Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

INTEGRAL ENERGY SPECTRA OF SOLAR SOFT X-RAY FLARES IN THE THREE SOLAR ACTIVITY CYCLES (1972-2001)

V. V. Kasinsky a, R. T. Sotnicova b

a Irkutsk State University of Railway, 664074 Irkutsk, Russia.
b Irkutsk State University, 664074 Irkutsk, Russia.

Online Publication Date: 01 June 2003
To link to this article: DOI: 10.1080/1055679031000079665
URL: http://dx.doi.org/10.1080/1055679031000079665

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: http://www.informaworld.com/terms-and-conditions-of-access.pdf

This article maybe used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.
INTEGRAL ENERGY SPECTRA OF SOLAR SOFT X-RAY FLARES IN THE THREE SOLAR ACTIVITY CYCLES (1972–2001)

V. V. KASINSKY\textsuperscript{a,}\textsuperscript{*} and R. T. SOTNICOVA\textsuperscript{b}

\textsuperscript{a}Irkutsk State University of Railway, 664074 Irkutsk, Russia; \\
\textsuperscript{b}Irkutsk State University, 664074 Irkutsk, Russia

(Received 3 October 2002)

The flare energy distribution of ultraviolet Ceti-type stars has a power-law spectrum \(N \propto E^{-b}\). Solar optical flares have an identical power energy spectrum with \(b \approx 0.80\). In a preliminary study (1972–1993), the X-ray flare energy spectrum was represented by a unique power law over the whole range of energies with \(b \approx 0.75\).

The purpose of this paper is to reveal the temporal variations in the power-law spectrum of X-ray flares over three solar cycles (1972–2001). A more extensive statistic (56,500 flares) allow us to revise earlier results and to find new data. The parameters \(b\) and \(\log E_m\) of the energy spectrum undergo variations associated with a 11-year cycle phase and correlate with the Wolf number. The 3-cycle-averaged \(b\) is 0.666, with variations in the range 0.50 < \(b\) < 0.80. The result obtained may be useful for the study of flare activity on red dwarf stars.

Keywords: X-ray flares; 11-Year variations of the power-law spectrum

1 INTRODUCTION

The solar flare consists of an explosive release of radiation and mechanical energy near the solar surface of the Sun with an upper limit of radiant energy of \(4 \times 10^{32}\) erg (Hudson, 1983). According to Hudson (1991), solar flare energy spectra in soft X-rays in the region of 20 keV and for the microwave range have a power-law form with spectral indices of 0.80–0.85.

A statistical study of the most active UV Ceti-type flare stars showed that their time-integrated flare energy at the optical wavelength can be represented by a power function with a wide variety of \(b\) induces: 0.5 < \(b\) < 0.9 (Gershberg, 1972). The Sun behaves as a UV Ceti star whose exponent of the energy spectrum for flare optical radiation in all spectral lines and continua of hydrogen series is 0.80 (Kurochka, 1987), which virtually coincides with that for X-ray flares.

Based on X-ray (1–8 A) flux data from 1977 to 1987 (19,700 flares), integral energy spectra of flares where computed for one 11-year cycle (Kasinsky and Sotnikova, 1989a,b). The cycle-averaged index of the power spectrum in X-rays, \(\langle b \rangle = 0.85\),
and a power-law energy spectrum approximation gives the following linear dependence: 
\[ \log E = 31.2 - 1.17 \log N. \]

Subsequently the X-ray flare energy spectrum was calculated for 1972–1993 with 
\[ \langle b \rangle = 0.76 \pm 0.01 \] (37,000 flares); the energy spectrum approximation gives the linear dependence 
\[ \log E = 30.9 - 1.35 \log N. \] (Kasinsky and Sotnikova, 1996).

By investigating the energy spectrum of solar flares in the soft-X-rays region Kasinsky and Sotnikova (1989a,b, 1996) also found clear variations in the spectral index \( b \) with the phase of the 11-year solar cycle. It was shown that the exponent \( b \) of the energy spectrum varies with the cycle phase, that is with the Wolf number \( W \). The correlation coefficients between \( W \) and the spectral indices \( b \) measured in the previous year (−1), simultaneously (0) with a determination of \( W \) and in the subsequent year (+1) are 0.39, 0.62 and 0.84 respectively. In other words a 1-year delay in comparison with the Wolf number takes place in the time variations of \( b \). These results need to be supported by a new data set related to the end of cycle 22 and the beginning of the cycle 23 (1993–2001).

2 DATA REDUCTION AND SELECTION

To obtain the flare energy \( E \), the flux \( F \) (1–8 Å) was integrated over the duration of the individual flare \( T \) and over the hemisphere from the flare site to the Earth by the formula 
\[ E = \pi R_{AE}^2 \int F(1–8 \text{ Å}) \, dt, \]
where \( R_{AE} \) is the distance from the Sun to the Earth and \( F \) was taken from the data of the National Oceanic and Atmospheric Administration – US Air Force Space Environment Service Center (1972–2001). The integral was calculated by the triangle approximation of the flare profile, that is approximately \( \frac{1}{2} FT \), which was sufficient for massive calculations for more than 1000 flares per year.

This study takes into account the light curves \( F(t) \). To estimate the flare energy \( E \), a more refined technique was adopted by differentiating the phase of the rise and decay of \( F(t) \). There exists a wide variety of flare light curves, but the main feature is a sharp increase in \( F(t) \) from \( t_0 \), to a maximum \( t_{\text{max}} \) and an exponential decay in \( F \) from \( t_{\text{max}} \) to \( t_{\text{end}} \). To obtain the full flare energy the flux \( F(\text{erg cm}^{-2} \text{s}^{-1}) \) was time integrated by taking into account the beginning \( t_0 \), the maximum \( t_{\text{max}} \) and the end \( t_{\text{end}} \) of the flare. Next the amount of flux was spatially integrated over the hemisphere \( 2\pi R^2 \) where \( R = 1 \text{ AU} \).

Assuming for the increasing part ‘the triangle’ approximation and for the decaying part the exponential approximation that \( t_{\text{end}} \) corresponds to \( e \) times the decay of \( F(t) \), we have the following empirical estimation formula:

\[ E = 4.22 \times 10^{28} (t_{\text{max}} - t_0) + 2.53 \times 10^{28} (t_{\text{end}} - t_{\text{max}}), \] (1)

where \( F \) is in ergs per square centimetre per second, \( t \) is in seconds and \( E \) is in ergs. The individual \( E \) values from Eq. (1) fit the range \( 10^{24} \text{ erg} < E < 10^{32} \text{ erg} \), in accordance with previous data. Again, the early evaluations of an energy spectrum suffer from selection since the data on faint flares are incomplete (Kasinsky and Sotnikova, 1989a,b). The new data were essentially added from 56,500 faint flare totals (1972–2001).

Thereupon, the accumulated number \( N \) of flares with energy \( E > E_m \) was approximated by the power-like function:

\[ N(E_m) = \int n(E) \, dE \sim CE^{-b}. \] (2)
A dependence similar to Eq. (2) means that, if \( E \) is plotted versus \( N \), one obtains on a log–log scale a linear dependence:

\[
\log E = \log E_m - \frac{1}{b} \log N.
\]

Equation (3) shows that there is one flare (in 1 year) with the limiting energy \( \log E = \log E_m \) which corresponds to the condition \( N = 1 \). The limiting energy is a maximum possible energy of flares according to the power-law approximation (2). In \( \log E \)–\( \log N \) coordinates in the linear part of the energy spectrum the index \( b \) of the power spectrum was determined by the angular coefficient of the straight line given by Eq. (3).

3 THE FEATURES OF THE ENERGY SPECTRUM IN THE X-RAY EMISSION OF FLARES

On a double-logarithmic \( \log E \)–\( \log N \) scale there are three features of the power spectrum. As an example, Figure 1 shows the power spectrum of X-ray flares for the year 1981 for \( N = 4005 \). As follows from Figure 1, the flares of intermediate energies reveals a relation between \( \log E \) and \( \log N \) close to the linear relation (3). As regards the fainter flares (approximately \( 10^{26} \) erg), however, a sharp break appears in the spectrum. \( E_{\text{break}} \) is near the flare detection threshold, and the break therefore is due to observational selection (Gershberg, 1972).

In the high-energy region a drop is observed in the energy spectrum as a deviation from the straight line (Figure 1). Systematic deviations from a linear dependence of the form (3) in the region of the strongest flares (greater than \( 10^{29} \) erg) should be attributed to the effect of saturation. In fact, whatever the physical nature of flare activity, a maximum allowed energy per flare should exist. As a maximum of flare energy (\( \log E_m \)) is approached, which is possible on the Sun, the energy spectrum should become a horizontal line. Therefore, the limiting energy of solar flares can be estimated with confidence only when the saturation of the energy spectrum is found.

Table 1 presents the summary of energy spectra parameters for 1972–2001 (a total of 56,500 flares). The data for 1974–1976 are approximate since the values of \( b \) were calculated using small statistics.

The high limiting energy of one solar flare for 30 years is estimated as \( \log E = 32.5 \) (1989) with the mean energy for all flares being \( \langle \log E \rangle = 31.01 \) in the “log” scale. Thus, the limiting energy of one flare in soft X-rays is much lower than the upper limit of the radiant energy of \( (4–5) \times 10^{32} \) ergs reported by Hudson (1983) and Kurochka (1987). The actual energy of a single flare is still lower owing to energy spectrum saturation as seen from Figure 1.

The energy spectrum parameters \( b \) and \( \log E_m \) are mutually poorly correlated: \( r \leq 0.37 \). They can be taken therefore as two independent parameters of the power spectrum. Their physical meanings are different. While the former parameter may be defined as the ‘slope of spectrum’ the latter defines the limiting energy, the intersection on the \( E \) axis. Separately, they show good correlation with the Wolf number \( W \), revealing a significant 11-year modulation at the following level (Figure 2):

\[
r(b, W) = 0.49–0.58, \quad r(\log E_m, W) = 0.80–0.72.
\]
The parameter \( \log E_m \) shows a relatively high correlation and 11-year modulation with the yearly number of flares as well (Figure 3):

\[
r(\log E_m, N) = 0.77 - 0.71.
\]

To describe the high-energy decrease in \( E \) (Figure 1) we introduced the new parameter

\[
\Delta(\log E) = \log E_m - \log E_F
\]

as the difference between the limiting ‘approximation’ energy and the actually observed energy maximum \( E_F \). Thus \( \Delta(\log E) > 0 \). This parameter means ‘the amount of saturation’ of \( E \) and is correlated with the Wolf number \( W \), as well as with the yearly number \( N \) of flares (Figure 4):

\[
r(\Delta(\log E), W) = 0.76 - 0.61,
\]

\[
r(\Delta(\log E), N) = 0.73 - 0.521.
\]
TABLE I  The Parameters of the Power-law Energy Spectrum in X-ray Solar Flares.

<table>
<thead>
<tr>
<th>Year</th>
<th>( \log E )</th>
<th>( \langle b \rangle_{\text{total}} )</th>
<th>( N/\text{years} )</th>
<th>( W )</th>
<th>( \log N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>31.2</td>
<td>0.68 ± 0.01</td>
<td>2723</td>
<td>69</td>
<td>4.22</td>
</tr>
<tr>
<td>1973</td>
<td>30.38</td>
<td>0.88 ± 0.02</td>
<td>1784</td>
<td>38</td>
<td>3.20</td>
</tr>
<tr>
<td>1974</td>
<td>31.35</td>
<td>0.50 ± 0.08</td>
<td>321</td>
<td>34</td>
<td>3.22</td>
</tr>
<tr>
<td>1975</td>
<td>28.56</td>
<td>0.57 ± 0.08</td>
<td>250</td>
<td>15</td>
<td>3.30</td>
</tr>
<tr>
<td>1976</td>
<td>29.7</td>
<td>0.64 ± 0.10</td>
<td>153</td>
<td>12</td>
<td>3.44</td>
</tr>
<tr>
<td>1977</td>
<td>30.3</td>
<td>0.60 ± 0.08</td>
<td>268</td>
<td>27</td>
<td>3.24</td>
</tr>
<tr>
<td>1978</td>
<td>31.6</td>
<td>0.60 ± 0.03</td>
<td>1134</td>
<td>92</td>
<td>4.24</td>
</tr>
<tr>
<td>1979</td>
<td>31.6</td>
<td>0.64 ± 0.03</td>
<td>1469</td>
<td>155</td>
<td>4.28</td>
</tr>
<tr>
<td>1980</td>
<td>31.6</td>
<td>0.69 ± 0.01</td>
<td>2463</td>
<td>155</td>
<td>4.60</td>
</tr>
<tr>
<td>1981</td>
<td>31.9</td>
<td>0.684 ± 0.005</td>
<td>4005</td>
<td>140</td>
<td>4.84</td>
</tr>
<tr>
<td>1982</td>
<td>32.3</td>
<td>0.631 ± 0.005</td>
<td>3852</td>
<td>116</td>
<td>4.64</td>
</tr>
<tr>
<td>1983</td>
<td>31.8</td>
<td>0.71</td>
<td>2583</td>
<td>67</td>
<td>4.36</td>
</tr>
<tr>
<td>1984</td>
<td>31.8</td>
<td>0.58</td>
<td>2176</td>
<td>46</td>
<td>3.95</td>
</tr>
<tr>
<td>1985</td>
<td>30.1</td>
<td>0.68</td>
<td>1065</td>
<td>18</td>
<td>3.45</td>
</tr>
<tr>
<td>1986</td>
<td>30.3</td>
<td>0.62</td>
<td>916</td>
<td>13</td>
<td>3.39</td>
</tr>
<tr>
<td>1987</td>
<td>30.4</td>
<td>0.68</td>
<td>1389</td>
<td>29</td>
<td>3.96</td>
</tr>
<tr>
<td>1988</td>
<td>31.6</td>
<td>0.64</td>
<td>2367</td>
<td>100</td>
<td>4.24</td>
</tr>
<tr>
<td>1989</td>
<td>32.5</td>
<td>0.60</td>
<td>2610</td>
<td>158</td>
<td>4.56</td>
</tr>
<tr>
<td>1990</td>
<td>31.5</td>
<td>0.72</td>
<td>2630</td>
<td>142</td>
<td>4.68</td>
</tr>
<tr>
<td>1991</td>
<td>32.2</td>
<td>0.659 ± 0.005</td>
<td>3324</td>
<td>145</td>
<td>4.08</td>
</tr>
<tr>
<td>1992</td>
<td>31.6</td>
<td>0.67 ± 0.01</td>
<td>2816</td>
<td>90</td>
<td>4.40</td>
</tr>
<tr>
<td>1993</td>
<td>31.1</td>
<td>0.69 ± 0.01</td>
<td>2429</td>
<td>56</td>
<td>4.04</td>
</tr>
<tr>
<td>1994</td>
<td>30.3</td>
<td>0.69 ± 0.02</td>
<td>1612</td>
<td>22</td>
<td>3.63</td>
</tr>
<tr>
<td>1995</td>
<td>29.2</td>
<td>0.72 ± 0.02</td>
<td>1124</td>
<td>16</td>
<td>3.45</td>
</tr>
<tr>
<td>1996</td>
<td>29.2</td>
<td>0.63 ± 0.07</td>
<td>510</td>
<td>9</td>
<td>3.03</td>
</tr>
<tr>
<td>1997</td>
<td>30.4</td>
<td>0.66 ± 0.03</td>
<td>1138</td>
<td>22</td>
<td>3.57</td>
</tr>
<tr>
<td>1998</td>
<td>31.1</td>
<td>0.68 ± 0.01</td>
<td>2244</td>
<td>62</td>
<td>4.32</td>
</tr>
<tr>
<td>1999</td>
<td>30.9</td>
<td>0.76 ± 0.01</td>
<td>2421</td>
<td>95</td>
<td>4.52</td>
</tr>
<tr>
<td>2000</td>
<td>30.9</td>
<td>0.80 ± 0.01</td>
<td>2260</td>
<td>130</td>
<td>4.76</td>
</tr>
<tr>
<td>2001</td>
<td>31.4</td>
<td>0.72 ± 0.01</td>
<td>2730</td>
<td>134</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Mean: 31.01   Mean: 0.666 ± 0.005   Sum: 56500   Mean: 4.01

FIGURE 2  The cross-correlation of the energy spectrum exponent \( b \) with the Wolf number \( W (\odot) \) and the limiting energy \( (\log E_m) \) with \( W (\odot) \). A strong 11-year modulation in the parameter \( \log E \) can be seen.
Another important fact following from evaluation of the power spectrum for an appreciable length of time (1972–2001) is the detection of a positive trend for the spectral index $b$ in the form (Figure 5)

$$b = 0.0041T + 0.5988,$$

FIGURE 3  The cross-correlation of the limiting energy $-\log E$ with $N$ (the number of flares per year). A strong 11-year modulation can be seen.

FIGURE 4  The cross-correlation of the ‘deficit’ of high-energy flares with the Wolf number $W$ (■) and the number $N$ of flares (◆). A clear 11-year modulation in the parameter $\Delta(\log E)$ can be seen.
where $T$ is the number of the years counted from $T_0$ taken as 1972. The positive trend of $b$ with $T$ over 30 years is a rather unexpected result. This means that, in general, the flare activity depends on the ‘secular’ variations in $W$. Our data show a slight positive secular regression of the numbers $N$ of flares per year with the time $T$. However, it is possible that the trend may have an explanation in the progressive increase in the number of flares because of improvement in the observational technique.

4 DISCUSSION

A systematic quantitative analysis of the limiting energy of flares ($\log E_m$) as well as the ‘slope’ $b$ of the energy spectrum allows us to draw some important conclusions on the general properties of flare activity on the Sun.

As follows from Table 1 and Figures 2–4, the power spectrum index $b$ varies with an 11-year cycle phase, that is with the Wolf number. The cycle-averaged index of the power spectrum in X-rays ($b = 0.666 \pm 0.005$) is found to be markedly less than that for optical flares (0.80) determined by Kurochka (1987). The cycle-averaged $\langle E \rangle$ in one X-ray flare on the Sun is estimated to be $1.02 \times 10^{31}$ erg. Therefore, on a log–log scale the linear dependence (3) for 30 years may be written as

$$\log N = a - b \log E, \quad a = 20.65, \quad b = 0.666. \quad (4)$$

The absolute maximum of limiting energy in one X-ray flare is estimated to be $E = 3.3 \times 10^{32}$ erg (1989). As was shown by Gershberg and Shakhovskaya (1983), from Eq. (4) it follows that the time-averaged power of flare radiations $\varepsilon/T$ is essentially dependent on the values of $E_{\text{max}}$ and $E_{\text{min}}$ ($T = 1$ year). The value of the spectral index $b$ indicates what
sorts of flare provide the main contributions to $e/T$. If $b < 1$, the rare but more powerful flares are the main contributors; if $b > 1$, frequent but faint flares play the main role.

The fact that $b(X) < b(\text{opt})$ over the three cycles means that the frequency of X-ray flares must be lower than that of optical flares, all other factors $E$ being equal. As the energy range in X-ray flares is less than or equal to the energy range in optical flares (Kurochka, 1987), this means that the flaring process in the chromosphere is dominant over the X-ray flare process in the corona, rather than the reverse.

4.1 The Largest Energy Flares

The absolute maximum of limiting energy in one solar flare was estimated as $E = 3.3 \times 10^{32}$ erg in 1989. An estimation of the largest energy (Table 1) that can be released during one flare is of primary significance since most powerful flares provide the most stringent constraints to the theory of the flare source of energy (Gershberg et al., 1987).

The time distribution of flares with different energies is rather random. A meaningful estimation of the largest energy flare is possible only from a statistical study of the flare activity over a sufficient interval, as made in this study. Irrespective of the physical nature of flare activity, a maximum allowed energy per flare should exist (Table 1, second and sixth columns). In the vicinity of this limiting energy ($\log E_m$) the energy spectrum of flares must be saturated (Figure 1). Therefore the limiting energy of solar flares can be estimated with confidence because the spectrum saturation region was found for each year (1972–2001).

From Eq. (4) it follows that the whole energy spectrum takes a long time to realize (Gershberg and Shakhovskaya, 1983). Taking into consideration the low energy activity of the Sun ($10^{32}$ erg) compared with that of the stellar flares ($10^{35}$ erg) (Gershberg, 1998), we expect that $T = 1$ year is sufficient time for realization of the flare energy spectrum.

Within the frameworks of modern gas-dynamic flare theory (Livshits and Katsova, 1996) the magnetic energy could be converted into the energy of flares by virtually instantly giving rise to numerous subflares. Hence the low and intermediate energies of flares can be realized within a time less than $T = 1$ year.

On the other hand, the lack of large magnetic structures may lead to a systematic scarcity of a large energy of flares and may explain the systematic drops in $E$ near the high region of spectrum saturation (Figure 1).

4.2 The Cyclic Variations in the Power-law Spectrum Parameters

The cyclic variation in the energy spectrum most strongly expresses itself in the limiting energy parameters, that is $\log E_m$ (Figure 2). This is a new result absent in the early studies (Kasinsky and Sotnikova, 1996).

The energy spectrum exponent $b$ has somewhat increased from the epoch of the cycle minimum to the epoch of the cycle maximum, which means a positive correlation with the Wolf number $W$ and flare occurrence rate. The mean of the epoch of the three minima is $(b) = 0.637 \pm 0.005$, and the mean of the epoch of the three maxima is $(b) = 0.715 \pm 0.005$. The increase in the energy spectrum exponent $b$ from the epoch of the minimum to the epoch of the maximum means an increase in the flare occurrence rate. Therefore, the epoch of the cycle maximum is characterized by a relative abundance of frequent but energetically weak flares, while the epoch of the cycle minimum is characterized by the predominance of rare but powerful events. Probably, this effect manifests itself best for optical flares since they are much more powerful than X-ray flares.

The cycle-like variation in the energy spectrum exponent $b$ with the phase of an 11-year solar cycle may serve as a fundamental reference dependence for identifying a similar cyclic
variability for UV Ceti red dwarf stars. In fact, a periodicity with a time of about 7.3 years was observed on the red dwarf EV LAC in distributions of $b$ and average flare energy parameters (Alekseev and Gershberg, 1997).

5 SUMMARY OF RESULTS

An investigation of the solar flare energy spectrum for three cycles, Nos 21–23 (1972–2001) has provided the following new results.

(1) The integral energy spectrum of solar flares has a power-law form. The cycle-averaged index of the power spectrum in X-rays is $b = 0.666 \pm 0.005$. The range of variations ($0.509 < b < 0.805$) is in reasonable agreement with the previous estimates.

(2) The exponent $b$ undergoes variations with the 11-year cycle phase, and correlates with the Wolf number $W$ and with the number $N$ of flares. The exponent $b$ increases from the epoch of the minimum to the epoch of the maximum of the cycle. The mean of the epoch of the minima is $\langle b \rangle = 0.637$ and the mean of the epoch of the maxima is $\langle b \rangle = 0.715 \pm 0.005$. When the $b$ index increases, the contributions of numerous weak flares grows.

(3) There is a strong 11-year modulation in the limiting energy of flares ($\log E_{\text{m}}$) and a sizeable modulation of the $b$ index with the Wolf number $W$ as well as with the number $N$ of flares per year. An 11-year modulation is also present in the ‘deficiency’ in the $E$ parameter (‘amount of drop in $E$’) with the Wolf number as well as with the number $N$ of flares. There is a positive trend of $b$ with a time of more than 30 years. The variations in the flare energy spectrum exponent $b$ and limiting energy with the phase of the 11-year cycle can serve as the reference dependences for identifying a similar cyclic variability of the energy spectrum of red dwarf stars.

Acknowledgement

We are indebted to Professor V. Mikhalkovsky for his assistance in preparing the English version of the manuscript.

References


