, 34) explicitly stated that according to Eratosthenes the circumference of the Earth is 252 000 stades. Also Theon of Smyrna [22] (p. 124) reported that Eratosthenes demonstrated that the great circle of the Earth is 252 000 stades. The same value was adopted by Pliny [29] (ii, 247) and is the most widely quoted in ancient writings. This value was adopted by later writers (see [19] (p. 653)) and used by Hipparchus and Ptolemy for their various measurements, although not by Posidonius of Rhodes, who calculated on his own the size of the Earth, using astronomical observations (see [20] (pp. 168–169), [24] (p. 345) and [30] (p. 177)). In his calculations of the size of Sun and Moon, as well as of their distances from the Earth, Posidonius used the value obtained by Dicaearchus, namely 300 000 stades, for the size of the Earth. The different value for the circumference of the Earth given by Cleomedes has been the topic of great discussion with respect to the reason for this discrepancy. It seems more plausible that Eratosthenes himself or even a later author corrected the value 250 000 to 252 000 because a round number of 700 stades was required for 1° on the surface of the Earth (see [12] (pp. 33 and 161), [13] (ii, p. 107), [27] (p. 410), [30] (p. 175) and [35] (p. 141)).

Goldstein's [36] (pp. 411–416) argued that Eratosthenes, in order to obtain the approximate value of 252 000 stades, did not carry out any observations for his estimate but, rather, performed a calculation based on estimates of various magnitudes that he had available. We should note that the same applies to the Posidonius, namely that, although Cleomedes mentions the value of 240 000 stades for the Earth's size, later sources ascribed the value of 180 000 stades to Posidonius (see [2] (2.2.2), [12] (p. 150), [30] (p. 177) and [33] (p. 34)). It is not very likely that both accounts given by Cleomedes were wrong, as Goldstein [36] stated for Eratosthenes. We would rather conclude the following.

Eratosthenes adopted the value of 252 000 stades so that 1° corresponds to 4200 stades. It is not certain that Eratosthenes himself knew how to divide a circle into 360°. This division seems to have appeared for the first time in the second century BC (see [12] (see pp. 32, 107 and 148–149) and [19] (p. 671)). Eratosthenes estimated the circumference of the Earth (of the circle) as 60° (see [2] (2.5.6, 7)); in this case, 4200 stades correspond to 1°. The approach using 252 000 stades resulted from the need that an integer number of 4200 stades should correspond to 1°. Eratosthenes used this round number (4200) for the measurement of the distances of various places, the length of a parallel as well as the extent of the inhabited world in order to derive a geographical map of the then-known world.

Also, Hipparchus some years later adopted the same value and, after dividing it by 360°, obtained the round number of 700 stades for 1°. Hipparchus, like others before him, divided the circle into 360° and he introduced this division into the circumference meridian of the Earth. In order to describe the celestial phenomena and to construct tables with astronomical data for every zone (band) of geographical latitude from the equator to the North Pole, he adopted the value of 252 000 stades used by Eratosthenes, which he considered accurate (see [2] (2.5.6, 7)). The division (252 000)/360° gives 700 stades per 1°. So Hipparchus described the celestial phenomena per 1° of latitude or per 700 stades.

Finally, although there has been some discussion regarding the exact length of the stade used by Eratosthenes the majority of authors such as Heath [13, 24], Dreyer [30], Dilke [33], Hultsch [34] and Thomson [37] accepted the value of 157.5 m. It appears that, in ancient Greece, various values were used in different periods. A well-known stade was the Olympic value of 185 m, while another was the Ptolemaic or Royal stade of 210 m. However, the stade used by Eratosthenes was (see [34] (p. 364)), according to Pliny [29] (xii, 53, c13), shorter than the Olympic value and equal to about 157.5 m. The Eratosthenian stade was a wayfaring measure, used in antiquity for distances counted with steps by professional steppers.

3.3 *The significance of the method*

Eratosthenes calculated with accuracy the size of the Earth, making a relative error of approximately 1%, if we adopt a stade equal to 157.5 m as his unit of length. This method causes admiration even today, since Eratosthenes did not have at his disposal modern instruments and since the errors we mentioned here are not due to his method but to other causes. This is because in general the contribution of these errors is small and because, for some of them, this contribution is positive while, for other errors, it is negative.

His deep knowledge of spherical geometry led him to two important discoveries: his method for the measurement of the size of the Earth and his map of the world (oikoumene). On this map he traced, using his own method, meridian and parallel circles. This discovery was very important, because he introduced useful coordinates to determine the position of a place on Earth's surface. Based on his method and on astronomical observations, he managed to determine the distance between various places and the position of constellations on the celestial globe.

As we have already mentioned, Hipparchus described the celestial phenomena for various zones of geographical latitude from the equator to the North Pole, giving in the form of tables the astronomical data for every 1° of latitude. However, he did not proceed to estimate the lengths of various arcs in different latitudes along one meridian. We suppose that this was due to the belief of the ancient philosophers, astronomers and mathematicians since Pythagoras that the Earth, like the planets, is spherical.

The method of Eratosthenes by itself is precise and is applied even today for the determination of the size and shape of the Earth. In our opinion, the work of Hipparchus combined with the method of Eratosthenes, who had calculated the latitudes and the distances of various places such as the distance from Alexandria to Rhodes as 3750 stades (see [2] (2.1.1)), were the factors that encouraged subsequent geographers to determine the size and shape of the Earth using better technology, which in turn supplemented the Eratosthenian method.

Finally, in another paper [38], we apply Eratosthenes' method in order to determine the shape and the size of the Earth. Our approach is based on curvature theory (see [39] (p. 48)). By measuring the lengths of various arcs of the same meridian which correspond to equal epicentric angles, corresponding to the closest circle (the circle of curvature), at different geographical latitudes, it is proved that these lengths increase as we proceed from the equator to the poles. The lengths of the arcs of the same meridian that correspond to an angle of 1° are not equal. Thus it can be proved that the Earth is oblate at its poles. On the contrary, along the equator (and at sea level) these lengths are equal. So, it can be proved theoretically that the meridians of the Earth are ellipses, because the radius of the circle of curvature corresponding to the poles is larger than the respective radius of the circle of curvature that corresponds to the equator (the centre of the ellipse does not coincide with the centres of the inscribed circles corresponding to the various points of the ellipse).

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