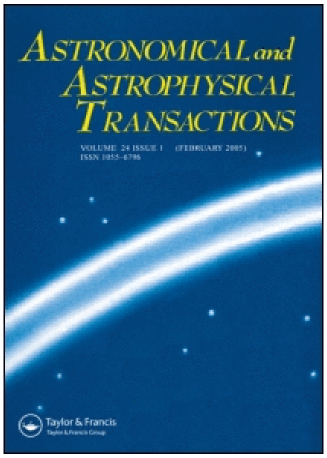

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MODIFICATION OF THE SANDERS METHOD FOR ESTIMATION OF THE MEMBERSHIP PROBABILITIES FOR STARS IN STELLAR CLUSTERS

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In this paper, a modification of the well-known Sanders method for estimation of the membership probabilities for stars in stellar clusters is proposed. It is shown that adding one more normal distribution to the approximation expression for the two-dimensional probability density of proper motions allows us to obtain more reliable values for membership probabilities.

Keywords: Stellar clusters; Probable members; Sanders method

1 INTRODUCTION

It is well known that the most useful method of the estimation of membership probabilities for stars in stellar clusters is the method proposed by Sanders (1971) based on the treatment of the proper motions of stars in cluster fields. This method uses the approximation of the two-dimensional probability density distribution of proper motions by the sum of two normal distributions separately for cluster and field stars, and the distribution of the proper motions of field stars is expressed by an elliptical normal distribution whereas for the distribution of cluster stars a rotationally symmetric normal distribution is used. This sum can be expressed as follows:

$$F(x, y) = \frac{N_\mathrm{f}}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{(x-x_\mathrm{f})^2}{2\sigma_x^2} - \frac{(y-y_\mathrm{f})^2}{2\sigma_y^2}\right) + \frac{L-N_\mathrm{f}}{2\pi\sigma_\mathrm{c}^2}\exp\left(-\frac{(x-x_\mathrm{c})^2+(y-y_\mathrm{c})^2}{2\sigma_\mathrm{c}^2}\right). \quad (1)$$

Here x and y are the components of the proper motion of stars, x_f and y_f denote the coordinates of the centre of distribution of the proper motions of field stars, σ_x and σ_y are the corresponding dispersions for field stars, L is the number of stars in the sample, N_f is the number of field stars in the sample, x_c and y_c are the coordinates of the centre of proper motion distribution of cluster stars and σ_c is the dispersion of the proper motions of cluster stars.

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For the evaluation of the parameters of distribution (1) for a particular cluster, appropriate equations have been derived by Sanders (1971) using the maximum-likelihood method. Eight parameters have to be determined from these equations. Later King (1979) derived additional equation for the inclination of the main axis of dispersion ellipses of the distribution of the proper motions of field stars to the x axes. King stressed the necessity for exclusion of stars with large proper motions from the samples, which make the procedure for estimation of the parameters more stable. In the paper by Sabogal-Martinez *et al.* (2001) the use of an elliptical distribution for the proper motions of cluster stars was proposed. This complication is justified when the departures from circular symmetry are obvious or when proper motions suffer from appreciable magnitude or colour equations. It must be mentioned that none of the methods mentioned above accounts for the departures of the proper motion distribution of field stars from the normal distribution. The departures of the proper motion distribution of cluster stars are not so important because it is substantially affected by the distribution of observational errors, which can be considered as circularly symmetrical with great distinctness and, for most clusters, the dispersion of errors exceeds the dispersion of the proper motions. To check how the distribution of the proper motions of field stars can differ from the normal distribution, let us consider the results of numerical experiment for the example of the open cluster M67 = NGC 2682.

2 NUMERICAL EXPERIMENT

For this target, one has to simulate proper motions in the field of the sky containing both background and cluster stars. For the representation of cluster stars the values of three-dimensional radius vectors of model stars are determined wit the help of a random-number generator, and then with the accepted value of cluster distance from the Sun (the mean distance 900 pc is used) the distance of each model star from the Sun is calculated. The distribution of the lengths of the radius vectors of cluster stars is taken to be exponential, and the parameters of distribution are selected so that the radius of a cluster core is approximately equal to 3 pc. The distribution of spatial velocities of cluster stars is accepted as spherically symmetric with a dispersion of $1.5\,\mathrm{km\,s^{-1}}$. From the calculated distance of a star from the Sun and the projections of spatial velocity, the components μ_x and μ_y of the proper motions of cluster stars are determined.

To imitate stars of a galactic field the set of distances from the Sun and spatial velocities of stars are calculated with the help of the random-number generator. A linearly increasing distribution density of star distances from the Sun with limiting value 1500 pc is used; the dispersion of the spatial velocity components of field stars is accepted as equal to $30\,\mathrm{km\,s^{-1}}$. To simulate the influence of space motion of the Sun, $10\,\mathrm{km\,s^{-1}}$ is added to the components of spatial velocities of all field stars. With the help of the calculated spatial velocities and distances, the proper motions of field stars are then evaluated. 500 model cluster stars and 300 field stars are used for one experiment, the observational errors of proper motions being taken equal to zero. Five experiments are carried out, all with a particular sequence random numbers.

In Figure 1 the frequency distributions of the proper motions of 1500 model field stars calculated by the technique described above are shown. As we consider a symmetric scheme, the distributions of μ_x and μ_y are the realizations of one distribution. The distributions of μ_x and μ_y are shown in the figure accordingly by solid and dotted curves respectively. For comparison the dashed curve shows the normal distribution curve for the mean value and dispersion, calculated for the distribution of the proper motions of model field stars. In the figure the differences of asymmetry and excess of distributions of the proper motions of field stars from the normal distributions are easily visible, thus showing the inadequacy of the representation of

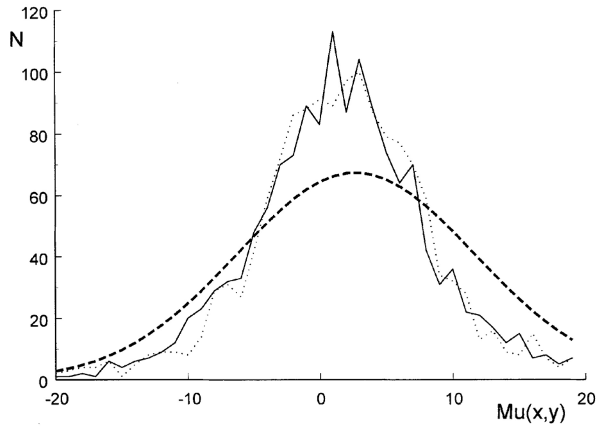

FIGURE 1 Frequency distributions of the proper motions of 1500 model field stars.

this distribution by a normal curve. Thus membership probabilities for cluster stars estimated by the Sanders method are only rough estimations of the real probabilities and depend on the form of distribution of the proper motions of field stars. It is interesting to note that, as the observed distributions are convolutions of true distributions with that of observational errors and the latter is usually close to the normal distribution, then for the observed proper motion distributions, the larger the errors, the smaller are the deviations from normality. In particular, this concerns cluster stars having small dispersions of proper motions.

3 MODIFICATION OF THE SANDERS METHOD

Owing to the poor representation of the distribution of the proper motions of field stars by the Sanders method an idea for modifying this method is implemented by adding a new term to Eq. (1) in order to allow non-zero asymmetry and an excess for the distribution of field stars. For simplicity, one more normal distribution is used as this new term, the dispersion of the new distribution being taken to be twice that used in first term of Eq. (1) to allow for the desired excess, and the displacement parameters to be determined so as to allow for the desired asymmetry. The new approximate expression takes the form

$$\begin{aligned} F_{\mathrm{M}}(x, y) = {} & \frac{N_{\mathrm{f}}}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{(x-x_{\mathrm{f}})^2}{2\sigma_x^2} - \frac{(y-y_{\mathrm{f}})^2}{2\sigma_y^2}\right) \\ & + \frac{M_{\mathrm{f}}}{8\pi\sigma_x\sigma_y} \exp\left(-\frac{(x-x_{\mathrm{f2}})^2}{8\sigma_x^2} - \frac{(y-y_{\mathrm{f2}})^2}{8\sigma_y^2}\right) \\ & + \frac{L-N_{\mathrm{f}}-M_{\mathrm{f}}}{2\pi\sigma_{\mathrm{c}}^2} \exp\left(-\frac{(x-x_{\mathrm{c}})^2+(y-y_{\mathrm{c}})^2}{2\sigma_{\mathrm{c}}^2}\right). \end{aligned} \tag{2}$$

Here M_{f} designates the number of stars, attributed to second distribution of the proper motions of field stars, and where x_{f2} and y_{f2} are the coordinates of the appropriate centre of distribution. From Eq. (2) the following system of the equations for estimation of parameters can be evaluated by the maximum-likelihood method as utilized to construct the equations for the Sanders method:

$$
\begin{aligned}
&N\colon \sum_{i=1}^{L}\left(\frac{A_i}{\sigma_x\sigma_y}-\frac{B_i}{\sigma_c}\right)\frac{1}{F_i}=0,\\
&x_f\colon \sum_{i=1}^{L}\frac{A_i}{F_i}(x_i-x_f)=0,\\
&y_f\colon \sum_{i=1}^{L}\frac{A_i}{F_i}(y_i-y_f)=0,\\
&\sigma_x\colon \sum_{i=1}^{L}\left(\frac{1}{\sigma_x^2}\left[N(x_i-x_f)^2A_i+\tfrac{1}{16}\,(x_i-x_{f2})^2\,C_i\right]-\left(NA_i+\tfrac{1}{4}MC_i\right)\right)\frac{1}{F_i}=0,\\
&\sigma_y\colon \sum_{i=1}^{L}\left(\frac{1}{\sigma_y^2}\left[N\,(y_i-y_f)^2\,A_i+\tfrac{1}{16}(y_i-y_{f2})^2C_i\right]-\left(NA_i+\tfrac{1}{4}MC_i\right)\right)\frac{1}{F_i}=0,\\
&x_{f2}\colon \sum_{i=1}^{L}\frac{C_i}{F_i}(x_i-x_{f2})=0,\\
&y_{f2}\colon \sum_{i=1}^{L}\frac{C_i}{F_i}(y_i-y_{f2})=0,\\
&x_c\colon \sum_{i=1}^{L}\frac{B_i}{F_i}(x_i-x_c)=0,\\
&y_c\colon \sum_{i=1}^{L}\frac{B_i}{F_i}(y_i-y_c)=0,\\
&\sigma_c\colon \sum_{i=1}^{L}\frac{B_i}{F_i}\left(\frac{(x_i-x_c)^2+(y_i-y_c)^2}{2\sigma_c^2}-1\right)=0,\\
&M\colon \sum_{i=1}^{L}\left(\frac{C_i}{4\sigma_x}-\frac{B_i}{\sigma_c^2}\right)\frac{1}{F_i}=0.
\end{aligned}
\tag{3}
$$

In the system of Eq. (3) the following designations are used:

$$
\begin{aligned}
A_i &= \exp\left(-\frac{(x_i-x_f)^2}{2\sigma_x^2}-\frac{(y_i-y_f)^2}{2\sigma_y^2}\right),\\
C_i &= \exp\left(-\frac{(x_i-x_{f2})^2}{8\sigma_x^2}-\frac{(y_i-y_{f2})^2}{8\sigma_y^2}\right),\\
B_i &= \exp\left(-\frac{(x_i-x_c)^2+(y_i-y_c)^2}{2\sigma_c^2}\right),
\end{aligned}
$$

F_i is given by Eq. (2), calculated for the components of proper motion of the ith star. Before each equation of system (3) the parameter is specified, the differentiation of which leads to the appropriate equation. It is obvious that the introduction of more parameters complicates their estimation and requires better initial values of the parameters for the procedure known as the method of consecutive approximations, by which the systems of the equations in the Sanders method are usually solved. However, a more adequate approximation for the distribution of the proper motions of field stars allows us to hope these evaluation gives more reliable estimates for the parameters of the distribution (2). In the new method there is no need to rotate the system of axes before solving the equations because the shift in the second term of Eq. (2) accounts for this effect.

4 DISCUSSION

In Figure 2 the comparison of the results of the members of the open cluster α Per by the classical Sanders method and the modified Sanders method using the photometric data from the catalogue Tycho-2 is given. Stars are chosen from rectangular area 4° wide with the centre coincident with the cluster centre. Figure 2(a) shows the colour–magnitude (CM) diagram constructed for the photometric data of the catalogue Tycho-2 for 56 probable members determined with the classical Sanders method: Figure 2(b) shows the CM diagram obtained with the modified method which gives 64 probable members. The numbers of members of a cluster here are determined as parameters of the distributions (1) and (2). From the figures it is easily visible that the positions of the additional eight stars, allocated with the modified method, well match the main sequence of the CM diagram and do not increase its dispersion; so the fact that these eight stars belong to the cluster is proved by the photometric test.

In Figure 3 the same comparison is shown for stars in the field of an old open cluster M67; the proper motions and photometric data used are taken from the work of Frolov and Ananievskaya (1996). By the classical Sanders method for a sample containing 1023 stars, for which in the work by Frolov and Ananievaskaya (1996) the photographic values V and $B - V$ are determined 775 probable members are extracted. The mean proper motion of the cluster appears to be given by $x_c = -6.48 \times 10^{-3''}$ year^{-1} and $y_c = -3.51 \times 10^{-3''}$ year^{-1}, and the dispersion of proper motions of probable members is equal to $\sigma_c = 4.54 \times 10^{-3''}$ year^{-1}. The fraction of the probable cluster members in the sample seems to be significantly overestimated,

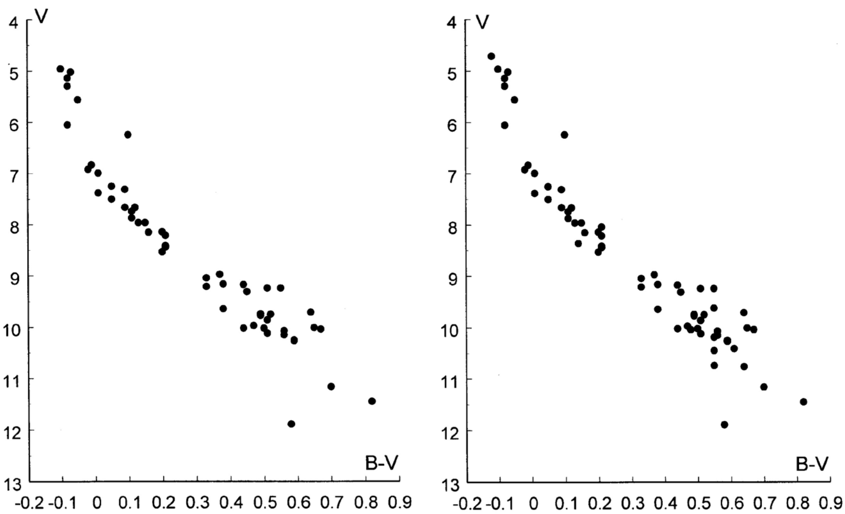

FIGURE 2 Comparison of V versus $B - V$ curves determined by (a) the classical Sanders method and (b) the modified sanders method for Tycho-2.

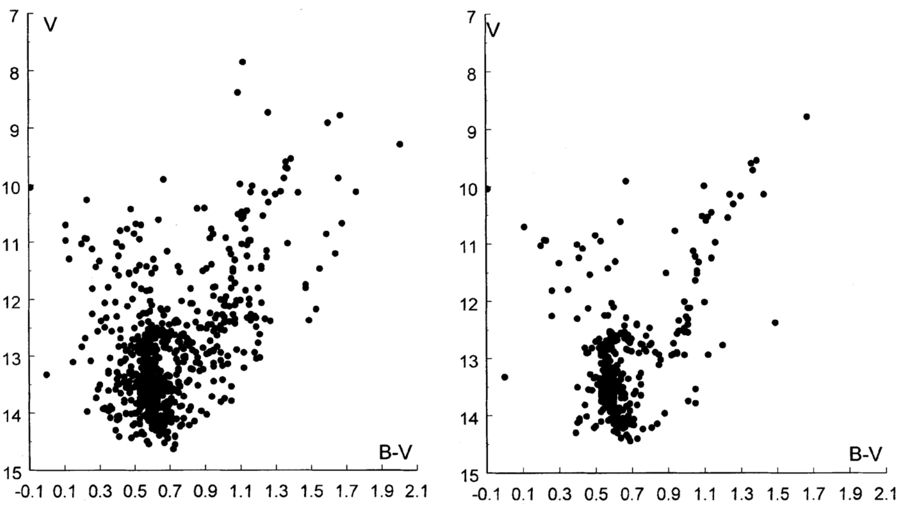

FIGURE 3 The same as Figure 2 but for the open cluster M67.

but the parameters of distribution (1) determined for different values of zero approximation remain the same.

The modified method has given the estimates of cluster parameters: $N_c = 413$; $x_c = -7.63 \times 10^{-3''}$ year^{-1}; $y_c = -4.16 \times 10^{-3''}$ year^{-1}; $\sigma_c = 2.26 \times 10^{-3''}$ year^{-1}. Probably, owing to a closer arrangement of modes of the proper motion distributions of cluster and field starts the classical Sanders method has not given good estimates of the parameters, and the advantage of the modified method has obviously been demonstrated. Note that stars with the largest proper motions are not excluded from the sample.

At the same time it is necessary to note that the greater number of parameters to be determined should worsen the reliability of the estimates of their values; so the modified method should require larger volumes of sample and a better initial approximation than the classical Sanders method. Probably the best decision is to use a set of programs realizing the Sanders method in the classical variant and its various modifications. As was specified by King (1979), in order to achieve stability of the solving procedure it may be necessary to exclude field stars with the largest proper motions from the sample. As the classical variant of the Sanders method requires the determination of the least number of parameters among all the considered modifications, the procedure for estimation of distribution parameters should be the most stable. Other modifications should be used when the samples are great and the errors of proper motions are small, and as initial approximation of parameters it is worth using the results estimated by the classical Sanders method.

For the practical use of the Sanders method in its classical variant and its modification from stated here the appropriate computer programs for systems Windows 9x have been prepared by the author, where the equations for the estimation of parameters of distributions are approximately solved by consecutive application of a secant method. The programs can be requested from the author (email: a1exhander.loktin@usu.ru).

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