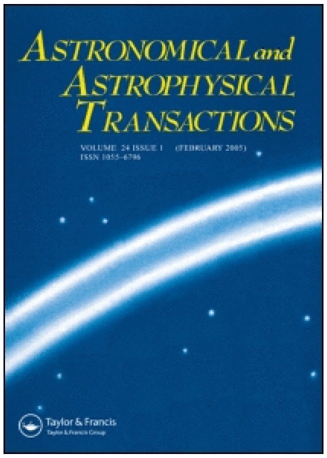

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

TURBULENT PRESSURE EVOLUTION IN COLLAPSING PROTOSTELLAR CLOUDS

A. E. Dudorov [a]; S. N. Zamozdra [a]
[a] Chelyabinsk State University, Br. Kashirinykh 129, Chelyabinsk, 454021, Russia.

To cite this Article: Dudorov, A. E. and Zamozdra, S. N. (2003) 'TURBULENT PRESSURE EVOLUTION IN COLLAPSING PROTOSTELLAR CLOUDS', Astronomical & Astrophysical Transactions, 22:1, 43 - 46
To link to this article: DOI: 10.1080/1055679021000040956
URL: http://dx.doi.org/10.1080/1055679021000040956

Astronomical and Astrophysical Transactions,
2003, Vol. 22(1), pp. 43–46

TURBULENT PRESSURE EVOLUTION IN COLLAPSING PROTOSTELLAR CLOUDS

A. E. DUDOROV* and S. N. ZAMOZDRA

Chelyabinsk State University, Br. Kashirinykh 129, Chelyabinsk, 454021, Russia

(Received 9 April 2002)

An influence of ambipolar diffusion on turbulent pressure (P_t) evolution in collapsing molecular cloud cores is investigated analytically. Exact solution for Alfven waves in free collapsing weakly ionized slab is found and then used to derive an equation for P_t. Taking ambipolar diffusion into account, we show that the pressure-density relation $P_t(\rho)$ is non-power-law in general, and can be much softer than $P_t \propto \rho^{3/2}$.

Keywords: Star formation; Collapse; Turbulence; Alfven waves

1 INTRODUCTION

Observations show that turbulent velocities inside the dense molecular cloud cores (protostellar clouds, hereafter PSC) often exceed thermal velocities, especially in the hot PSC (see Caselli and Myers, 1995). It means that turbulent pressure, P_t, influences the dynamics of PSC formation (see Myers and Lazarian, 1998). The role of P_t during subsequent stages of PSC evolution, such as gravitational collapse, is not clear.

Preliminary analysis can be made comparing dependences of the turbulent, thermal, and magnetic pressures on density $\rho(\vec{r}, t)$ in a collapsing cloud. The turbulent pressure is important if it grows similarly to the others pressures, *i.e.* approximately not slower than $P_t \propto \rho$. McKee and Zweibel (1995) found analytically that turbulent pressure created by undamped Alfven waves may grow as $P_t \propto \rho^{3/2}$ under certain conditions. Recent numerical investigations (Vazquez-Semadeni *et al.*, 1998) confirm this conclusion, but simultaneously point out the importance of dissipative effects.

In this paper we show analytically that taking ambipolar diffusion into account, one can obtain more soft pressure-density relation than $P_t \propto \rho^{3/2}$. The model is based on the exact solution (Section 2) for Alfven waves in free collapsing weakly ionized slab. In Section 3, the pressure-density relations are described. Conclusions are presented in Section 4.

2 ALFVEN WAVES IN FREE COLLAPSING SLAB

Turbulent energy density inside PSC is comparable or smaller than energy density of large-scale magnetic field, and so turbulence in PSC must be treated as MHD turbulence.

---

* Corresponding author.

ISSN 1055-6796 print; ISSN 1476-3540 online
DOI: 10.1080/1055679021000040956

Alfven wave is the dominant mode of MHD turbulence both in compressible and incompressible cases (Lithwick and Goldreich, 2001). Therefore turbulent pressure, P_t, approximately equals to the pressure of Alfven waves, which in turn is half of the wave energy density ε_w. Consequently, we can put $P_t = \varepsilon_w/2$. Equation for ε_w may be derived directly from MHD equations, but must contain undetermined quantities. To determine these quantities, we should specify a spatial behavior of Alfven waves.

Let us consider Alfven waves propagation in direction z of the large scale magnetic field that is perpendicular to the plane-parallel slab. Denote dimensionless Alfven perturbations of magnetic field by $\vec{b}$. Assume that (1) initially homogeneous slab undergoes to a free collapse; (2) the waves do not influence the collapse; (3) the waves are damped due to ion-neutral friction in the process of stationary ambipolar diffusion; (4) ion density scales as a power law of neutral density: $n_i \propto n_n^{\alpha}$. It can be shown that evolution of b_x (or b_y) under such conditions can be described by the following equation in Lagrangian form:

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}\tau^2} = q\Phi'' + K_\lambda \frac{\mathrm{d}}{\mathrm{d}\tau}(q^{1-\alpha}\Phi''), \tag{1}$$

where $\Phi = b_x/q$ is the new dependent variable, $\tau = t/t_{A0}$, and $q = \rho(\tau)/\rho(0)$ are the dimensionless time and density respectively, K_λ is the initial ratio of neutral-ion friction time t_{ni0} to Alfven crossing time t_{A0}. The primes denote $\partial/\partial z$.

General solution of the Eq. (1) can be found via separation of variables. Let, $\Phi(z,\tau) = \psi(z)\phi(\tau)$. Then

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\tau^2} + k_1(\tau)\frac{\mathrm{d}\phi}{\mathrm{d}\tau} + k_2(\tau)\phi = 0, \tag{2}$$

$$\psi'' = \Lambda\psi, \tag{3}$$

where Λ is a separation constant. Since Λ is a complex number, Eq. (3) describes decaying harmonical oscillations.

Figure 1a shows temporal behavior of Alfven waves in a given Lagrangian element. If ambipolar diffusion is not too fast, Alfven fluctuations slightly dampen during a slow initial stage of the collapse and grow at its rapid final stage. Comoving wave frequency increases with density as: $\omega_c \propto \rho^{1/2}$. The equipartition between kinetic and magnetic energies is fulfilled only on average, because ambipolar diffusion leads to a small phase shift between fluctuations of mass velocity and magnetic field. However the equipartition violation is not significant: a few percentages usually.

3 PRESSURE–DENSITY RELATIONS

Assume that magnetic and kinetic energies are in equipartition. From equations of a two component MHD (see Dudorov, 1990), using Eq. (3), one can obtain the following equation for dimensionless wave energy density ε:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} = \frac{3\varepsilon}{2}\frac{\mathrm{d}\ln q}{\mathrm{d}\tau} \mp q^{1/2}\varepsilon' + K_\lambda q^{1-\alpha}\left(\frac{\varepsilon''}{2} - K\varepsilon\right), \tag{4}$$

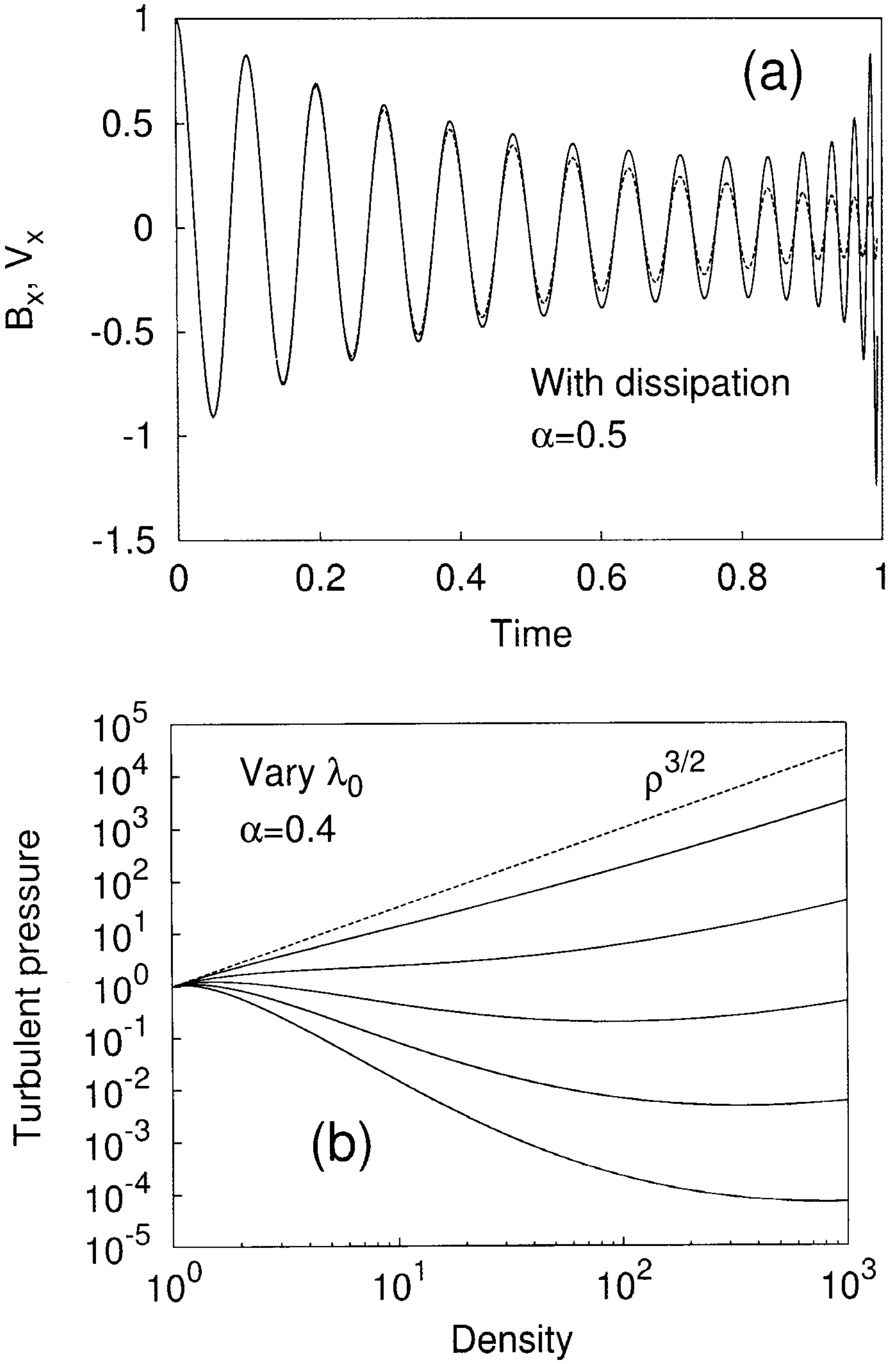

FIGURE 1 Alfven waves in the free collapsing plane-parallel slab in the case of moderate ambipolar diffusion. (a) Velocity (dashed line) and magnetic field (solid line) pulsations in a given Lagrangian element. (b) Turbulent pressure (solid lines) versus density; lower lines correspond to shorter initial waves; dashed line shows power law with exponent 3/2.

where $K = \omega_0^2 t_{A0}^2$, and ω_0 is the initial wave frequency. The right terms in Eq. (4) correspond to (1) growth of ε_w due to compression of fluctuating magnetic field, (2) redistribution of ε_w due to wave propagation, (3) diffusion and dissipation of ε_w in the process of ambipolar drift.

Equation (4) can be solved by separation of variables in several cases. Neglect, for example, the energy diffusion ($\varepsilon''/2$ term) and assume that the waves are generated on the cloud boundary and lose their energy while propagating to the cloud center. It can be readily shown that, in this case, the turbulent pressure P_t depends on the density q as:

$$P_t \propto q^{3/2} \exp\left\{ C_1 \int_1^q \frac{x^{1/2} - x^{1-\alpha}}{x^2\sqrt{1 - 1/x}}\, \mathrm{d}x \right\}, \tag{5}$$

where $C_1 = \omega_0^2 t_{ff} t_{ni0}$, and t_{ff} is the free fall time. One can see that, at $\alpha = 1/2$, the integral in (5) is always zero and consequently $P_t \propto q^{3/2}$. This corresponds to the case when dissipation is exactly compensated by energy redistribution. Figure 1b shows that in more realistic cases, when $\alpha < 1/2$, the dissipation dominates, and so P_t grows slower than $q^{3/2}$. Contribution from short waves to the turbulent pressure decreases during the collapse because of their strong damping.

For the set of parameters that corresponds to real PSC, the Alfven wave damping is strong enough, therefore only the longest waves can survive during the PSC collapse. Turbulent pressure can grow, but not significantly, and it becomes negligible in comparison with magnetic and thermal pressures.

4 CONCLUSIONS

In this paper, a rigorous modeling of turbulent pressure (P_t) evolution in collapsing clouds is performed, with a view to forecast the dynamical role of P_t during the unobserved stages of star formation. We conclude that P_t ($\propto \rho v^2$) can grow but mainly due to the density increase. Velocity fluctuations, v, usually slow down and therefore turbulent pressure in protostars is negligible in comparison with magnetic and thermal pressures. The main restriction of the presented model is the approximation of the collapse homogeneity. In future work, we shall demonstrate that, in the inhomogeneously collapsing clouds, the pressure of damped Alfven waves grows slower than $P_t \propto \rho$, so the above conclusions remain valid.

Acknowledgements

The work is supported partially by grant RFBR N 02-02-17642. Travel grant from RFBR (01-02-26139-g) is acknowledged.

References

Caselli, P. and Myers, P. C. (1995). *Astrophys. J.*, **446**, 665.
Myers, P. C. and Lazarian, A. (1998). *Astrophys. J. Let.*, **507**, L157.
McKee, C. F. and Zweibel, E. G. (1995). *Astrophys. J.*, **440**, 686.
Vazquez-Semadeni, E., Canto, J. and Lizano, S. (1998). *Astrophys. J.*, **492**, 596.
Lithwick, Y. and Goldreich, P. (2001). *Astrophys. J.*, **562**, 279.
Dudorov, A. E. (1990). *Itogi nauki i tehniki. Astronomiya*, **39**, 77.