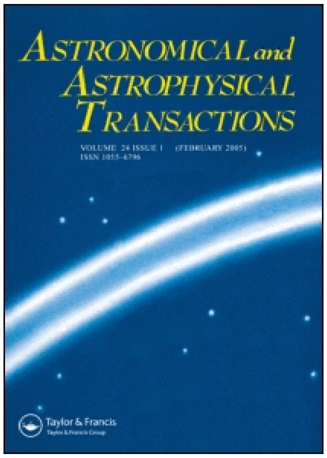

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THE DYNAMICS OF COSMIC STRINGS IN THE LARGE METAGALAXY

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The wide range of problems related to the dynamics of cosmic strings in the large Metagalaxy as an observable part of the Universe is presented. In this article we emphasize the evolution of a network of cosmic strings, the self-consistent dynamics of N cosmic strings, and the behaviour of a single cosmic string in external gravitational backgrounds. Some results obtained by authors are also noted.

Keywords: Dynamics of cosmic strings; large Metagalaxy

1 INTRODUCTION

Gauge theories with spontaneously broken symmetry predict the existence of string-like vacuum structures. The simplest theory of this type is the Abelian Higgs model, which when combined with the gravitational field theory leads to the vortex solutions of the Einstein equations.

Hence, cosmic strings are the strongly stretched one-dimensional configurations of a scalar field which have microscopic cross-sectional sizes. These strings are topologically stable objects and according to the inflational Universe model appeared in phase transitions during the evolution of the Universe (Kibble, 1976; Zel'dovich, 1980; Everett, 1981; Vilenkin, 1981a; Kibble *et al.*, 1982; Vachaspati and Vilenkin, 1984; Garfinkle, 1985; Stein-Shabes, 1988; Scherrer and Vilenkin, 1997; Meierovich, 2001).

Initially the strings were Brownian-like trajectories joined at cosmic networks. Then, owing to their tension, they became stretched and began to move with a speed close to the speed of light. This resulted in an cross-section and led to the appearance of closed loops. The closed cosmic threads crossed, shrank and emitted gravitational radiation. However, the open strings became almost straight and formed a system of N cosmic strings that evolved owing to vacuum and classical interactions. Finally the single strings, which radically changed the space–time structure and the physical processes near them, were still in the Metagalaxy.

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For all these cases the trajectory of a string can be written

$$x^\alpha = x^\alpha(\tau, \rho) \quad (\alpha = 0, 1, 2, 3) \tag{1}$$

where τ and ρ are the time-like and space-like parameters respectively. The string action in flat space–time is (Barbashov and Chernikov, 1974)

$$S_0 = -\mu \int\int_\Omega [(u^\alpha u_\alpha)(l^\beta l_\beta) - (u^\alpha l_\alpha)^2]^{1/2}\, \mathrm{d}\tau\, \mathrm{d}\rho, \tag{2}$$

where $u^\alpha = \mathrm{d}x/\mathrm{d}\tau = \dot{x}^\alpha, l^\alpha = \mathrm{d}x^\alpha/\mathrm{d}\rho = x'^\alpha, \mu$ is the linear mass density and Ω is the two-dimensional world hypersurface which is covered by the string. Using the reparametrization $\tau \to \overset{*}{\tau}(\tau, \rho), \rho \to \overset{*}{\rho}(\tau, \rho)$ for coordinates on the surface it is possible to rewrite action (2) as

$$S_0 = -\mu \int\int_\Omega (- g)^{1/2}\, \mathrm{d}\overset{*}{\tau}\, \mathrm{d}\overset{*}{\rho}, \tag{3}$$

where g is the metric of the space–time manifold. By varying equation (3) it is possible to obtain the string equation of motion (the asterisk over parameters will be omitted everywhere)

$$\dot{u}^\alpha - \acute{l}^\alpha = 0 \tag{4}$$

with the constraint $(u^\alpha \pm l^\alpha)^2 = 0$. Equation (4) can be derived from the conservation law $\partial_\beta T^{\alpha\beta} = 0$, if the energy–momentum tensor is

$$T^{\alpha\beta} = \mu \int_V (u^\alpha u^\beta - l^\alpha l^\beta)\delta_4(x - x_0)\, \mathrm{d}V. \tag{5}$$

For an arbitrary curved four-dimensional space–time equation of motion, equation (4) allows the natural generalization

$$\dot{u}^\alpha - \acute{l}^\alpha + \Gamma^\alpha_{\beta\gamma}(u^\beta u^\gamma - l^\beta l^\gamma) = 0. \tag{6}$$

That is why the equation of motion for a cosmic string in general relativity becomes nonlinear. For instance, in de Sitter space–time, four equation (6), as shown by Barbashov and Chernikov (1974), reduce to two nonlinear Liouville equations. Using the geometrical approach to the dynamics of a relativistic string (the Plato problem for finding the minimal world surface) it is possible to reduce equation (4) to the nonlinear Liouville equation for a complex function [Barbashov and Nesterenko (1987)] also. Moreover, the string equation (4), as argued by Kobayashi (1985) and Barbashov and Nesterenko (1987), may be converted to a nonlinear equation of sine–Gordon type, for example, to describe waves of a widerange of classes.

On the other hand, Chechin and Omarov (1999) have proposed a new version of nonlinear strings. This approach is based on the generalization of the stress–energy tensor (5) by primordial accounting of the nonlinear terms in it:

$$T^{\alpha\beta} = \mu \int_V \left[u^\alpha u^\beta - l^\alpha l^\beta \left(1 - \frac{\varepsilon^2}{2} l^\gamma l_\gamma\right)\right]\delta_4(x - x_0)\, \mathrm{d}V, \tag{7}$$

where ε is the coupling constant of order not smaller than the parameter in the Higgs model. Thus, we obtain the following modification of the string equation of motion:

$$\dot{u}^\alpha - \acute{l}^\alpha \left(1 - \frac{\varepsilon^2}{2} l^\gamma l_\gamma\right) = 0, \tag{8}$$

which is drastically nonlinear in flat space–time, yet.

2 EVOLUTION OF THE NETWORK OF COSMIC STRINGS

When the strings formed, their motion was damped by friction with the surrounding matter. However, after the Universe cooler to 10^{12} GeV, matter friction became negligible, and the strings move freely, according to the equations of motion described by the Nambu–Goto action. The concept when friction for a cosmic string is no larger important is the scaling solution.

After cooling, the system of cosmic strings looks like Brownian strings or Brownian trajectories with a persistent length ξ_0. Hence, the energy density of strings at formation is inversely proportional to ξ_0^2. As the Universe expanded, the strings tended to straighten out and at time t the energy density can be written down as follows:

$$\epsilon(t) \geq \mu\xi^{-2} \sim \mu t^{-2}. \tag{9}$$

However, the regime for the evolution of strings, as is absolutely clear, strongly depends on the expansion parameter $a(t)$ in the Friedmann model. Moreover, it also depends on string intercommutations, loop fragmentation, gravitational wave radiation and other cosmological factors. That is why the main aim is to determine the most adequate expression for the evolution of the linear energy density with time and to find the scaling solution.

The first researcher to use this method was apparently Vilenkin (1981b), who found the dependence of the energy density on the expansion parameter $a(t) \sim t^{\alpha}$. He argued that equation (9) relates to $\alpha > \frac{1}{2}$ (the matter-dominated Universe) but, for $\alpha = \frac{1}{2}$ (the radiation-dominated Universe),

$$\epsilon(t) \sim \mu t^{-2} \ln\left(\frac{t}{\xi}\right), \tag{10}$$

and, for $\alpha < \frac{1}{2}$,

$$\epsilon(t) \sim \mu t^{-2} \left(\frac{t}{\xi}\right)^{2(1-2\alpha)/(1-\alpha)}. \tag{11}$$

It has been said that the effect of cosmic loop fragmentation is very important for understanding the evolution of the network of cosmic strings. It is possible to investigate this by examining the statistical properties of the system of strings at formation. In particular, the properties studied are the number density and size distribution of closed loops. To obtain these results the hypothesis that the system of strings is scale invariant, that is that the system looks the same (in terms of statistical properties) at all resolutions, was put forward. So, the number of closed loops with sizes from r to $r + \mathrm{d}r$ per unit volume and the length distribution of loops at time t are (Vachaspati and Vilenkin, 1984)

$$\mathrm{d}n \sim r^{-4}\,\mathrm{d}r \tag{12}$$

and

$$\mathrm{d}l \sim t^{-3/2} r^{-5/2}\,\mathrm{d}r \tag{13}$$

respectively.

What is the subsequent fate of a loops? There are several possible scenarios for their evolution. For example, according to Smith and Vilenkin (1987) the ancestral loop has a certain probability of becoming an unending cascade in which an infinite number of loops are produced.

Another, apparently more probable scenario is the stretching of cosmic strings. The simplest description of this dynamic process in the Friedmann Universe has been given in the article by Turok and Bhattacharjee (1984) on the basis of the string equation (2). As a result it was found that the scaling equation connects the rate of energy change and the rate of change in the scale parameter of the Universe in a direct proportional dependence. For moving and interacting cosmic loops this equation was then generalized by Bennet (1986). Thus, the interaction of cosmic strings described by a special loop production function arose from the plausible assumption that a loop has a constant probability that it will be split into two equal-sized pieces. (Another form of the loop production function was suggested by Turok† on the hypothesis that a 'parent' loop fragments in such a way that the resulting 'child' loops have approximately the same sizes.) However, these functions have a number of adjustable parameters and hence do not provide any firm predictions in on the evolution of the network of cosmic strings using an analytical model only.

To overcome this drawback, Albrecht and Turok (1989) devised on improved method for describing the interaction of cosmic strings. The main idea of their method is to use a natural single scale: the Hubble radius. All other scales (the typical radius of curvature of the string, the typical distance between strings, etc.) are determined in terms of R_{H}. Thus the total length of strings is $\rho_{\mathrm{L}}\alpha\mu/R_{\mathrm{H}}^2$. So, the scaling equation is

$$\frac{\mathrm{d}\rho_{\mathrm{L}}}{\mathrm{d}t} = -3H\rho_{\mathrm{L}} + H\frac{\xi}{R_{\mathrm{H}}}\rho_{\mathrm{L}} - C\frac{\rho_{\mathrm{L}}}{\xi}, \tag{14}$$

where the number ξ of correlation lengths per Hubble radius is

$$\frac{\xi}{R_{\mathrm{H}}} = \left(\frac{\mu}{\rho_{\mathrm{L}}R_{\mathrm{H}}^2}\right)^{1/2}, \tag{15}$$

H is the Hubble constant and C is the loop's chopping efficiency. From numerical calculations it was found that for the radiation-dominated era, $C = 0.074$. The main theoretical significance of this model is the possibility of predicting the scaling density in the matter-dominated era and at the transition between the two eras based on the scaling density of the radiation-dominated era only.

In spite of obtaining the single-valued scaling from equation (14), the very important question remains: does the evolutionary process really lead to a 'scaling' regime in which the characteristic length scale describing the string network increase proportionally to the horizon distances? In particular it concerns the evolutionary processes at small-scale distances. To investigate this question, Austin, Copeland and Kibble (ACK) (1993; 1995) considered for an arbitrary length scale l along a string the probability distribution for the end-to-end distance (or extension) $\boldsymbol{r}$. Using this probability distribution $P[\boldsymbol{r}(l)]$ they derived the evolution equation for this quantity, which includes terms representing the effect of the stretching of cosmic strings, their gravitational radiation, the intercommutation of long strings and the formation of loops. Making that probability distribution Gaussian, they introduced the variance $K(l) = \boldsymbol{r}^2$, which depends on the types (double, triple, etc.) of joint probabilities and on the types (intercommutation, loop formation, stretching, emission of gravitational waves, etc.) of evolution of strings. The evolution equation for l, for instance, has the form (referred to hereafter as the ACK equation)

$$\dot{l} = \alpha(\boldsymbol{r}, l)\frac{\dot{R}}{R}l, \tag{16}$$

†See paper by Bennet (1986).

with the coefficient

$$\alpha(\boldsymbol{r}, l) = \bar{\alpha} + \hat{\alpha}(l)\frac{r^2 - K(l)}{K(l)}, \tag{17}$$

where, in turn, $\bar{\alpha}$ and $\hat{\alpha}$ depend in complicated forms on the correlation between the left and right wave movers along a string, on the distances $\xi, \bar{\xi}$ and ζ including the different types of scale (from large to small) that are typical of the evolution of the cosmic string network.

Because the ACK equation is complicated, another scheme in which all the relevant nonlinearities and non-localities of loops (of length l) evolution were combined into a single quantity was put forward (Vilenkin, 1990; Embacher, 1994). This was realised by changing from a standard to an average string whose coordinates were on a given scale ζ. As a result, the average string dynamics are described by the system of equations (Embacher, 1994)

$$\begin{aligned} \dot{x}(l) &= p(l) + lAx' + \tfrac{1}{12}\zeta^2\left(A + \frac{\dot{\zeta}}{\zeta}\right)x''(l), \\ \dot{p}(l) &= x'' + lAp'(l) + \tfrac{1}{12}\zeta^2\left(A + \frac{\dot{\zeta}}{\zeta}\right)p''(l), \end{aligned} \tag{18}$$

with the corresponding equation for the rate of change in ζ.

Other aspects of the dynamics of a cosmic string network (a network with monopoles, a dependence on the number of infinite strings in the special field index, etc.) have been discussed by Aryal *et al.* (1986), Vachaspati and Vilenkin (1987), Allen and Cardwell (1990), Robinson and Yetes (1996) and Copeland *et al.* (1998).

3 DYNAMICS OF *N* COSMIC STRINGS

The problem of N straight cosmic strings was considered, probably for the first time, by Letelier (1979). He put forward a concept in general relativity about a cloud of strings or N, infinitely long crossless strings that occurred continuously at every point of the world surface. Naturally, any interaction between strings was equal to zero.

This idea was developed by Letelier (1983), who considered a cloud of strings with some interaction between them and which was described by different equations of state. These states relate to the different types of string (Nambu strings and Takabayasi strings).

The central idea of these papers consists in constructing appropriate energy–momentum tensors to describe cosmological models of different types (Bianchi type I, Kantowski–Sachs type, etc.).

More complicated equations of state for a gas of classical strings have been proposed by de Vega and Sánchez (1994). The equations involve space–time multidimensionality and the generalized perfect fluid relation $p = (\gamma - 1)p$ for the various types of γ. Bearing in mind that γ itself depends on the dimensionality D they considered unstable $\gamma = (D-2)/\,(D-1)$, dual $\gamma = D/(D-1)$ and stable $(\gamma - 1)$ Universe models. Recently Larsen and Sánchez (1996a,b) studied multistrings silutions in a number of curved space–time situations (de Sitter, anti-de Sitter, etc.).

To search for the dynamics of cosmic strings in the large Metagalaxy the dynamics of N, discretely located strings are highly important. This problem has been investigated from different viewpoints by a number of workers.

One of the first articles was devoted to search for the $N = 2$ global $U(1)$ cosmic strings interaction (Shellard, 1987). Based on the Lagrangian of a simple Goldstone model for the 'Mexican hat' potential approximate energy per unit of single string was found. The interaction energy of two strings was calculated under the usual assumption that a two-string scalar field Φ can be approximated by the product of their wavefunctions:

$$\Phi(\boldsymbol{r}, \boldsymbol{r}_1, \boldsymbol{r}_2) \approx \Phi_1(\boldsymbol{r} - \boldsymbol{r}_1)\Phi_2(\boldsymbol{r} - \boldsymbol{r}_2). \tag{19}$$

Hence, the interaction energy of two parallel global $U(1)$ cosmic strings is

$$E_{\text{int}} = 2\pi\eta \ln\left(\frac{R}{\lambda}\right), \tag{20}$$

where $R = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$ is the string core width and η is the energy specific to the scale of symmetry breach. This interaction energy leads to parallel string–antistring annihilation, to string–string repulsion, to intercommutation of two perpendicular cosmic strings, etc. All these effects have been demonstrated by numerical calculations.

The generalization of these results in the presence of an electromagnetic field has been considered by Bettencourt and Rivers (1995). The bundle of N motionless gauge cosmic strings in the Riemann–Cartan manifold was investigated by Prokof'ev (1995).

Another type of cosmic strings interaction, namely vacuum interaction, may be considered in the framework of quantum field theory. So, the paper by Bordag (1990) is devoted to the search for the vacuum interaction of two parallel gauge cosmic strings (an analogue of the Casimir effect).

To obtain the energy interaction mentioned above, it is necessary to write the field equations in the metric

$$\mathrm{d}s^2 = \mathrm{e}^{-\Omega(x)}(\mathrm{d}x_1^2 + \mathrm{d}x_2^2) + \mathrm{d}x_3^2 - \mathrm{d}x_0^2 \tag{21}$$

with $\Omega = \sum_{i=1}^{2} 8\mu_i \ln r_i$, where μ_i and r_i are the linear mass density and distance from the current field point x to the position of the ith string. Further reducing this to the Poisson equation allows us to find the corresponding Green function and with its aid to calculate the vacuum expectation of the energy per unit string length as the second-order perturbation. The final result is very simple:

$$E_{\text{int}} = -\frac{32}{45\pi}\frac{\mu_1\mu_2}{R^2}. \tag{22}$$

This expression shows the existence of an attractive force between two strings, which decreases with increasing distance as R^{-3}.

The generalization of this approach to the case of N motionless parallel cosmic strings (multistrings) has been performed by Galt'tsov *et al.* (1995). (Note that their result is different from that of Bordag in the coefficient $8/3$.) This procedure has been carried out by spreading the conformal power in equation (21) over N strings.

Moreover these results, as is obviously clear, are the consequence of the self-consistent approach to the dynamics of N cosmic strings (Omarov and Chechin, 1995).

For this we proceeded from the standard Lagrange method. Let us write the string action as follows (see equation (3)):

$$S_0 = -\int_{\Omega} \Lambda_0 \,\mathrm{d}\Omega, \tag{23}$$

where Λ_0 is the Lagrangian density and $\mathrm{d}\Omega$ is the four-dimensional invariant element of integration. For a free string the Lagrangian density is

$$\Lambda_0 = (-g)^{1/2}\mu. \tag{24}$$

Using the continuity conditions for a string (space-like curve) and for a string trajectory (time-like curve) we obtain two expressions:

$$\boldsymbol{\nabla}_\alpha(\mu l^\alpha) = \mathbf{0}, \quad \boldsymbol{\nabla}_\alpha(\mu u^\alpha) = \mathbf{0}. \tag{25}$$

To calculate the variation in equation (24), it is necessary to take into account that

$$\delta(-g)^{1/2} = (-g)^{1/2}\boldsymbol{\nabla}_\alpha\, \delta x^\alpha \tag{26}$$

and

$$\delta\mu = \mu(u_\alpha u^\beta - l_\alpha l^\beta)\boldsymbol{\nabla}_\alpha\, \delta x^\alpha. \tag{27}$$

Thus the variation in eqaution (23) may be written as

$$\delta S = -\int_\Omega \mu(u_\alpha u^\beta - l_\alpha l^\beta)\boldsymbol{\nabla}_\beta\, \delta x^\alpha\, \mathrm{d}\Omega. \tag{28}$$

From equation (28) we obtained the conservation law $\boldsymbol{\nabla}_\alpha T^{\alpha\beta} = 0$ with the energy–momentum tensor (5). In the case of N non-interacting strings the appropriate tensor is of the form

$$T^{\alpha\beta} = \sum_{i=1}^{N} \mu_i \int_{V_i} (u^{i_\alpha} u^{i_\beta} - l^{i_\alpha} l^{i_\beta})\delta_4(x - x_0^i)\, \mathrm{d}V_i \tag{29}$$

under the condition that their linear mass densities are constants.

To obtain the (approximate) metric of the gravitational field of N massive gauge cosmic strings it is sufficient to specify the general form of the permissible coordinate expansions within the framework of the method of Infeld and Plebansky (1960):

$$x'^0 = x^0 + \underbrace{a^0}_{1} + \underbrace{a^0}_{3} + \cdots, \quad x'^k = x^k + \underbrace{a^k}_{2} + \underbrace{a^k}_{4} + \cdots \tag{30}$$

Using the definition of the space-like and time-like vectors for string we obtain the following power series:

$$u^0 = 1 + \underbrace{u^0}_{2} + \underbrace{u^0}_{4} + \cdots, \quad u^k = \underbrace{u^k}_{1} + \underbrace{u^k}_{3} + \cdots \tag{31}$$

and

$$l^0 = 1 + \underbrace{l^0}_{1} + \underbrace{l^0}_{3} + \cdots, \quad l^k = \underbrace{l^k}_{0} + \underbrace{l^k}_{2} + \cdots. \tag{32}$$

Inserting these expansions into the Einstein equation we reduce them to the three-dimensional Poisson equations

$$\Delta \underbrace{h_{00}}_{2} = 0, \quad \Delta \underbrace{h_{kl}}_{2} = 8\pi \sum_{i=1}^{N} \underbrace{\overset{i}{\mu}}_{2} (\delta_{kl} + \underbrace{\overset{i}{l^k}}_{0}\underbrace{\overset{i}{l^l}}_{0})\delta_3(x - \overset{i}{x_0}). \tag{33}$$

Hence the gravitational field of N massive motionless cosmic strings takes the form

$$\mathrm{d}s^2 = \mathrm{d}x_0^2 - \left[1 + 8\sum_{i=1}^{N} \underbrace{\overset{i}{\mu}}_{2} \ln\left(\frac{r_i}{r_0}\right)\right](\mathrm{d}x_1^2 + \mathrm{d}x_2^2) - \mathrm{d}x_3^2, \tag{34}$$

which is similar (with reasonable accuracy and in the first approximation) to equation (21).

The self-consistent approach allows us to take into account naturally the movement of cosmic strings, together with their rotation and other physical characteristics. In other words it allows us to treat the dynamics of N cosmic strings with completeness characterizing N-body dynamics in general relativity (Infeld and Plebansky, 1960) and to search for its cosmological consequences (galaxy formation, gravitational lensing, etc.) in detail.

Some other approaches to the dynamics of N cosmic strings (binary cosmic strings, clouds of cosmic strings with monopoles, multistring solutions, etc.) have also been made (Stakel, 1980; Letelier, 1987; Culter, 1992; Combes *et al.*, 1994; Rohm and Dasgupta, 1996; Sánchez, 1997).

4 PHYSICS OF SINGLE COSMIC STRINGS

Most investigations on the dynamics of cosmic strings have been devoted to searching for the physics of a single cosmic string.

To search for the quantum dynamics of cosmic strings in curved space–times the induced action

$$S_0 = -\mu \int\int_{\Omega} \mathrm{d}^2 x\, g^{1/2} G^{\mathrm{AB}} g_{\mu\nu}\, \partial_{\mathrm{A}} x^{\mu}\, \partial_{\mathrm{B}} x^{\nu} \tag{35}$$

is used. (This action for classical strings is entirely equivalent to Nambu–Goto action (3)). Here G_{AB} describes the world-surface metric.

The main feature of string dynamics in a curved manifold is that the equations of motion (just as equation (4)) are nonlinear in x^{μ}. Thus, determination of their exact solutions entails great difficulties. To overcome this drawback a general perturbative scheme was proposed by de Vega and Sánchez (1987). They assumed that the centre-of-mass point of a closed string $q_{\mu}(\tau)$ yields the geodesic line. That is why it is possible to set the general coordinate expansion

$$x^{\mu}(\tau,\rho) = q^{\mu}(\tau) + \underbrace{\xi^{\mu}}_{1}(\tau,\rho) + \underbrace{\xi^{\mu}}_{2}(\tau,\rho) + \cdots \tag{36}$$

for a high-energy string. In Eq. (36) we limited the first-order and the second-order perturbations to powers of a small parameter $l_{\mathrm{Pl}}/R_{\mathrm{c}}$ only. Here l_{Pl} is the Planck length, and the length R_{c} characterizes the curvature radius of the external space–time.

This perturbative scheme has been successfully utilized to search for the dynamics of cosmic strings in different space–times. For instance, by examining string dynamics in a black-hole gravitational field and de Vega and Egusquiza (1993), Larsen and Sánches (1996a,b), de Vega and Sánchez (1988) found the impact parameters corresponding to the scattering amplitudes; Sánchez and Veneziano (1990) and de Vega and Egusquiza (1996) by searching for string propagation in cosmological backgrounds, determined the distances between string growth in the Friedmann Universe. Besides the string motion in a plane gravitational wave (de Vega *et al.* 1993), the string motion in a shock wave background of cosmic scales (Costa and de Vega, 1991) was also investigated.

Later the above-mentioned approximation was applied to give a law for the energy of a string. Within this method, namely the null-string approach (Larsen, 1994; Lousto and Sánchez, 1996), the string equation of motion and constraints were systematically expanded in powers of the parameter c (the speed of light on the world surface). The basis of the null-string method is the expansion of the string coordinates:

$$x^{\mu}(\tau,\rho) = q^{\mu}(\tau) + \frac{1}{c^2}\,\xi^{\mu}(\tau,\rho) + \frac{1}{c^4}\,\xi^{\mu}(\tau,\rho) + \cdots, \tag{37}$$

which is similar to the series (30). Using this weak-field approximation it was shown, in particular, that in the standard cosmological model the string length grows like a conformal factor.

The de Vega–Sánchez method was further developed by Zheltukhin and Roschupkin (1997) and Roschupkin and Zheltukhin (1999) (who obtained an easily managed expanding form of the functional string action).

A new type of cosmic string, instead of all these cited above, has been explored by Curtright *et al.* (1986). This is the so-called 'string with rigidity' that is described by the natural generalization of the string action (23), by including an extrinsic curve-dependent term in it. Hence, the total string action is

$$S_{\text{tot}} = S_0 + k\int\!\!\int_{\Omega} (-g)^{1/2} g^{\alpha\beta} K_{\alpha\beta}\,\mathrm{d}\Omega, \tag{38}$$

where k is the rigidity (or stiffness) coefficient and $K_{\alpha\beta}$ is the Gauss curvature tensor of the world surface. Using this action, new solutions that include finite-energy static configurations for the closed rotating loops in the flat Universe have been found.

In the search for cosmic string behaviour in the large Metagalaxy it is necessary to mention its movement in gaseous medium. In fact, the main cosmological consequence of the existence of cosmic strings is galaxy formation due to accretion of matter on their loops. Early discussions of this problem were based on the standard action and the assumption of one-to-one correspondence between string loops of a given radius and cosmological objects (galaxies) of a given mass (Vilenkin and Shafi, 1983; Stebbins, 1986; Turok and Brandenberger, 1986). Later it was found that the majority of clusters (of galaxies) are not seeded by single loops, but by a large mass concentration of small loops (Shellard and Brandenberger, 1988).

However, the dynamic consequences of the existence of cosmic strings in the large Metagalaxy are not confined by the formation of galaxies. In fact, Gasilov *et al.* (1984) considered that the process of luminosity of galaxy matter is due to the intersection of it by a cosmic string. In this the considered gas-dynamic process turns out to be unimportant for clusters of galaxies. If it is regarded as an intersection effect on galaxies, then it becomes essential. This is because the initial galaxy density increases by up to 600 times owing to its intersection by massive cosmic strings and this creates favourable conditions for the speeding up the process for producing stars.

The passage of a cosmic string though a galaxy leads to a back-reaction effect or a dynamic friction effect on the motion of the cosmic string. This effect has been treated by Garfinkle and Will (1987), who considered the motion of a long cosmic string though two types of field in the fiat Friedmann Universe. (The effect of dynamic friction on the centre-of-mass motion of the loops of a string, in particular, was calculated by Hogan (1984).) Thus, the following relativistic equation of motion was deduced.

$$\frac{\partial^2 \boldsymbol{r}}{\partial\eta^2} + \frac{1}{\chi^2}\left(\frac{2}{a}\frac{\mathrm{d}a}{\mathrm{d}\eta} + 16\pi^2\mu\varepsilon\Lambda aF(\chi)\right)\frac{\partial\boldsymbol{r}}{\partial\eta} = \frac{1}{\epsilon}\frac{\partial}{\partial\rho}\left(\frac{1}{\epsilon}\frac{\partial\boldsymbol{r}}{\partial\eta}\right). \tag{39}$$

Here the following notation is used: μ is the mass per unit length of the string; ε is the energy density of the fluid; the parameter Λ is the transverse size of that part of fluid whose motion correlated with the motion of a given segment of string; the quantity $F(\chi)$ describes the state of the fluid ($\frac{1}{3}\chi(4-\chi^{-2})$ for radiation, and $(1-\chi^{-2})^{1/2}$ for non-relativistic particles), where $\chi = [1-(\partial \boldsymbol{r}/\partial\eta)^2]^{-1/2}$; the scalar $\epsilon = \gamma[(\partial \boldsymbol{r}/\partial\rho)^2]^{1/2}$; $a = a(\eta)$ is the conformal factor of the Friedmann metric. Based on equation (39) it has been shown that both for the radiation-dominated Metagalaxy and for the matter-dominated Metagalaxy the term describing the dynamic friction becomes negligible. Thus the motion of the string is well approximated by its motion in curved space–time only. (Other aspects of dynamic friction during string motion have been discussed by Vilenkin (1991) and Avelino and Shellard (1995).)

Considering a cosmic string's motion in an expanded Universe, Thompson (1988) concentrated attention on the dynamics of the cusps. He found that the cusps are the result of string self-intersection, are not smoothed by the self-gravity of the string, emitted gravitational energy due to the reactive force, are annihilated by anticusps, etc.

Other aspects of the physics of a single cosmic string in the Metagalaxy are the production of ultahigh-energy cosmic rays (Hill and Schramm, 1987; Berezinsky and Vilenkin, 1997), radiation of Goldstone bosons (Vilenkin and Vachaspati, 1987), fragmentation of the cosmic string's loop (Scherrer and Press, 1989), gravitational radiation from cosmic strings (Vachaspati and Vilenkin, 1985; Sakellariadou, 1990; Allen *et al.* 1995), etc.

An essentially new method of studying the physics of a single cosmic string is the exploration of a test thread's dynamics in the gravitational field produced by a massive cosmic string (two-string problem) (Omarov and Chechin, 1999).

After the pioneering paper by Vilenkin (1981a) where the metric of a rectilinear massive cosmic string was deduced, many articles generalizing his result appeared. The metrics of a rotating charged superconducting cosmic string, a cosmic string with kink, a hollow cosmic string, etc., were also derived. However, all these disregarded one of the main features of cosmic string behaviour, namely their oscillations. The special case of this type of metric describing the space–time interval in the vicinity of a cosmic string with weak disturbance waves travelling along it with speed of light was deduced by Vachaspati (1986).

Omarov and Chechin (1999) derived the space–time interval of a finite cosmic string due to the family of standing waves applied to it:

$$\begin{aligned} \mathrm{d}S^2 = \mathrm{d}x^{02} &- \left[1 + 8\gamma\mu \ln\left(\frac{(x^2+y^2)^{1/2}}{r_0}\right)\right]\mathrm{d}x^2 \\ &- \left[1 + 8\gamma\mu \ln\left(\frac{(x^2+y^2)^{1/2}}{r_0}\right) + 8\gamma\mu\ln\left(\frac{y}{r_0}\right)\right]\mathrm{d}y^2 - \mathrm{d}z^2, \end{aligned} \tag{40}$$

where

$$M(x^0, z) = M\sum_{n=1}^{\infty}\frac{\pi^2 n^2}{L^2}\left[A_n \cos\left(\frac{\pi n}{L}\omega x^0\right) + B_n \sin\left(\frac{\pi n}{L}\omega x^0\right)\right]^2 \cos^2\left(\frac{\pi n}{L}z'\delta(z-z')\right) \tag{41}$$

is the time-variable mass density.

The search for a test thread's behaviour in the background (40) leads to the result that it will perform constrained oscillations but with a linear time-dependent amplitude. Therefore, the cosmic thread that moves as a massive oscillating string will perform forced oscillations with a constantly increasing amplitude. Hence, the gravitational energy flux in the Metagalaxy has to be greater than predicted earlier (Vachaspati and Vilenkin, 1985; Sakellariadou, 1990; Allen *et al.*, 1995).

A thread motion in conical space–time produced by a massive cosmic string was considered by de Vega *et al.* (1992).

Other aspects of the physics of a solitary cosmic string (cosmic starting fragmentation, cosmological density fluctuations produced by the string, rigidly moving cosmic string, tidal force from a cosmic string, cosmic string self-interaction, etc.) have been examined by Vilenkin, (1981a,b,c), Smith and Vilenkin (1987), Lonsdale and Moss (1988), Quashnock and Spergel (1989), Garfinkle (1989), Spergel *et al.* (1989), Grats (1991), Egusquiza (1995), Mahapatra (1997), Avelino *et al.* (1997), and Frolov *et al.* (1997).

5 CONCLUSIONS

So, there is a wide spectrum of problems that are related to the dynamics of cosmic strings that are of interest both in themselves and in the context of the evolution of the large Metagalaxy.

In fact, the following features were demonstrated: the cosmological density fluctuations produced by cosmic strings ($\delta\rho/\rho \approx 10^{-5}$) lead to the origin of galaxies; the origin of clusters of galaxies depends on the large mass concentration of small loops of a cosmic string; the movement of a long cosmic string through a gaseous substance creates favourable conditions for speeding up the production of stars; the evolution of a network of cosmic strings determines the number and distribution of galaxies in the Metagalaxy; the angular momentum and mass function of the galaxies originate from the cosmic strings; the microwave anisotropy relates to the cosmic string gravity; etc. (Gibbons *et al.*, 1990; Linde, 1990; Vilenkin and Shellard, 1994).

All the listed consequences of cosmic string dynamics (as argued by the various workers investigating them) have satisfactory empirical representations. Thus the search for the dynamics of cosmic strings now looks rather promising to explain the properties of the large Metagalaxy.

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