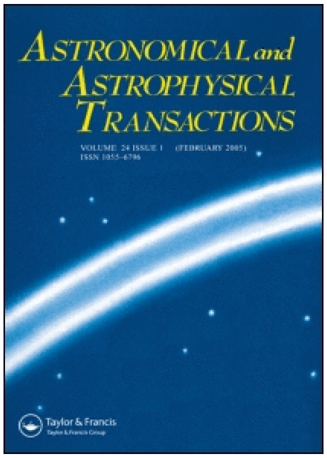

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THE THEORY OF ORBITS IN NON-STATIONARY STELLAR SYSTEMS

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The review of results obtained on the investigation of orbits in non-stationary stellar systems at the Astrophysical Institute is presented. The main interest is the theory of orbits in binary and triple systems of stars and the theory of orbits in regular potentials of galaxies with variable mass.

Keywords: Orbits; Non-stationary systems; Stability

1 INTRODUCTION

In reviews on the theory of orbits in stellar systems (Timoshkova and Kholshevnikov, 1982; Antonov, 1985) the work of the Fessenkov Astrophysical Institute on this problem is notable in that it considers the non-stationarity of gravitational fields. The development of this work at the Institute began from the investigation of the evolution of orbits of wide binary stars with intensive corpuscular radiation in a new formulation which additionally takes into account the effects of the behaviour of isotropically radiating particles (Idlis and Omarov, 1960). This 'physical' two-body problem with decreasing masses is sometimes called the Idlis–Omarov problem (Gelfgat and Berkovic, 1975). Also a description of that problem by other workers with reference to the original research by Idlis and Omarov may be found in the fundamental review investigations by Hadjidemetriou (1967) and Mikhailov (1985). In the analytical theory of that problem, Omarov (1964a) used for the first time the perturbed variables of the aperiodic motion on conic section, later called Omarov–Hadjidemetriou elements (Polyakhova, 1989). Omarov (1964b) also obtained qualitative estimates for the value of the corresponding perturbed functions for real observable binary stars with stationary mass loss. The importance of Omarov's result was also noted in the scientific literature (Schrivastava and Ishwar, 1983). For stellar dynamics the integrated problem (Omarov, 1972a) on the motion of particle in the gravitational field of point mass and extending gravitational background, the density of which varies with time proportional to t^{-2}, is of interest (the solution evidently is valid according to the same law in the case of a pressing background) (Antonov, 1985). On the basis of that problem the gravitational influence of

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the cosmological background with a critical density on the dynamics of 'supergalactic' scale systems was investigated (Omarov, 1972b; 1975).

The indicated references suggested the development of investigations of non-stationary dynamic problems of astronomy as one of the main directions of the activity of the laboratory of the dynamics of gravitating systems of the Fessenkov Astrophysical Institute. Now we give some of our results.

2 INTEGRABLE CASES OF THE NON-STATIONARY HAMILTON–JACOBI EQUATION

For problems of celestial mechanics and stellar dynamics the integrability of canonical equations of motion for corresponding non-stationary model schemes is important. Bekov and Omarov (1978a) and Bekov (1986a) described the new integrable cases for the Hamilton-Jacobi equation in the form

$$p + \frac{1}{2}\sum_{i=1}^{n} g^{ii} p_i^2 + \sum_{i=1}^{n} h^i p_i - U = 0 \left(p = \frac{\partial V}{\partial t}, p_i = \frac{\partial V}{\partial q_i} \right). \quad (1)$$

They are the generalization of Jarov-Jarovoy's (1963) results for the equation under consideration and include the Demin (1968), the Liouville and the Stackel (Duboshin, 1975) cases of integrability.

Here the Hamiltonian function of system in the form

$$H = \frac{1}{2}\frac{\gamma(t)}{b}\sum_{i=1}^{n}\frac{1}{a_i(q_i)}\left(p_i - \varphi\frac{\partial \Phi}{\partial q_i}\right)^2 - \dot{\varphi}(t)\Phi - \frac{\gamma(t)}{b}\sum_{i=1}^{n} U_i(q_i), \quad (2)$$

in which

$$b = \sum_{i=1}^{n} b_i(q_i),$$

where a_i, b_i, U_i and $\Phi(q_1, q_2, \ldots, q_n)$ are arbitrary functions of the generalized coordinates q_i and γ and φ are continuous functions of time, gives the generalization of the Liouville cases of integrability; also the Hamiltonian function of system in the form

$$H = \frac{1}{2}\gamma\sum_{i=1}^{n}\left[\frac{A_i}{a_i}\left(p_i - \varphi\frac{\partial \Phi}{\partial q_i} - \frac{\partial B}{\partial q_i}\right)^2 - A_i b_i\right] - \dot{\varphi}\Phi - \gamma\sum_{i=1}^{n} A_i U_i(q_i), \quad (3)$$

where

$$A_i = \frac{1}{\Delta}\frac{\partial \Delta}{\partial \varphi_{i1}},$$

and a_i, b_i, $\Phi(q_1, q_2, \ldots, q_n)$ and $B(q_1, q_2, \ldots, q_n)$ are arbitrary functions of the generalized coordinates q_i, the determinant $\Delta = |\varphi_{ij}(q_i)|$ is unequal identically to zero and $\gamma(t)$ and $\varphi(t)$ are functions of time, gives the generalization of the Stackel cases of integrability.

Further development of the Hamiltonian formalism was obtained by Bekov (1986a). The method of reducing of one-class non-autonomous dynamic systems to the canonical form is given, and their integrable cases shown. The comparison theorems permitted by the form of the Hamiltonian function to determine the integrability of dynamic systems are given. As applications, the two-body problem, the problem of two fixed centres and the straight-line version of the restricted three-body problem with variable masses in addition to a resisting and gravitating background are considered.

In the real astronomical two-body problem with variable mass the role of the factor additional to the Newtonian gravitation factor, which is not directly connected to the mass variation process but invariantly coexistent with concrete conditions of that process, for example the resistance of accreting or radiating matter may be important. The Hamiltonian formalism is used (Demchenko and Omarov, 1977) for the problem of two similar bodies with variable mass. On this basis (Omarov and Omarkulov, 1982; Minglibaev, 1992) the equations for the two-body problem with variable mass analogous to Jacobi, Delaunay and Poincaré canonical elements are obtained.

The solution of non-stationary scheme of the generalized two-fixed-centres problem with variable gravitation constant G is of interest (Bekov and Omarov, 1978b). The force function U of the problem of a barycentric system of Cartesian coordinates Oxyz with an applicate axis along the line of centres P_1P_2 have the form

$$U = \frac{G(t)m}{2}\left(\frac{1+\sigma \mathrm{i}}{r_1} + \frac{1-\sigma \mathrm{i}}{r_2}\right), \tag{4}$$

where

$$\begin{aligned} r_1 &= \left\{x^2 + y^2 + [z - c(\sigma + i)]^2\right\}^{1/2}, \\ r_2 &= \left\{x^2 + y^2 + [z - c(\sigma - \mathrm{i}]^2\right\}^{1/2} \\ \mathrm{i} &= (-1)^{1/2}, \end{aligned}$$

where m, σ and c are constants and $G(t)$ is a gravitational constant variable in time. On the basis of this problem an intermediate orbit of a test body that moves in the gravitational field of a non-spherical body taking into account the variability of the gravitational constant is constructed (Bekov and Nurgaliev, 1979). The differential equations for elements of an intermediate orbit are obtained. The importance of this result was noted in the review by Timoshkova and Kholshevnikov (1982). The solution and results obtained are valid for the problem under consideration with variable masses $m(t)$ of centres and for the more common case of variability of gravitational parameter of system $\mu(t) = Gm$ (Bekov and Omarov, 1978a; Bekov, 1986a,b).

3 THE EVOLUTION OF ORBITS IN NON-STATIONARY MODELS OF MULTIPLE STARS AND STAR CLUSTERS

The non-stationary scheme of the three-body problem with Newtonian–elastic interaction (Omarov, 1972a, 1975) may also be used for analysis of dynamics of the first-generation binary stars in a freely pressing protogalaxy (Kozhanov, 1982). As result it is possible to investigate the character of the test body motion at a sufficiently large distance from the central gravitating body. The criteria for escape of a star from the system are found; the motion of stars that had been in the deep regions of the system considered, are analysed.

Qualitative estimates for the coordinates of the relative orbits of the two bodies were obtained by Glikman (1978). In this case an analysis of the motion of the two bodies is made when the mass is decreasing according to the Eddington–Jeans law $\dot{M} = -\alpha M^n$. The additional question of trapping in the two-body problem with variable masses was dealt with by Omarov and Minglibaev (1983).

Using the autonomization method (Bekov, 1989) an analysis of integrable cases and trajectories of motion in the Gylden–Mestschersky problem was made. For the laws of mass variation obtained, all possible trajectories of motion were indicated; the class of orbits with variable parameter and constant eccentricity were considered in detail. For this class of orbits the law of mass variation both in the parameter form and in the explicit dependence on time were established. For the case of the periodic law of mass variation in the same problem (Bekov, 1993a)

$$[\mu(t)]^{2/3} = A + B\sin(at + b), \tag{5}$$

where $\mu(t) = GM(t)$ and A, B, a and b are constants, the orbit and its elements were obtained, the qualitative peculiarities of the motion were adduced, and the disintegration and capture time scales in the system were estimated. The parametric solutions of this problem were considered by Bekov (1990a) and Mychelkin (1990). Here, the mass $\mu(t)$ of binary system may be chosen, in particular, as the parameter. The connection between the Bertran and the Gylden–Mestschersky problems and the analysis of the character of orbits was investigated by Mychelkin and Mychelkin (1994, 1997).

In the Gylden–Mestschersky problem the intermediate motion, that is the aperiodic motion on a quasiconical section with variable parameter

$$r = \left(\frac{\mu}{\mu_0}\right)^k \frac{p}{1 + e\cos\varphi}, \quad \int_0^{\varphi} \frac{\mathrm{d}\varphi}{(1 + e\cos\varphi)^2} = \int_0^t \left(\frac{\mu}{\mu_0}\right)^{(1-3k)/2} \frac{\mu_0^{1/2}}{p^{3/2}}\,\mathrm{d}t, \tag{6}$$

where r and φ are the polar coordinates, $\mu(t)$ is the gravitational parameter and k is a constant, was found (Bekov, 1993b). This intermediate motion is the most general of those proposed earlier, and the Newton and Lagrange equations for osculating elements of that intermediate motion obtained in this work contain, as special cases, the well-known results given earlier (Minglibaev and Omarov, 1984; Bekov, 1993b).

In non-stationary gravitating systems with axis symmetry there is a definite class of circular and spiral orbits, which play an important role in the dynamics of those systems. Bekov (1981, 1982) and Bekov *et al.* (1997b) considered circular and spiral orbits in gravitational fields with an axial symmetry. Conditions of the existence and the stability of the circular and spiral orbits were obtained. The development of these results was given by Bekov and Omarkulov (1986), where the stability criteria of ring galaxies were established and the models of peculiar ring galaxies were generalized, taking into account the gravitational influence of the corona of system.

The results of analysis of the straight-line restricted three-body problem with variable mass are used to interpret the structure of some peculiar galaxies with cerns, the masses of which vary with time. The exact solutions of the problem in different formulations and the detailed analysis of particular solutions (the collinear L_1, L_2 and L_3 solutions and the special L_0 Lagrange ring) for various time dependences of the masses have been given by Bekov (1987, 1991).

The collinear L_1, L_2 and L_3 and the triangular L_4 and L_5 solutions in the classical three-body problem with constant masses are well known. The meanings of these solutions, namely the libration points in the analysis of motion in the restricted three-body problem, are also well known. The existence of five libration points (collinear and triangular), analogous to the classical L_i ($i = 1, 2, , 5$), in a restricted three-body problem with variable masses was established by Gelfgat (1973). Here it was proposed that the isotropic variation in the masses of the main bodies takes place in accordance with the united Mestschersky law. In this problem the new coplanar libration points L_6 and L_7, located outside the rotation plane of the main bodies were found (Bekov, 1986c, 1988a). In the symmetrical case (equal masses of the main bodies), in the rotating coordinate system O$\xi\eta\zeta$, the libration points L_6 and L_7 settle down on the rotation axis Oζ symmetrically relative to the origin of coordinates (the barycentre), with the coordinates

$$\begin{array}{l} L_6\left(0,\, 0,\, +\,\rho_{12}\left[\left(\dfrac{\kappa}{\kappa-1}\right)^{2/3}-\dfrac{1}{4}\right]^{1/2}\right) \\ L_7\left(0,\, 0,\, -\,\rho_{12}\left[\left(\dfrac{\kappa}{\kappa-1}\right)^{2/3}-\dfrac{1}{4}\right]^{1/2}\right) \end{array} (\kappa > 1), \tag{7}$$

where ρ_{12} and κ are constants determined by the motion of main bodies. In the general case, the solutions

$$L_6(\xi^*, 0, +\,\zeta^*), \quad L_7(\xi^*, 0, -\,\zeta^*) \tag{8}$$

with coordinates $\xi = \xi^*$, $\eta = 0$ and $\zeta = \pm\zeta^*$, where ξ^* and ζ^* are constants, gives the libration points L_6 and L_7, different from the classical L_i ($i = 1, 2, \ldots, 5$). The importance of this result has been noted by Lukyanov (1989, 1992) and El-Shaboury (1990). Possible interpretations for near-star bipolar jets (flows due to mass flow from coplanar libration points of a close binary stellar system) were pointed out by Kardopolov *et al.* (1991). Further study of the appearance and disappearance of collinear L_1, L_2 and L_3, triangular L_4 and L_5, coplanar L_6 and L_7, ring L_0 and infinitely distant $L_{\pm\infty}$ solutions in the restricted problem of three variable-mass bodies for different time dependences of the masses of the main bodies and for some additional conditions imposed on the system parameters were considered by Bekov (1990b, 1993c).

An important role in the dynamic evolution of real gravitating systems is their non-stationarity, connected with mass variation of the system and the additional influence of the variable light pressure from the system's components. The motion of a test body in the field of the gravitating and radiating main bodies was considered on the basis of the non-stationary photogravitational three-body problem, in which the masses and reduction parameters of main bodies varied with time (Bekov *et al.*, 2001). The existence of collinear L_1, L_2 and L_3, triangular L_4 and L_5 and coplanar L_6 and L_7 solutions in this problem was shown. Accordingly the results of investigations on the three-body problem with variable masses were generalized.

Minglibaev (1990) investigated the averaged restricted three-body problem with variable masses.

The evolution of an n-body stellar system with variable mass, describing the behaviour of the interacting groups of galaxies, was studied by means of numerical methods (Omarov and Mukhametkalieva, 1982). It was shown that, if the module of the initial potential energy is much greater than the initial kinetic energy, the evolution of system will explode and, as a result, the velocities of individual galaxies may increase.

The Lagrange–Jacobi generalized virial equation for a dispersed stellar cluster plunged into a gravitating background and rotating in the Galaxy plane on a circular orbit with respect to its centre was determined by Kozhanov (1990a). From this a class of stellar orbits in the plane of an ellipsoidal cluster, coinciding with the Galaxy plane, is based. The new approach to investigation of stellar clusters dynamics, rotating in an elliptical orbit with respect to the Galaxy centre, was proposed (Kozhanov, 1990b, 1992a). By the Poincaré small-parameter method the solution of the third-order differential equation with periodic coefficients for the inertia moment of an ellipsoidal stellar cluster was found. The necessary conditions of equilibrium were established and the dependence of the stability zones of the considered systems on their geometric forms was discovered. The equations of the stellar cluster components rotating in an elliptical orbit with respect to the Galaxy centre, taking into account its tidal forces and the regular field of clusters and averaged with respect to the true anomaly, were derived (Kozhanov, 1992b, 1993). For this problem the analogy of the Jacobi integral, the generalization of virial equation and the Sundman inequality were obtained. On this basis the bifurcated curves, which separate the regions of possible and impossible motions of cluster stars, are built.

4 INVERSE PROBLEM OF DYNAMICS WITH A NON-STATIONARY LAGRANGIAN

The papers by Omarova and Kozhanov (1988), Omarov and Omarova (1998) and Omarova and Omarov (2002) were devoted to constructing the intermediate orbits and to determining the potentials for non-stationary problems of celestial mechanics and stellar dynamics by generalized Szebehely methods. The inverse problem of dynamics for systems with a non-stationary Lagrangian under a known integral of motion and given single-parameter family of evolving orbits was studied. The analogy of the Szebehely equation (Omarov and Omarova, 1998) for a non-stationary regular potential $U(r, t)$ of a stellar system was derived:

$$\begin{aligned}\frac{\partial U}{\partial r} = &-\frac{\beta^2}{r^5 f_r^3}(rf_{rr}f_\theta^2 + rf_r^2 f_{\theta\theta} - 2rf_r f_\theta f_{r\theta} + r^2 f_r^3 + 2f_r f_\theta^2)\\ &-\frac{2\beta}{r^2 f_r^2}(f_r f_{\theta t} - f_\theta f_{rt}) - \frac{2\beta f_t}{r^3 f_r^3}(f_r f_\theta - rf_r f_{r\theta} + rf_{rr} f_\theta)\\ &-\frac{f_t^2 f_{rr}}{f_r^3} + \frac{2f_t f_{rt}}{f_r^2} - \frac{f_{tt} - \alpha f_t}{f_r},\end{aligned} \tag{9}$$

where an integral of the form $m(t)r^2\dot{\theta} = k$, the family of orbits $f(r, \ \theta, \ t) = c$ in polar coordinates r and θ, $\beta = k/m(t)$, and $\alpha = \dot{m}/m$ (the subscripts denote partial derivatives). With the aid of this equation, one can obtain an integrable non-stationary problem of celestial mechanics, in which various dissipative factors are taken into account (exact solutions may be used as the intermediate motion for analysis of the evolution of orbits in real binary systems). In particular, on the basis of this approach to the two-body problem with variable mass, that is

$$\ddot{\boldsymbol{r}} = -\mu(t)\frac{\boldsymbol{r}}{r^3} - \frac{\dot{m}}{m}\dot{\boldsymbol{r}}, \tag{10}$$

its adiabatic invariants were obtained. One should especially emphasize a version of the considered problem when the magnitude m is constant in analogy to the Szebehely equation. This will enable a non-stationary spatially symmetric potential in the dynamics of central motions, which might be of independent interest for the problems of evolution of stellar and galactic systems, to be reconstructed. The general form of the evolving orbit which is used to write the differential equations for non-stationary potential (Omarova and Omarov, 2002),

$$\begin{aligned}\frac{\partial U}{\partial r} = &-\frac{k^2}{p_0 r^2} - \frac{2k}{p_0^2}[-r\dot{e}\sin\varphi + re\dot{\omega}\cos\varphi + (\dot{e}\cos\varphi + e\dot{\omega}\sin\varphi)re\sin\varphi]\\ &+\frac{2r^3}{p_0^2}(\dot{e}\cos\varphi + e\dot{\omega}\sin\varphi)^2 - \frac{r^2}{p_0}(\ddot{e}\cos\varphi + 2\dot{e}\dot{\omega}\sin\varphi - e\dot{\omega}^2\cos\varphi + e\ddot{\omega}\sin\varphi),\end{aligned} \tag{11}$$

may also be interpreted as an osculating orbit of the perturbed Keplerian motion. In this case an additional transformation of the basic equation of the problem was made and an appropriate example of construction of a non-stationary potential of the gravitating system was demonstrated. Thus, in the general case of the family of orbits $f(x, y, t) = c$, the analogy of the Szebehely equation for the inverse problem can be used also for constructing non-stationary gravitational potentials, allowing a manifold of known osculating orbits under particular conditions. In particular, on the basis of this approach the non-stationary potential of ring galaxies was reconstructed.

5 CONCLUSION

We now mention briefly some other non-stationary problems not included in the items under consideration.

The generalized virial theorem for a system of fluctuating composition was obtained (Omarov, 1985). For the case of a collection of gravitational particles having a non-stationary composition and with the condition of self-coordination of these particles, the appropriate generalization of the known Poincaré–Eddington equation was given. The development of peculiar motion in an increasing gravitating environment taking into account the inhomogeneously distribution of mass was investigated (Kozhanov and Omarov, 1994). The formulae describing the different between the kinetic energies of inhomogeneously gravitating masses and its peculiar motions, depending on the inertia momentum of system, was deduced.

Now we consider the actual problems in which the mass variation and at the same time the variations in other physical parameters of interacting bodies are taken into account. On this topic the investigations by Bekov (1984, 1993d), Minglibaev and Mailybaev (1990) and Bekov *et al.* (1997a) are of interest. New model problem of celestial mechanics and stellar dynamics, namely the generalized problem of two centres with variable masses and variable distance between centres, were studied by Bekov (1984, 1986b, 1993d). On this basis the motion of the material point in the gravitational field of a non-spherical body with variable mass, size and form were considered. The solution of the problem was given, and the intermediate orbit of a test body in the gravitational field of a non-stationary non-spherical body was constructed. Differential equations for the elements of an intermediate orbit were deduced. The theory of motion of a material point (Bekov, 1993d) in the gravitational field of a non-spherical body with variable mass, size and form was also constructed.

To investigate the structural and dynamic peculiarities in the vicinity of evolutionary stars and galaxies the motion of test body in the external gravitational field of a rotating triaxial ellipsoid with a slowly varying mass, size and form was considered. Families of the equatorial, polar, ring, coplanar and z solutions were found for this problem (Bekov, 1988b, 1992). A qualitative analysis of the motion regions in this problem was given by Bekov (1990c). The obtained solutions may be used in different problems of stellar dynamics, and also in astrophysical applications. For example, the polar and z solutions may be considered as a possible gravitational model of jets (astrophysical mass flow in the vicinity of young stars and galaxies) (Bekov, 1995a), and the obtained libration points may serve as indicators of the potential variations of the formed star (Bekov, 1995b); the positions of the libration points simultaneously follow possible variations in the mass, size and form of the gravitating body.

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